To The SciPost Team.

Thank you for your message of February 04, regarding our manuscript submitted to SciPost Physics, scipost_202408_00023v1, entitled "Coherent deflection of atomic samples and positional mesoscopic superpositions", authored by L. F. Alves da Silva, L. M. R. Rocha, and M. H. Y. Moussa. We also thank the referee (Report #2) for the criticisms raised, which helped us to improve the manuscript. Below we present our answers to the questions raised in Report #2 and the corresponding changes made to the manuscript, starting with the referee's specific questions and them addressing the issues regarding the presentation of the manuscript.

Specific Questions:

1) Regarding the first question of referee #2: "how the author characterize the atomic decay factor?", we first mention that we are considering typical values for the atomic spontaneous emission in both optical and microwave regimes. In the microwave regime, for example, circular Rydberg states are considered, due to their long lifetimes, and elaborate techniques are used for their preparation. We expect the reader to access this really important issue in the references provided in the manuscript for the experimental implementations of the optical and microwave regimes. On the other hand, we note that for the treatment of the atomic decay, we resort to the formalism of the master equation, as derived in Ref. [14].

2) In his second question, the referee #2 requires "the authors to address better (and separately) the (main steps of the) derivation of the MF equations". On this regard, after Eq. (3), we have introduced a sentence, where we note that "Basically, the mean-field approximation is used, as an method to approach the master equation of our many-body system, composed of the atomic sample and the field. This method consists in tracing out all the degrees of freedom of $N-1$ atoms, leaving us with the reduced master equation for a single representative atom interacting with the cavity field as described by Hamiltonians (2a) and (2b)."

3) The referee also asks us whether in the derivation of the MF equations "the ω_k of the bath enter in the values of the averages on that?". On this regard, we note that the

multimodal frequencies of the bath enter the definition of the atomic decay rate γ, as can be verified through the techniques of derivation of the master equation.

4) The referee observe that "the expression for ϵ after eq. 12 can be simplified in N", and this observation is perfectly correct. However, we decided to keep the form $\epsilon = 4\sqrt{N}g/N\gamma$ to make clear the competition between the effective oscillation frequency $\sqrt{N}g$ and the effective damping factor $N\gamma/4$. To clarify this competition in the definition of the parameter ϵ, in the new version of the manuscript, after Eq. (12), we have introduced the following sentence: "The definition of these regimes becomes clear by noting that the parameter ϵ follows from the competition between the effective oscillation frequency $\sqrt{N}g$ and the effective damping factor $N\gamma/4$, as shown in Eq. (12)."

5) Regarding the referee question "the authors could recall briefly the basis general features of the 3 discussed regimes ?", we believe that the sentence introduced after Eq. (12) —as anticipated in the answer to question (4) raised by the referee— makes it clear that these regimes are equivalent to those of a damped pendulum.

6) In his sixth question, the referee observes: "in the same three regimes, can the author identify optimal numbers for the superradiance-superabsorption cycles ? How do these reflect on the allowed time-scales for the experiment ?" On this regard we note that in the new version of the manuscript we have provided an extensive discussion about the implementation of our suggested experiment, in all three regimes, considering the optical or microwave cavity QED experiment. This important question was also raised by referre #1, asking us to provide typical values for the parameters used in the experimental implementation of the protocol presented in the manuscript. To better address this question from both referees, in the new version of the manuscript we decided to modify the presentation of Figs. 2 to 4, noting that we have inserted a new figure (Fig. 1), as requested by referee #1, with a schematic illustration of the experimental realization. Essentially, instead of defining the overdamped, damped and underdamped regimes by fixing the number of atoms and varying their relaxation rates, as we did previously, in the new version we did the opposite: we fixed the relaxation rate of the atoms and varied the number of atoms in the sample. This procedure seems more suitable for the purposes of defining the parameters to be used in a cavity QED experiment; in fact, the relaxation rate of the atoms is defined a priori,

by the choice of the atomic states, while the number of atoms can be modified through trapping techniques. Based on this fact, in the new version of the manuscript we modified the paragraphs, below Eq. (28), where we present figures 2, 3 and 4. We have now written:

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For the implementation of the overdamped, damped, and underdamped regimes, we consider $\omega = 10^5 g$ and $\gamma = 5 \times 10^{-3} g$, in order to contemplate both microwave [24] and optical [25] cavity QED regimes, simultaneously. In fact, depending on the value of the Rabi frequency, $g \approx 10^5 Hz$ or $\approx 10^9 Hz$, we are in the microwave or optical regime, respectively. Starting with the overdamped regime, considering a sample with $N = 10^8$ atoms, such that $\epsilon \approx 0.1$, in Fig. 2(a, b and c) we set $\alpha_0 = 0.1$, $\theta_0 = 2/N$, and $\phi_0 = \pi/2$, to draw the curves for the numerical and analytical solutions for $\theta(t)$, $|\alpha(t)|$, $\mathcal{I}_a$, and $\mathcal{I}_f$, respectively. Whereas the circles and squares represent the numerical solutions, the full and dotted lines represent the analytical ones. As we observe, the analytical solutions match very well for the overdamped regime where we basically observe, in Fig. 2(c), an atomic superradiant pulse with intensity of about $10^{18} g^2$ and delay time $\tau_D \approx 2.65 \times 10^{-4} g^{-1}$, in perfect agreement with the analytical value coming from Eq. (15). The field superabsorption, presenting negative intensity [14], is inhibited by the small coherence parameter ϵ. In Fig. 3(a, b, and c), we plot the same functions as in Fig. 2 for the damped regime, with $N = 10^6$ such that $\epsilon \approx 1$, and all other parameters equal to those in Fig. 2. As anticipated above, our overdamped solutions apply with much less accuracy to the damped regime. We now observe a superradiant-superabsorption cycle, although the superabsorption occurs slightly less intensely than the superradiance ($10^{14} g^2$). Moreover, the delay time for superabsorption is slightly greater than that for the superradiance, the latter being around $\tau_D \approx 1.55 \times 10^{-3} g^{-1}$.

In Fig. 4(a, b, and c), we again plot the same functions as in Fig. 2, considering the underdamped regime for $N = 10^4$ such that $\epsilon \approx 10$. We again consider all other parameters equal to those in Fig. 2, except for $\theta_0 = \pi/2$ due to the linearization procedure. Now, we observe around 4 superradiant-superabsorption cycles, with intensities starting at around $10^{10} g^2$, as the strong coherence parameter leads to a slow damping of the initial atomic excitation. The number of superradiance-superabsorption cycles can be controlled by Stark shifting the sample out of resonance with the field. From Ref. [14] it follows that the time interval for a superradiant-superabsorption cycle is around two times the characteristic emission time $2/\sqrt{N}g$, which is in excellent agreement with Fig. $4(c)$.

From Fig. $4(c)$ it follows that the time required for the 4 superradiance-superabsorption cycles is around $10/\sqrt{N}g$, resulting in the values $10^{-6}s$ and $10^{-10}s$, for the microwave and optical cavity QED regimes, respectively. In the microwave regime the decay time of a high-finesse cavity is around a thousand times greater than $10^{-6}s$, while in the optical regime it is around 10 times greater than $10^{-10}s$, making it possible to carry out the experiment in both regimes, with advantage for microwave cavities.

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7) The referee #2 has also observed that "as far as I understand, the discussion is performed in the limit $T = 0K$. Can the authors include temperature effects (also from optical heating, perhaps) ? I have in mind also possible decoherence effects related to T. Other decoherence effects are expected ?".

On this regard, we note that in the first version of the manuscript, in the second paragraph below Eq. (20), we have written that "our experiment does not require a high finesse cavity as far as the necessary superradiant-superabsorption cycle occurs in a short time interval of the order of $\tau_D + \tau \ll 1/\gamma$." In the new version of the manuscript, we return to this important point by writing, in the third paragraph below Eq. (28), that "From Fig. $4(c)$ it follows that the time required for 4 superradiance-superabsorption cycles is around $10/\sqrt{N}g$, resulting in the values $10^{-6}s$ and $10^{-10}s$, for the microwave and optical cavity QED regimes, respectively. In the microwave regime the decay time of a high finesse cavity is around a thousand times greater than $10^{-6}s$, while in the optical regime it is around 10 times greater than $10^{-10}s$, making it possible to carry out the experiment in both regimes, with advantage for microwave cavity."

Therefore, we do not see the need to consider, at least for this first work on the subject, a more detailed analysis of the effects of the temperature of the environment on the decoherence of our mesoscopic positional superposition. It is worth nothing that, in Ref. [14], a master equation was derived from which the Hamiltonian $H(t)$ of Eq. (2) comprised the von Neumann term. But since we are dealing with a short-time problem involving superradiance and superabsorption, we simply chose to disregard the terms of the irreversible evolution of the system associated with dissipative-diffusive effects. Taking these terms into account would make the analysis of the momentum transfer between the atomic sample and the field much more complex, without however adding significant gains.

Finally, we observe that in addition to dissipation and diffusion, the latter coming from finite temperatures, there is in fact another decoherence effect in our proposal resulting from the dispersion in the atomic positions after the trap is turned off. This dispersion affects the approximation $kx \ll 1$, as time progresses, and consequently the coherence of the sample deflection.

In the last but one paragraph of the new version of the manuscript, we have inserted the following comment about decoherence sources:

"It is worth nothing that, in Ref. [14], a master equation was derived from which the Hamiltonian $H(t)$ of Eq. (2) comprised the von Neumann term. Since we are dealing with a short-time problem involving superradiance and superabsorption, we simply chose to disregard the terms of the irreversible evolution of the system associated with dissipative-diffusive effects. Taking these terms into account would make the analysis of the momentum transfer between the atomic sample and the field much more complex, without however adding significant gains. In addition to dissipation and diffusion (for finite temperatures), there is another decoherence effect in our proposal resulting from the dispersion in the atomic positions after the trap is turned off. This dispersion affects the approximation $kx \ll 1$, as time progresses, and consequently the coherence of the sample deflection."

8) As a final specific question, referee #2 noted that: "since, as correctly admitted by the authors, the present proposal poses a challenge to the experimental physics of radiation-matter interaction, can the authors themselves comment further on the required experimental strategies and developments?" In this regard, we note that Fig. 1, introduced in the new version of the manuscript as a suggestion of referee #1, puts into perspective the challenges necessary to carry out the experiment we suggested. Basically, the use of the trap as described in Fig. 1, first confining the atoms, then compressing them in a moderately dense sample and finally proceeding to the population inversion, is a problem as interesting as it is challenging for experimental physics.

Presentation of the Manuscript

Regarding the presentation of the manuscript, referee #2 have written:

"I find the presentation style unsatisfactory, especially concerning the resulting clarity to read. In particular, I find unpleasant that the derivations of the main equations, although not so involved, are put together with the discussion of the results stemming from them. This fact makes the presentation quite tiring to follow, also because the relative weight devoted for the results turns out with almost a minor importance (that cannot be, clearly). The same problem holds for the description of preparation of the set-up, in my opinion. I also find inconvenient to locate all the pictures after the end of the text; perhaps this is intended just a feature of the preprint. However, this choice does not simplify the reading process."

In the new version of the manuscript we have addressed some of the problems pointed out by referee #2 regarding the presentation style. We believe that some of these problems occurred because the manuscript was first designed as a letter. Now, we have introduced Sections I to VII, with the aim of making the development of the theoretical calculations clearer. After the Introduction in Section I, we then define the nonlinear mean-field Hamiltonians in Section II, which have been derived in Ref. [14]. These Hamiltonians govern the evolution of the interaction between the atomic sample and the cavity field. We then introduce the Lewis & Riesenfeld dynamical invariants in Section III, from which the obtain the evolution of the state vector of the system. In Section IV we define the three regimes of solutions for the interplay between superradiance and superabsorption: the underdamped, damped and overdamped regimes, and in Section V, we approach the coherent deflection of the atomic sample that leads to the positional mesoscopic superpositions. In Section VI the numerical analysis for the interplay between superradiance and superabsorption and the vality of our analytical solutions are presented considering figures 2 to 4. Finally, our conclusions are presented in Section VII.

In addition, as already anticipated above, in the new version we decided to modify the presentation of Figs. 2 to 4, now fixing the relaxation rate of the atoms and varying the number of atoms in the sample. This procedure seems more suitable for the purposes of defining the parameters to be used both in microwave and optical cavity QED experiments. We have thus verified that although the experiment can be carried out in both regimes, the microwave experiment is more advantageous for preserving the coherence of the sample deflection.

The sentences introduced throughout the text, with the aim of answering the specific questions raised by referee #2, also helped to improve the presentation of the manuscript.

Finally, instead of presenting all the figures at the end of the manuscript, they were inserted throughout the manuscript. We believe that all the criticisms and suggestions made by referee #2 were observed in the preparation of the new version of the manuscript, which certainly made it clearer, more precise and complete.

Best regards,

L. F. Alves da Silva, L. M. R. Rocha, and M. H. Y. Moussa.