Reply to the report

Report

I appreciate the authors' sincere responses. I am satisfied with the current manuscript, but I have a few additional comments before it can be considered for publication:

We are very grateful to the reviewer for reading our manuscript so carefully. His/Her thoughtful and constructive comments have greatly improved our work. The reviewer also offered additional remarks in their report, which we address below. In compliance with the editorial policy of SciPost Physics, we cannot revise our manuscript before receiving the official editorial recommendation; however, we stand ready to revise it as soon as that recommendation is issued.

I asked in the previous round that "On the other hand, the result seems dependent on the initial state of the ancilla qubits used for input injection.". I might be unclear and let me clarify again. My point was that the initial state with which the input-dependent state is created could be dependent on the performance. In the numerical experiments, the authors used the computational basis $|00\rangle$ *and* $|11\rangle$ *to construct* $|\psi_{in}(s^k)\rangle$*. That is, every time step, the input is injected on the* $|00\rangle = \left(\frac{I+Z}{2}\right)^{\otimes 2}$ *or* $|11\rangle = \left(\frac{I-Z}{2}\right)^{\otimes 2}$*, meaning the way that the input comes into the system is always limited, i.e., through tensor products of (combinations of)* I *and* Z*. Here* I *is identity operator and* Z *is Pauli* Z*. I know this is the conventional way of input-injection in QRC, but considering the fact that the types of input affect the performance, I assume this also have an effect on the precision for probing the property of quantum systems. It would be nice if the authors clarify this.*

We thank the reviewer for providing further clarification. We acknowledge that the choice of input scheme can affect the performance, as it determines the type of excitation generated in the system. Specifically, different initial states (or equivalently, different quantum quench protocols) excite different dynamics, which can lead to varying levels of sensitivity to the property being probed. In our manuscript, we employed a conventional input scheme in the QRC and successfully revealed notable information propagation physics, reflecting either the free-fermionic or chaotic nature of the system. However, we recognize that using alternative input schemes could, in principle, obscure these distinguishing dynamical signatures, simply because they would yield different resultant dynamics.

It is important to emphasize that this does not compromise the effectiveness of the QRP. In general, any experiment in physics requires carefully chosen methods and conditions to reveal specific properties. For example, applying a magnetic field is not helpful in distinguishing an insulator from a metal, whereas electric current is. In the same way, we need to choose an appropriate input scheme to reveal certain phenomena in the QRP. Importantly, our method is not restricted to a single choice of input or output settings; rather, it accommodates a variety of inputs (and other hyperparameters) tailored to different physical targets. This design flexibility broadens the applicability of the QRP to a wide range of quantum phenomena.

We will clarify these points in the second paragraph of the Discussion and conclusion section of our revised manuscript once the editorial recommendations have been prepared.

Related to the comment above, I would recommend the authors to examine the effect of hyperparameters such as types of input, evolution time, virtual time, initial state of the ancilla state and so on, because this would be an important point to see if the QRP can robustly produce the property of quantum systems regardless of practitioners' choice of hyperparameters. Then, it would be nice if you summarize this, e.g., in certain section or a paragraph in Discussion and conclusion. If these really affect the performance, I need to rethink the novelty of this work. (I mean, I would like to make sure if the method can perform the task well with only access to practically-reasonable prior knowledge on the system.)

We thank the reviewer for raising these important points. As discussed in our previous response, the QRP may fail to detect specific properties of the system under certain conditions, and it is indeed necessary to explore suitable hyperparameter settings. Nonetheless, we would like to emphasize that this kind of tuning is a common requirement for any probing scheme.

For instance, when investigating a new material without prior knowledge, researchers routinely vary experimental conditions (e.g., the direction, amplitude, and frequency of applied currents, or the orientation of external magnetic fields) to uncover the material's properties. The same principle applies to QRP: systematically scanning through different configurations is a standard and necessary step for extracting information about the system. Far from undermining the novelty of the QRP framework, this design flexibility is in fact one of its key strengths, as it allows us to tailor the framework to the specific system under investigation.

We will clarify and further elaborate on this point in the second paragraph of the Discussion and conclusion section once the editorial recommendations have been prepared.

Minor comment

I would recommend to double check typos: I found a typo, e.g., missing punctuation in a sentence "... estimate the information stored in $O(\tau)$ Employing..." in line 202, page 7.

We sincerely thank the reviewer for carefully reading our manuscript and identifying typos. We will thoroughly proofread the manuscript and correct any remaining typos.