RESPONSE TO REFEREE 2

Referee 2: The manuscript *Phases and dynamics of quantum droplets in the crossover to two-dimensions"* describes a theoretical analysis of quantum droplets formed within a bosonic binary mixture of equal mass atoms, in 2D and quasi 2D configurations realized by imposing a tight box potential along the z-direction. The Authors recall earlier results from literature to write the energy functionals and the wavefunction equations, which are solved numerically and by means of a variational ansatz for the wavefunction in the form of a generalized Gaussian function...

... Generally, I find the analysis interesting with a good balance of analytical results, mainly from literature, and numerical simulations. The former are used to illustrate the latter and help developing insight. The proposed variational approach/ansatz is very pertinent for this purpose. These strengths are evident in the analysis of the ground state properties, less so in the results about dynamics.

I think the paper is worth of publication, provided the Authors address the following remarks.

Response: We thank the referee for their positive evaluation of our work and interesting comments which helped us to improve our presentation. Below, we address in detail all the issues raised and provide a list of changes at the end of the reply letter.

Referee 2: 1. At beginning of Sec. 3, the Authors give some numbers, e.g. χ_{q2D} and trace some boundaries of the regions where the agreement between 2D and quasi-2D occurs, e.g. in Fig. 1. It is unclear how this number is obtained and what is the level of agreement. Some clarification occurs later with the plots of the energy and the droplet size, but I recommend the Authors to state earlier on what they consider an agreement, e.g. in terms of the fractional difference of energy.

Response: We thank the referee for this important question. The agreement between the quasi-2D and the 2D description is initially determined by the χ_{q2D} parameter. The latter is defined as the ratio between the characteristic intraspecies interaction energy of the mixture ($\sim g_{11}n_1 + g_{22}n_2$) and the box energy ($\epsilon_0$) in the transverse dimension. The smaller that number is, the more suppressed the excitations are in the strongly confined direction rendering the two geometries almost equivalent.

In order to trace a boundary for χ_{q2D} so that the two descriptions are the same, we rely on the analytical calculations performed within the thermodynamic limit, as outlined in Ref. *PRA*

***98**, 051603 (2018)*. Specifically, we demand that the relative difference between the 2D and quasi-2D equilibrium densities becomes less than 20 %, i.e. $\left(\left|n_0^{(2D)} - n_0^{(q2D)}\right|\right)/n_0^{(2D)} \lesssim 0.2$. This corresponds to $\chi_{q2D} \lesssim 0.05$, resulting in the blue dashed line in Fig. 1, i.e. $L_z = -4a_{11}a_{22}/\delta a$. Below this line, the χ_{q2D} ratio becomes significantly smaller, and the agreement between the two geometries becomes even better as can be seen from our analysis in the subsequent sections. More details regarding this boundary and the associated energy difference have been included in the revised manuscript, see also the list of changes.

Referee 2: 2. What is the energy scale of the collective oscillation frequencies in the absence of an harmonic potential, like in the isotropic case? The interaction energy? The frequencies should be compared to this scale.

Response: We thank the referee for their insightful remark. In the absence of a harmonic trap, the absolute value of the chemical potential, $|\mu|$, sets the scale for the breathing frequency of a 2D droplet, ω_B, as shown in *PRA **103**, 053302 (2021)* . In particular, for small gN, the frequency is comparable to the chemical potential, i.e. $\hbar\omega_B/|\mu| \sim 1$, while for large gN it gradually decreases, namely $\hbar\omega_B/|\mu| < 1$. In our case, the breathing frequencies of the quasi-2D and 2D droplets resulting from the interaction quenches are approximately $\hbar\omega_B/|\mu| \sim 10^{-1}$. We have clarified this issue in our revised manuscript (see also the list of changes). However, we prefer to still report the breathing frequencies in the main text in terms of Hz in order to facilitate a direct connection with experiments as done in the rest of our work.

Referee 2: 3. Eq. (8) is dimensionally wrong, the LHS being an energy, the RHS a volume.

Response: We thank the referee for spotting this typographical error. We have now corrected this in our revised manuscript, see also the list of changes.

Referee 2: 4. Eq. (9): it is stated that *Notice that the first LHY contribution is negative for* $\delta a \gtrsim 0$": this must be explained since the first LHY term does not contain δa.

Response: We thank the referee for their comment. In the energy functional of the quasi-2D geometry described by Eq. 9 of the main text, it becomes evident that the $E^{(1)}_{LHY}$ contribution becomes negative when the density n satisfies $n < (4\pi\sqrt{a_{11}a_{22}e}L_z^2)^{-1}$. This is due to the logarithmic dependence of $E^{(1)}_{LHY}$. Small densities occur at $\delta a > 0$, where the droplet size increases to relatively large values as suggested by Figs. 3(b) and 4 of the main text. To clarify this point, we have incorporated a relevant discussion in the revised manuscript (see also the list of changes).

Referee 2: 5. Eq. (14) contains $\Gamma(...)$ that, presumably, is the Gamma function: I think the Gamma function should not be taken for granted and must be named explicitly

Response: We agree with the referee. Accordingly, we have explicitly introduced the notion of the Gamma function in the revised manuscript, see the list of changes.

Referee 2: 6. Fig. (1): for clarity, I recommend stating that these are contour plots of χ_{q2D} and $|\chi_{2D} - \chi_{q2D}|$, otherwise they might be confused for ordinary plots of L_z vs δa at different values of χ_{q2D}.

Response: We agree with the referee's comment and have now explicitly stated in the caption of Figure 1 that these refer to contour plots of the appropriate quantity (see also the list of changes).

Referee 2: 7. Fig. (2a): in the inset the three lines touch at $L_z = 0.1\mu m$ implying that the three energy per particle are equal, and so also the lines in the main plot should touch: why not?

Response: We thank the referee for their remark. The three energy contributions, E_{MF}, $E^{(1)}_{LHY}$ and $E^{(2)}_{LHY}$, depicted in the inset of Fig. 2(a) of the main text are not the same at $L_z = 0.1\ \mu m$. This can be seen by restricting the inset of Fig. 2(a) in the manuscript to smaller values of L_z. To avoid any possible confusion we have updated the inset of Fig. 2(a) in the manuscript by restricting the range of L_z (see also the list of changes).

To further clarify this point we also provide a magnification of the inset of Fig. 2(a) of the manuscript in Fig. 1 of the current reply letter. As it can be seen, the individual energy contribu-

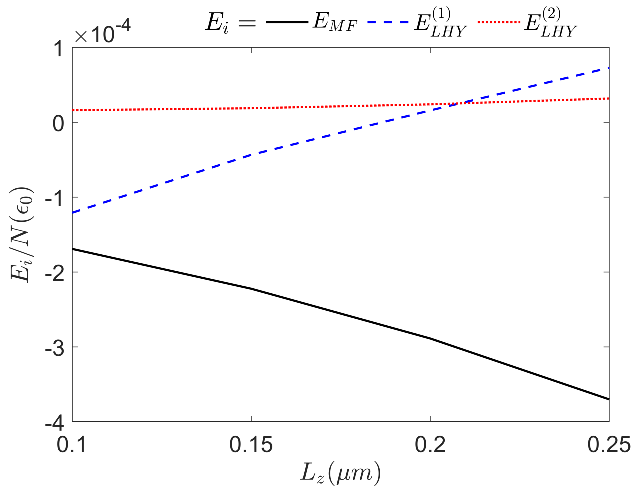

FIG. 1. Magnification of the inset of Fig. 2(a) in the manuscript in terms of L_z. The individual energy terms per particle, i.e. E_i/N at $\delta a = -1.94$, are depicted. It is evident that the individual contributions are not the same at $L_z = 0.1\mu m$.

tions are different at $L_z = 0.1\mu m$.

Referee 2: 8. Page 9: it is stated that *one can see from Fig. 2 ... that for $\delta a < 0$ the mean-field (logarithmic LHY) interaction energy is negative (positive), while for $\delta a > 0$ this is reversed.* I suggest to explicitly state that it is seen in the insets, the main plots having the magnitudes of the various terms.

Response: We agree with the referee that pinpointing to the corresponding insets of Fig. 2 supports our statement more clearly. As such, we have now explicitly stated that the aforementioned distinction can be seen in the insets (see also the list of changes).

Referee 2: 9. Figure (5): there is some confusion about colors, some lines are orange color whereas they should be green, I think.

Response: We thank the referee for noticing this color mismatch. In our revised manuscript, we have updated Fig. 5 and all colors appear to be consistent, see also the corresponding list of changes.