## List of changes

All the changes listed below are highlighted in red in the manuscript.

**Sec. 2**

[1] 1st paragraph in Sec. 2.2: We rephrased a sentence to emphasize that the washout process ensures independence from the initial state.

[2] 1st paragraph in Sec. 2.2: We added sentences to further clarify the concept of virtual time in the QRP.

[3] 1st paragraph in Sec. 2.2: We revised a previously confusing sentence (line 162 in the prior manuscript).

**Sec. 3**

[4] 1st paragraph in Sec. 3.1: We added a description explaining that our input protocol preserves to the intrinsic symmetry of the system.

**Sec. 4**

[5] 3rd paragraph in Sec. 4: We included a sentence to emphasize the potential applicability of information processing capacity and non-local observables as promising avenues for future research.

**Others**

[6] We resolved the inconsistency in Refs. [5, 17].

[7] We added a new reference [71].

[8] For greater clarity, we revised the wording in the manuscript as necessary.

## Reply to the report

***Report***

*This paper introduces a new research question on the connection between properties of quantum systems and their information processing capabilities. To answer this question, the authors propose a new paradigm called quantum reservoir probing, which uses the framework of quantum reservoir computing to diagnose the property of quantum systems. The proof of concept is shown using numerical simulations for quantum Ising chain with transverse and longitudinal magnetic fields. The manuscript is well-written, but some concerns prevent me from recommending the manuscript for publication in the journal.*

We would like to express our gratitude to the reviewer for his/her valuable time reading manuscript carefully and thoughtful reviews. He/She made nine comments in his/her report. We have answered them below and revised the manuscript accordingly.

*- In line 133, the authors mentioned that "Successful (unsuccessful) estimation indicates that the input information does (does not) influence the read-out operator". However, does unsuccessful estimation always mean input information does not influence operator? The error could arise in the weight optimization. Does the error have an effect on the judge of the information propagation? Also, does a finite number of measurement shots affects the judge?*

In complicated machine learning models such as deep neural networks, optimizing the numerous weight parameters is a challenging task and the errors in weight optimization are not negligible. In contrast, our QRP framework requires the optimization of only two parameters ($w_O$ and $w_c$) using the linear regression scheme, which guarantees a theoretically optimal solution [Eq. (3)] for the provided dataset. Consequently, unsuccessful estimation indicates that, even with such optimal weights, the input information cannot be faithfully reproduced. We attribute this failure to the insufficient influence of the input information. Indeed, as illustrated in Fig. 5, while $\langle \sigma_i^x \sigma_{i+1}^x \rangle$ succeeds information estimation, $\langle \sigma_i^z \sigma_{i+1}^z \rangle$ does not, consistent with the quasiparticle picture of the model. This observation strongly supports the validity of our interpretation.

As the reviewer pointed out, estimation performance inevitably degrades in the presence of statistical fluctuations, as previously reported in studies on QRC (e.g., Ref. [18]). Nevertheless, those studies also demonstrate that, with a sufficient number of measurements, the QRC can still perform information processing, and we expect the same to hold for the QRP. Specifically, when the number of measurements is too small, estimation is expected to universally fail with all operators, making it impossible to assess information propagation. However, as the number of measurements increases, performance should improve and converge to the results we calculated in the limit of infinite measurements. Therefore, as long as a sufficiently large number of shots is taken, information propagation can be reliably judged, and the operation of QRP is not hindered by a finite number of measurements, similar to the QRC.

*- Why do the authors consider 2-local observables at most? I agree that the increase of the number of operators does not necessarily mean the improvement of performance for quantum reservoir computing. On the other hand, I assume global observables could provide some valuable information for certain tasks. For instance, although it is a task with static quantum data, the phase recognition task could require information of global information. For quantum dynamical systems, is there no situation where the global information is needed? If it is not true, I think it is important to see if the QRP framework can efficiently capture the global information as well. It would be great if the authors could elaborate on it.*

We thank the reviewer for the insightful suggestion. As the reviewer pointed out, the QRP framework is not restricted to local observables. We certainly agree that there may be applications of the QRP where employing more nonlocal operators proves advantageous. In our current application, however, we focus on local observables because they are more appropriate than highly nonlocal ones for capturing information propagation. Nonetheless, exploring the possibility of using nonlocal observables to investigate global physics within the QRP framework is indeed an exciting avenue for future research, which has been highlighted in the revised manuscript. (See the summary of changes [5].)

*- Due to the definition of QRC, the result does not depend on the initial state of the quantum reservoir system. On the other hand, the result seems dependent on the initial state of the ancilla qubits used for input injection. How can we interpret this? Could it be possible to eliminate the effect of the dependence on the input-initial state?*

As detailed in Eq. (1), the ancilla qubit is reinitialized at each input step and does not participate in the system's dynamics because it remains decoupled from the reservoir under the Hamiltonian in Eq. (5). This means that although the ancilla qubit is present, the evolution of the reservoir, and hence the QRP results, are independent of the ancilla's initial state. In our setup, the ancilla qubit is involved only in the computation of the tripartite mutual information. Consequently, the results of the QRP do not depend on the ancilla qubit's initial state.

*- Why do the authors start with the ground state, despite the fact that the QRC is not influenced by the initial state. Is the washout time not enough to guarantee the independence on the initial state?*

We apologize for any confusion. Indeed, as the reviewer pointed out, starting the simulation from a random state yields identical results, and it is not necessary to initiate the simulation from the ground state. This clarification has been incorporated into the revised manuscript; please refer to the summary of changes [1].

To substantiate this, we analyzed the distance between two quantum states: one evolving from a random initial state and the other from the ground state, both subjected to the same input sequence $\{s_k\}$, as depicted in the figure below. The Frobenius norm, defined as $||\Delta\rho|| = \sqrt{\mathrm{Tr}[\Delta\rho(\Delta\rho)^\dagger]}$ with $\Delta\rho \equiv \rho_1 - \rho_2$, was employed to quantify the distance between the two states, $\rho_1$ and $\rho_2$. Our results demonstrate that the distance rapidly decreases, and by the washout step $l^\mathrm{w} = 1000$ used in our simulations, the distance converges to zero. As this is a standard approach to assess the dependence on initial states (e.g., see Ref. [21]), we conclude that the chosen washout step is sufficiently robust.

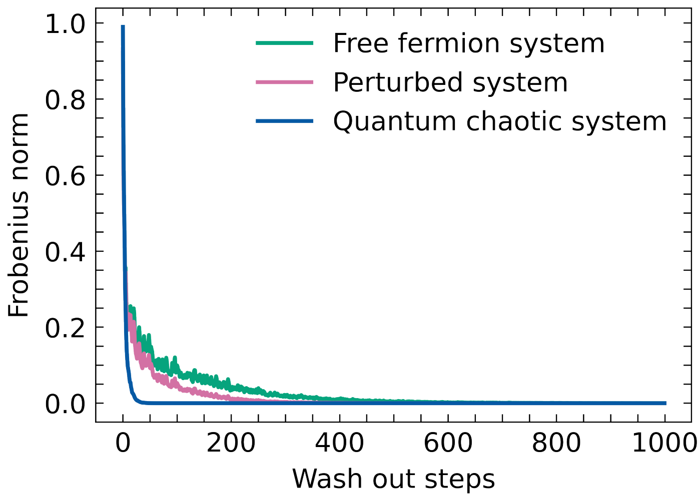


Fig. 1: Wash out process. The Frobenius norm is computed between quantum states evolving from a random initial state and the ground state under the same input sequence. The colors represent different systems.

*- As shown in the literature of reservoir computing, the range of input have an impact on the information capability. E.g., see a work by Kubota et al on the information processing capacity: https://journals.aps.org/prresearch/abstract/10.1103/PhysRevResearch.3.043135. Does it affect the performance of the QRP as well? If these factors including ones mentioned above matter, is it fair to say the QRP always tell the property of quantum systems? Could it be possible to provide certain theoretical guarantee?*

We thank the reviewer for drawing our attention to the impact of input range. In QRP scheme, the input range directly determines the spin-angle variation induced by the input operation. Specifically, we design the input such that the $x$-component of the input spin varies symmetrically from $-1$ to $1$, while the $z$-component varies from $0$ to $1$. This configuration preserves the system's intrinsic symmetry with respect to the $x$ axis in spin space ($\sigma_i^x \leftrightarrow -\sigma_i^x$ when $h_x = 0$). If the input range is biased, such that the spin $x$-component varies from $0$ to $1$, the inherent symmetry of the quantum system is disrupted, which may alter the reservoir performance. Such changes, however, should be regarded as artifacts of the input design rather than as intrinsic properties of the quantum system. In contrast, when symmetry is already broken in a system $(h_x \neq 0)$, employing a biased input protocol does not further distort its fundamental characteristics.

In this sense, it is critical that the input protocol and input range align with the underlying symmetry of the quantum system. Our input definition is optimal in achieving maximal spin variation ($-1$ to $1$) while preserving symmetry along the $x$ axis in spin space. These descriptions have been incorporated in the revised manuscript. (See the summary of changes [4].)

*- The short-term memory is used to see the information propagation in this work. Actually, there is a metric called the information processing capacity (IPC) in the context of reservoir computing that is used to see the profile of the reservoir's ability to process the time-series data. Can it be used to see the nonlinear processing of the information through the quantum channels?*

We appreciate the reviewer's insightful suggestion. Indeed, the IPC is a valuable metric for quantifying a reservoir's ability to process time-series data, including its nonlinear aspects. While our current study primarily focuses on information propagation rather than on nonlinear effects, we agree that IPC could be a potent tool for future exploration of the nonlinear quantum processes. We have added a brief comment on this promising direction in the revised manuscript. (See the summary of changes [5]).

*- The authors mentioned that an advantage of the framework is efficient operation compared to OTOC and TMI. I feel it boils down to the 2-localness of the observables considered for QRP, in contrast to these methods requiring the global information. In case the target property of the system is global, does the statement that the QRP is efficient still hold? In addition, does the result for QRP mean that observing local operator is enough to perform the task in the manuscript? Also, do the additional washout time or longer training and testing period lead to less efficiency of the proposal compared to other method?*

We appreciate the reviewer acknowledging the wide applicability of QRP. Among its applications, we discussed the advantages of QRP from the perspective of investigating information propagation, in comparison to other methods designed for the same purpose (e.g., OTOC and TMI). When other properties, including global ones, are of interest, it would be necessary to adapt both the QRP protocol itself and the corresponding indicators used for comparison. Accordingly, any claims regarding efficiency would need to be re-evaluated in that new context. While it is not feasible at this stage to discuss the advantages and limitations across all possible applications, we believe that the ability to accommodate such a wide range of scenarios is a significant advantage of the QRP.

Returning to the current scope of information propagation, we acknowledge that the additional washout, training, and testing periods increase the overall runtime of the protocol. Nevertheless, the fundamental experimental simplicity of the QRP approach remains intact. Unlike methods that require intricate procedures such as inverse time evolution (OTOC) or complete state tomography (TMI), the QRP only relies on measuring the expectation values of local operators. Therefore, even when these extra steps are added, the QRP preserves its core experimental advantages for studying information propagation.

*- As for the applicability, how likely do we have the information of the input in practical situations? To perform QRP, we always need to have a*

*supervised-learning-like setting. Thus, I wonder if it is likely to happen in practical settings. Moreover, in QRP, the dynamics should be expressed as Eq.(1). Can we extend this assumption to the case of unitary evolution, which could also be a common target property in quantum physics.*

In the QRC, external signals, such as financial or climatic data, are introduced for computational purposes. However, in certain practical scenarios, these signals may not be available beforehand, thus preventing a straightforward supervised-learning approach. In contrast, the objective of QRP is to investigate the quantum system itself, rather than to process externally defined signals. Hence, the choice of input is entirely under our control. In practice, we can always generate a random input sequence in advance and feed it into the quantum reservoir; this guarantees the availability of labeled data for supervised learning and ensures the applicability of the QRP approach.

Regarding the dynamics in Eq. (1), our assumption follows a standard framework in QRC research. Nonetheless, we acknowledge the interest in extending such a framework to purely unitary evolutions. We explore this possibility in a separate work (arXiv:2402.07097), as the technical and conceptual considerations differ from those presented here.

*- Why do the authors consider the STM task as a function of the virtual time \tau, not k. In the original QRC, the STM objective function is a function of k. Thus, if we follow the concept of QRC straightforwardly, I think it makes sense to regard the objective function in the same way. It would be great if the authors could elaborate on the reason why the virtual time is introduced and it is the main parameter of the STM task. The way the virtual time is used is also different from the QRC perspective; QRC uses the virtual node to improve the expressivity. Therefore, I would recommend to note the difference in the manuscript.*

We apologize for any confusion. In our approach, we indeed consider the STM task as a function of $d, \tau,$ and $k$. In particular, to capture the dynamics of information propagation, we focus on the operator $O$ at a time

$\tau$ after an input is provided. This can be evaluated by the STM task with delay $d$ using $\langle O(kt_{\text{in}} + \tau)\rangle$. In this formulation, one seeks weights that satisfy $s_{k-d} = w_o \langle O(kt_{\text{in}} + \tau)\rangle + w_c$ [Eq. (2)], which explicitly depends on $k, d,$ and $\tau$. Subsequently, the performance is evaluated by statistically treating different instances over $k$, so $R^2$ becomes a function of $\tau$ and $d$. This setup precisely follows the STM task defined in the original QRC.

The key distinction, as the referee pointed out, lies in how we employ the virtual time. In the QRC, the virtual time τ is used to increase the number of nodes; for example, $y = w_{o,1}\langle O(kt_{\text{in}} + \tau_1)\rangle + w_{o,2}\langle O(kt_{\text{in}} + \tau_2)\rangle + \cdots + w_c$. Although this approach improves expressivity, it simultaneously reduces time resolution, since $R^2$ then depends only on $d$, not on $\tau$. In contrast, the QRP leverages virtual time to provide temporal resolution in its computational capability. Evaluating performance at various values of $\tau$ directly links the computational output to the underlying physics at each specific elapsed time, using $R_d^2(\tau)$ as a probe.

We have clarified this difference in the revised manuscript. (See the summary of changes [2].)

***Requested changes***

1. *There is an inconsistency in reference; e.g., Initials are used for the first and family names in [5], while others use the initials for first names only.*

We would like to thank the reviewer for pointing out. We have resolved the inconsistency; please see the summary of changes [6].

2. *Some sentences are confusing; e.g., what does "general" mean in "Hereafter, we denote ⟨O(ktin +τ)⟩ for general k by ⟨O(τ)" in line 162?*

We would like to thank the reviewer for this comment. We have revised the sentence to enhance clarity. (See the summary of changes [3].)

***Recommendation:*** *Ask for minor revision*

We extend our sincere gratitude to the reviewer for the positive assessment of our work. His/Her numerous insightful and valuable suggestions have significantly improved the manuscript. We have diligently revised the manuscript to address all requests for further clarification. We are confident that the revised manuscript will meet with the reviewer's approval.