We thank the referee for the comments. Below we address the questions and list the modifications in the revised version of the manuscript.

1. *The agreement between the symplectic forms in eq (64) is necessary in order to make the TsT deformed theory equivalent to one in auxiliary* AdS_3 *and the authors say that this agreement is something they require ("In order to make the TsT string theory (40) and the auxiliary AdS3 string theory (59) equivalent, we need to require that the symplectic forms (63) agree"), but it would be good if they could clarify why such agreement is motivated. Since the whole theory in the bulk even in the absence of the TsT deformation is not a symmetric orbifold for* $k > 1$*, the perfect matching with single-trace* $T\bar{T}$ *would be very surprising. However, this perfect matching follows from the requirement that the full quantum theory agrees with the auxiliary* AdS_3 *one (not only the quantum algebra, but also the integration constants entering the classical computation are fixed in this way). In case there is some deeper motivation to impose this, the result is indeed something to be further explored, but in case there is no such motivation, the agreement is not surprising but rooted in the construction and I think this should be stated more clearly.*

   Reply: We thank the referee for bringing this up. As we commented in footnote 2 on page 4 of the manuscript, it is indeed important to keep in mind that the undeformed theory is not a symmetric product, and therefore, we do not expect the single-trace $T\bar{T}$ deformed symmetric product theory to be the complete holographic dual to the TsT deformed string theory. Nevertheless, we still expect matching in the long string sector, which is the main focus of this paper. In fact, the non-local coordinate $\hat{X}$ is only defined in a fixed w-sector. Subsequently, the equivalence between eq.(40) and the auxiliary AdS_3 (59) is only valid in the fixed w sector, not in the full theory (either classically or quantum mechanically).

   At the moment, the requirement that the symplectic forms agree in the two theories is mainly due to technical reasons. Matching the symplectic form is necessary to set up an equivalence relation between the TsT string theory and AdS_3 string theory, the latter of which is well studied. It is interesting to understand if there is any deeper reason. In particular, there are also some zero modes showing up in the discussion of symmetries in the (double-trace) $T\bar{T}$-deformed CFTs in arXiv 2212.09768, which we commented below eq.(68). We leave this interesting question to future study though.

   Revisions: We added some comments below equation (54) and (64). We also added "in the fixed w sector" to the cited sentence on page 15 in the revised version.

2. *The full theory dual to type IIB string theory in this background is proposed to be Little string theory compactified to 2d (for large values of the deformation parameter). It would be good if the authors could comment a bit about the implications of these symmetries for LST. Is it expected that these symmetries are indeed symmetries of this theory, namely that LST observables should be constrained by the existence of these infinite dimensional symmetry algebras?*

   Reply: We thank the referee for this interesting question. We expect the symmetries will play a role in LST theories as well. Unfortunately, at the moment we can not make any specific comments yet.

   Revisions: We added some comments above section 5.4 in the revised version.

3. *It is not clear to me how to motivate the choice of fall-offs in eq (88) (are they chosen in order to get the known result in eq (93)?)*

Reply: In the revised version, equation (88) is now (94). The first line in (94) in the old is to make sure that the generators $\mathcal{J}_F$ are asymptotically conserved in the worldsheet theory. The second line in (94) is a consequence of the Jacobi identity and the first line. The fall-off $\mathcal{O}(e^{-2\hat{\Phi}})$ is motivated by the Fefferman-Graham expansion and the Brown-Henneaux boundary conditions. More details can be found in section 5 of our previous work, arXiv 2403.18396.

Revisions: We added a sentence below equation (94) to clarify this.

4. Finally, we thank the referee for pointing out some typos, which we have all corrected in the revised version.