Probing δ_{CP} phase and Charged Lepton Flavour Violation with A_4 Flavor Symmetry and Deviations to Tri-Bi-Maximal mixing via $z_2 \times z_2$ invariant perturbation in the Neutrino sector.

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Abstract

In this work, a flavour theory of a neutrino mass model based on A_4 symmetry is considered to explain the phenomenology of CP violation phase and charged lepton flavour violation. The spontaneous symmetry breaking of A_4 symmetry in this model leads to tribimaximal mixing in the neutrino sector at a leading order. A$z_2 \times z_2$ invariant perturbations in this model is introduced in the neutrino sector which leads to testable predictions of θ_{13} and **CP** violation. We consider an effective theory with an $A_4 \times z_2 \times z_2$ symmetry, which after spontaneous symmetry breaking at high scale which is much higher than the electroweak scale leads to charged lepton flavour violation processes once the heavy Majorana neutrino mass degeneracy is lifted either by renormalization group effects or by a soft breaking of the A_4 symmetry.

Contents

1 Introduction

Since the flavour mixing happens due to the mixing between mass and flavour eigenstates, neutrinos have nondegenerate mass. To put into effect this idea into a renormalisable field theory, for any symmetry used in generating neutrino massess, the degeneracy must be broken. In this work A_4 symmetry [1,2] is broken spontaneously to produce the spectrum of different charged lepton masses. Solar and atmospheric angle as conferred by accelerator and reactor data indicate that the mixing in the lepton sector is very different from quark mixings, given the large values of θ_{12} and θ_{23}. These observations were soon encrypted in the tribimaximal (TBM) mixing ansatz presented by Harrison, Perkins, and Scott [3] described by.

$$U_{PMNS} \simeq \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} = U_{TBM} \tag{1}$$

where $\text{Sin}\,\theta_{13} = 0$. In this educated guess, mixing angles have $\text{Sin}\,\theta_{12} = \frac{1}{3}, \theta_{23} = \frac{\pi}{4}$ and $\text{Sin}^2\theta_{23} = \frac{1}{2}$ whose perspective is good bearing in mind the latest neutrino oscillation global fit. In fact, data from reactors have stipulated that such sterling TBM ansatz can not be the correct description of nature, since the reactor mixing angle θ_{13} has been confirmed to be non-zero to a very high significant content [4,5].

In this work we propose a A_4 family symmetry - the symmetry group of even permutations of 4 objects or equivalently that of a tetrahedron, which is used here to obtain neutrino mixing predictions within fundamental theories of neutrino mass. We also carry out studies on lepton flavour violation decay $\mu \to e\gamma, \tau \to \mu\gamma$ and $\tau \to e\gamma$ in $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbation in both charged lepton sector and neutrino sector as discussed below, and hence one can guess the sensitivity to test the observation of sleptons and sparticles at future run of LHC. These charged lepton flavour violation rates depend on the form of Dirac neutrino yukawa couplings as fixed by most favourable predicted value of Dirac CPV phase of this work and on the details of soft SUSY breaking parameters and $\tan\beta$ [6].

2 The A_4 model

We take a type I SeeSaw model based on A_4 symmetry. Let us limit ourselves to only leptonic sector. The field consists of three left handed $SU(2)_L$ gauge doublets, three right handed charged gauge singlets, three right handed neutrino gauge singlets. In addition there exists also four Higgs doublets $\phi_i (i = 1, 2, 3)$ and ϕ_0 and three scalar singlets.

The Yukawa Lagrangian of the leptonic fields of the model $G_{SM} \times A_4 \times U(1)_X$ [7] is

$$L = L_{\text{Charged leptons Dirac}} + L_{\text{Neutrino Dirac}} + L_{\text{Neutrino Majorana}} \tag{2}$$

where G_{SM} is the standard model gauge symmetry, $G_{SM} = U(1)_Y \times SU(2)_L \times SU(3)_C$. Now,

$$\begin{aligned} L_{\text{Charged leptons Dirac}} = & -\left[h_1\left(\overline{Y_{1L}}\phi_1\right)l_{1R} + h_1\left(\overline{Y_{2L}}\phi_2\right)l_{1R} + h_1\left(\overline{Y_{3L}}\phi_3\right)l_{1R} + h_2\left(\overline{Y_{1L}}\phi_1\right)l_{2R}\right. \\ & + \omega^2\left\{h_2\left(\overline{Y_{2L}}\phi_2\right)l_{2R}\right\} + \omega\left\{h_2\left(\overline{Y_{3L}}\phi_3\right)l_{2R}\right\} + h_3\left(\overline{Y_{1L}}\phi_1\right)l_{3R} \\ & \left.+\omega\left\{h_3\left(\overline{Y_{2L}}\phi_2\right)l_{3R}\right\} + \omega^2\left\{h_3\left(\overline{Y_{3L}}\phi_3\right)l_{3R}\right\}\right] + hc \qquad (3) \end{aligned}$$

where,

$$\omega = \exp\left(\frac{2\pi i}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$L_{\text{Neutrino Dirac}} = -h_0\left(\overline{Y_{1L}}v_{1R}\right)\overline{\phi_0} - h_0\left(\overline{Y_{2L}}v_{2R}\right)\overline{\phi_0} - h_0\left(\overline{Y_{3L}}v_{2R}\right)\overline{\phi_0} + h.c \qquad (4)$$

$$\begin{aligned} L_{\text{Neutrino Majorana}} = & -\frac{1}{2}\left[\left\{Mv_{1R}^TC^{-1}v_{1R} + Mv_{2R}^TC^{-1}v_{2R} + Mv_{3R}^TC^{-1}v_{3R}\right\} + h.c + h_sF_1v_{2R}^TC^{-1}v_{3R}\right. \\ & + h_sF_1v_{3R}^TC^{-1}v_{2R} + h_sF_2v_{3R}^TC^{-1}v_{1R} \\ & \left.+h_sF_2v_{1R}^TC^{-1}v_{3R} + h_sF_3v_{1R}^TC^{-1}v_{2R} + h_sF_3v_{2R}^TC^{-1}v_{1R}\right] \qquad (5) \end{aligned}$$

where C is the charge conjugation matrix. Here,

$$M_l^0 = \begin{bmatrix} h_1v_1 & h_2v_1 & h_3v_1 \\ h_1v_2 & h_2\omega^2v_2 & h_3\omega v_2 \\ h_1v_3 & h_2\omega v_3 & h_3\omega^2v_3 \end{bmatrix} \qquad (6)$$

$$M_R = \begin{bmatrix} M & h_su_3 & h_su_2 \\ h_su_3 & M & h_su_1 \\ h_su_2 & h_su_1 & M \end{bmatrix}$$

,

$$M_D = h_0v_0I \qquad (8)$$

For,

$$v_1 = v_2 = v_3 = v \qquad (9)$$

$$u_1 = u_3 = 0 \qquad (10)$$

$$h_su_2 = M' \qquad (11)$$

$$M_l^{0d} = U_\omega M_l^0 I \qquad (12)$$

where, M_l^{0d} is the diagonal form of M_l^0 and

$$M_l^0 = \begin{bmatrix} h_1 v_1 & h_2 v_1 & h_3 v_1 \\ h_1 v_2 & h_2\omega^2 v_2 & h_3\omega v_2 \\ h_1 v_3 & h_2 \omega v_3 & h_3\omega^2 v_3 \end{bmatrix} \tag{13}$$

and

$$U_\omega^{CW} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \tag{14}$$

$$\omega = \exp\left(\frac{i2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2} \tag{15}$$

$$M_l^{0d} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \begin{bmatrix} h_1 v & h_2 v & h_3 v \\ h_1 v & h_2\omega^2 v & h_3\omega v \\ h_1 v & h_2\omega v & h_3\omega^2 v \end{bmatrix} \tag{16}$$

$$M_l^{0d} = \frac{1}{\sqrt{3}} \begin{bmatrix} 3h_1 v & 0 & 0 \\ 0 & 3h_2 v & 0 \\ 0 & 0 & 3h_3 v \end{bmatrix} = \begin{bmatrix} \sqrt{3}h_1 v & 0 & 0 \\ 0 & \sqrt{3}h_2 v & 0 \\ 0 & 0 & \sqrt{3}h_3 v \end{bmatrix} = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{bmatrix} \tag{17}$$

M_R is diagonalised by the orthogonal transformation,

$$U_\nu M_R U_v^\dagger = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} M & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} M-M' & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M+M' \end{bmatrix} \tag{18}$$

The vacuum alignment breaks A_4 in charged lepton sector coupling only with ϕ_i to Z_3 group. Also the vacuum alignment breaks A_4 in neutrino sector coupling only with ϕ_0 and χ, the residual symmetry is Z_2 group. The PMNS matrix, tribimaximal with phases is

$$\frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{bmatrix} \begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{bmatrix} \tag{19}$$

3 Perturbations in Neutrino Sector

We introduce a $Z_2 \times Z_2$ invariant perturbation in the neutrino sector, and study its influence on θ_{13} and δ_{CP}. The perturbing matrix is diagonal since it should satisfy $Z_2 \times Z_2$ symmetry. We chose the perturbation [8] to be as follows:

$$M v_R^T C^{-1} \begin{bmatrix} \frac{1}{\rho} e^- i\varphi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\rho} e^- i\varphi \end{bmatrix} v_R$$

where, $\frac{1}{\rho}e^{-i\varphi}$ characterises the soft brreaking of A_4.M is A_4 invariant soft term in the Lagrangian. The perturbing term is A_4 breaking but $Z_2 \times Z_2$ invariant soft term in the Lagrangian. The perturbed matrix is now

$$\begin{bmatrix} M + \frac{1}{\rho}e^{-}i\varphi M & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M - \frac{1}{\rho}e^{-}i\varphi M \end{bmatrix} v_R \tag{20}$$

We can diagonalise it by rotation angle x, where,

$$\tan 2x = \frac{M'}{\frac{1}{\rho}e^{-i\varphi M}} \tag{21}$$

Thus we see that the ratio, $\frac{M'}{M}$ is a physical observable in the rotation angle, tan $2x$ which helps us to diagonalise the perturbing matrix. The introduction of the perturbing terms like $\frac{1}{\rho}e^{-}i\varphi$ and the ratio, $\frac{M'}{M}$ has helped us to derive non zero θ_{13} and other neutrino oscillation parameters in terms of x, α, ρ. The input range of Majorana phases used in our calculation is from 0 to 2π. The heavy right handed Majorana neutrino used here for the computation of various LFV decay rates like $\mu \to e\gamma, \tau \to \mu\gamma, \tau \to e\gamma$ is 10^{15}GeV.

The most interesting feature of our work is that, we can extract meaningful extract of current pattern of neutrino flavour mixing in the sense that the favoured value of δ_{CP} phase from our results attached with non zero θ_{13} can induce signatures of various decay rates of charged lepton flavour violation processes like $\mu \to e\gamma, \tau \to \mu\gamma, \tau \to e\gamma$ after spontaneous symmetry breaking of our model $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbation into account. The sleptons and gauginos so constrained are shown in Fig 3. The prospect to test these sparticles at future run of LHC will favour or rule out our model.

After introducing $Z_2 \times Z_2$ perturbations in both charged leptonic sector and neutrino sector [8-10], we have,

$$U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix}$$

PMNS matrix after perturbation becomes

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix} \begin{bmatrix} \mathrm{Cos}\, x & 0 & -\mathrm{Sin}\, x \\ 0 & 1 & 0 \\ \mathrm{Sin}\, x & 0 & \mathrm{Cos}\, x \end{bmatrix} \tag{22}$$

From above it is seen that, after computation matching with the actual PMNS matix one gets,

$$\mathrm{Sin}^2\,\theta_{13} = \frac{1}{3}(1 - \mathrm{Cos}\,2\alpha\,\mathrm{Sin}\,2x) = \frac{\kappa^2}{6} + \frac{2}{3}S^2 - \frac{\kappa^2 S^2}{3} \tag{23}$$

where perturbations in neutrino sector is defined by

$$\kappa = \frac{1}{\rho} e^{-i\varphi} \frac{M}{M'} = \mathrm{Cot}\, 2x \tag{24}$$

and $S = \mathrm{Sin}\,\alpha$. Therefore, in terms of soft breaking parameters, one gets

$$\mathrm{Sin}^2\,\theta_{13} = \frac{1}{6\rho^2} e^{-}2i\varphi \frac{M^2}{M'^2} + \frac{2}{3}\,\mathrm{Sin}^2\,\alpha - \frac{1}{3\rho^2} e^{-}2i\varphi \frac{M^2}{M'^2}\,\mathrm{Sin}^2\,\alpha \tag{25}$$

Finally,

$$Cos\ \delta_{CP} = \sqrt{1 - \frac{(-\kappa)^2}{4Sin^2\alpha + \kappa^2 - 16\frac{\kappa^2 Sin^2\alpha}{3}}} = \sqrt{1 - \frac{\frac{1}{\rho^2} e^{-2i\varphi} \frac{M^2}{M'^2}^2}{4Sin^2\alpha + \frac{1}{\rho^2} e^{-2i\varphi} \frac{M^2}{M'^2}^2 - 16\frac{\frac{1}{\rho^2} e^{-2i\varphi} \frac{M^2}{M'^2}^2 Sin^2\alpha}{3}}}$$

keeping the leading powers in numerators and denominator.
The most interesting feature of this work is that we predicted form of Dirac neutrino Yukawa coupling Y_v corresponding to favoured value of $\alpha \sim 60^\circ$ which could realise signatures of rare cLFV decays like $\mu \to e\gamma, \tau \to \mu\gamma, \tau \to e\gamma$ after spontaneous symmetry breaking of our model $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbation into account. In this context we used the value of Higgs mass as measured at LHC, latest global data on the reactor mixing angle θ_{13} for neutrinos, and latest constraints on BR$(\mu \to e\gamma)$ as projected by MEG at PSI and MEG II PSI [6] planning to achieve sensitivity to BR$(\mu \to e\gamma) \sim 10^{-14}$.

The value of δ_{CP} depends on the relative predominance between the parameters $\mathrm{Sin}\,\alpha$ and κ or ρ. This dependance is plotted in Fig 1,2. From Fig 1 the results of our present analysis suggests δ_{CP} violation phase to be around 144°, corresponding to $\frac{M'}{M} = 10^{-3}$ with $\alpha \sim 60^\circ$. The analysis of No v A results shows a preferance for $\delta_{CP} \sim 0.8\pi$ suggests our present analysis of δ_{CP} phase $\sim 144^\circ$ exactly coincides with the preferred value. The separate analysis of neutrino and antineutrino channels can not provide, at present, a sensitive measurement of δ_{CP} phase. The CPV phase can therefore be measured by the long-baseline accelerator experiments T2K and NO v A, and also by Super-Kamiokande atmospheric neutrino data. Similarly, for $\frac{M'}{M} = 10^{-2}$, we obtain the best fit value of $\delta_{CP} \sim 0.8\pi$ in our present analysis corresponding to $\alpha \sim 310^\circ$. Similarly from Fig 2 , one finds that NO v A preferance of $\delta_{CP} \sim 0.8\pi$ propounds the parameter of perturbation in neutrino sector $\frac{1}{\rho}$ to be around 5×10^{-4} and 2×10^{-3}.

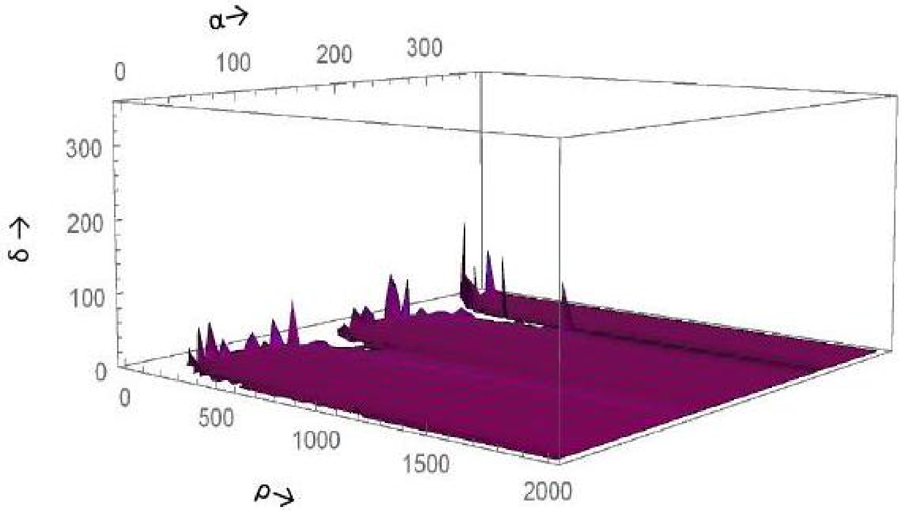

Figure 1: The values of δ_{CP} within its 3σ bounds phase for different regions in α space for $\frac{M'}{M} = 10^{-2}$

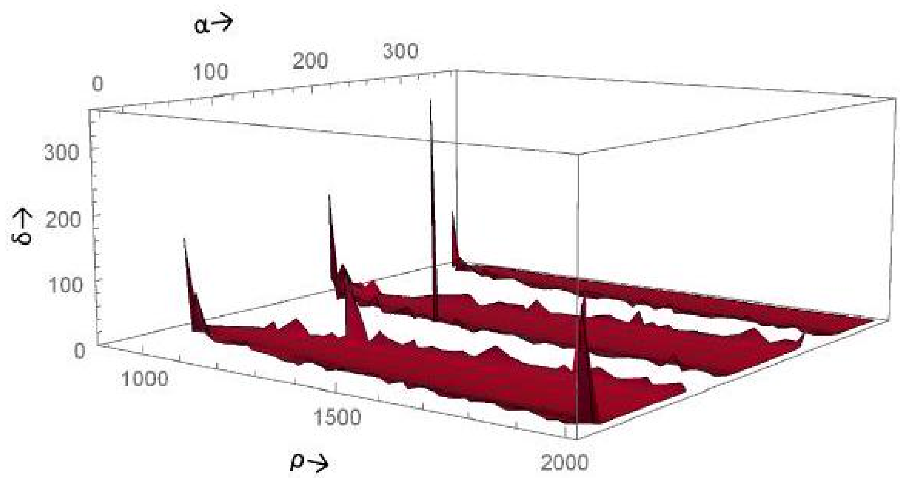

Figure 2: The values of δ_{CP} within its 3σ bounds as indicated by current neutrino oscillation global fit [11] for different regions in α space for $\frac{M'}{M} = 10^{-3}$.

4 Conclusion

We have considered leading order corrections in the form of $Z_2 \times Z_2$ invariant perturbations in neutrino sector after spontaneous breaking of A_4 symmetry. The CP violating phase δ_{CP} is around $\sim 144^{\circ}$ in this model. We show that our predicted value of $\delta_{CP} \sim 144^{0}$ corresponding to $\frac{M'}{M} = 10^{-3}$ and $\alpha = 60^{\circ}$ indicates signatures of various charged LFV channels in a class of $G_{SM} \times A_4 \times$

$U(1)_X$ model incorporating $Z_2 \times Z_2$ invariant perturbation in charged lepton and neutrino sector, which is the most interesting feature of our work. We find that very heavy m_0 region is allowed by future MEG bound of BR$(\mu \to e\gamma) \sim 10^{-14}$

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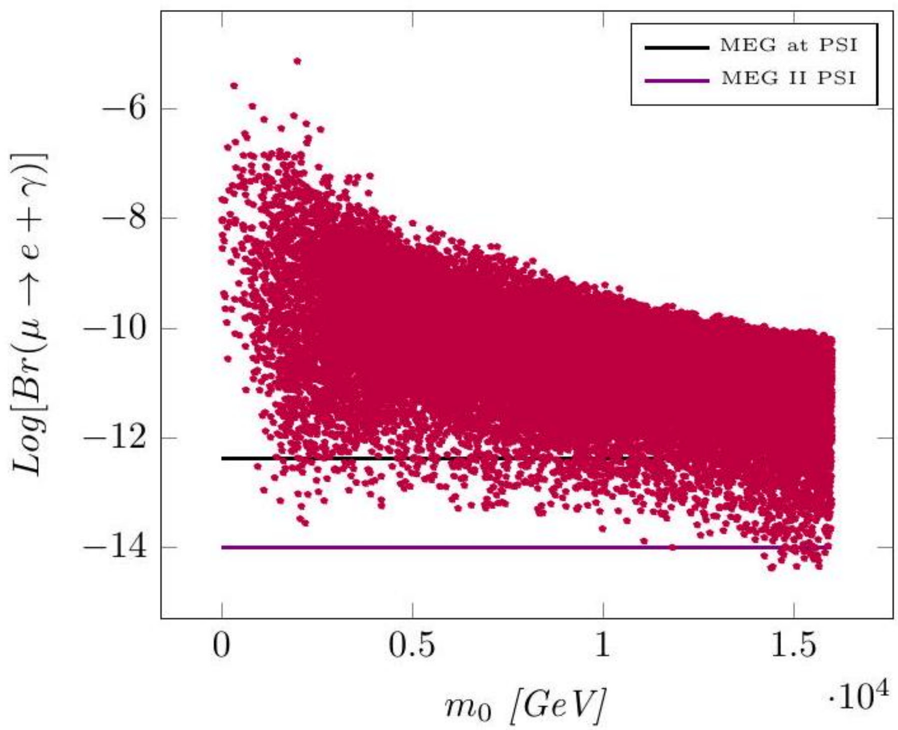

Figure 3: Different horizontal lines black and violet represent the present MEG bound at PSI and future MEG II PSI bounds for BR$(\mu \to \mathrm{e} + \gamma)$.

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