Response to Report

Dear Editor & Referee,

We would like to thank the Referee for their review of the manuscript. Below we respond the different Referee's queries and summarize the changes made to the document, which will be colored red in the paper resubmission.

1. Equation (2.1) seems a little redundant. I think you can define N_1' and N_2' in such a way that M is zero. I am not this is useful, however, but I will leave that to the authors to decide.

Reply: As the Referee is pointing out, one could always rotate the $N_{1,2}'$ fields and go into a basis in which their masses are diagonal. We chose to start with the Lagrangian in a generic basis to prepare the discussion of symmetries (section 2.1), and in particular to spell out possible sources of lepton number violation. Later (section 2.2) all the details of the diagonalization are provided.

2. Below Equation (2.5), I presume $A = 1, 2, 3$ and that $(m_A m_A)^{1/2}$ has an implied sum over A?

Reply: Yes, a sum is indeed implied over A. To make this clearer, we have added the sum explicitly in the expressions.

3. Throughout the model discussion, I assume the authors have assumed that contributions from the Weinberg operator are negligible. Since they include the dipole operators and the same physics that contributes to the dipole operators can also contribute to the Weinberg operator, they may want to comment on the self-consistency of this choice. I don't think there are any problems here, but it is useful to discuss it.

Reply: Yes, it is true that the unspecified UV physics that generates the dipole operator may also generate the Weinberg operator. The latter acts as an additional source of active neutrino masses and mixing and can thus decorrelate neutrino oscillations from the minimal seesaw contribution induced by the two sterile neutrinos. We actually mentioned the Weinberg operator and other sources of neutrino masses above Eq. (2.2) and in the last paragraph of section 2.4 (there we added a line to mention Weinberg explicitly again). In most of our analysis, we did not insist to fit neutrino oscillation data with the minimal seesaw contribution only. This minimal possibility is considered in the last paragraph of section 2.4 and later, when in the lower panel of Fig. 5 we consider the $e + \mu$ and $e + \mu + \tau$ cases, we are requiring the minimal seesaw to be responsible also for the generation of neutrino masses and mixing.

4. Are there any constraints on the invisible dipole from the Z? For example, $Z \to N_1 N_2$? I suspect the answer is no, but I am not at all sure. I don't recall an analysis along these lines and the effect could be very tiny...

Reply: In our setup, the decay $Z \to N_1 N_2$ is generated in two ways: (i) at tree-level from the active-sterile mixing and (ii) via the dipole, receiving a contribution from both the dimension-five operator and the electroweak loop. The dominant effect will be due to the tree-level contribution, but this depends only on the mixing angle (and the right-handed neutrino masses, of course) and the bound is already taken into account in the choice of $\theta_{i\alpha}$. Turning to the contribution due to the dipole, it was shown in Ref. [52] that, in the $\theta_{i\alpha} \to 0$ limit and assuming $N_{1,2}$ to be stable on the scales of LEP detectors, the region excluded by the Z invisible width would be $d \gtrsim 2.5 \times 10^{-4}\,\text{GeV}^{-1}$. Nevertheless, this limit is not meaningful because, for such large values of the dipole, the lifetime of right-handed neutrinos is quite short, so that we expect that most of them will actually decay inside the detector and would not contribute to the Z invisible signal. To the best of our knowledge, the signal generated by the decays of a pair of sterile neutrinos have not been searched for at LEP, so that even the region $d \gtrsim 2.5 \times 10^{-4}\,\text{GeV}^{-1}$ actually turns out to be unbounded. We now added a footnote in section 3.1 to mention that constraints from the Z are negligible.

Before concluding, we would like to point out that, in the upcoming paper resubmission, we will add the effect of the electroweak loops that generate a dipole between a sterile and an active neutrino. Although this source of dipole is well-known in the community that studies keV-scale sterile neutrinos as dark matter candidates, it seems that its effect has not been considered in the community studying experiments at the intensity frontier, probably because the effect was thought to be small. This turns out not to be the case: as we will show in our updated figures 3–5, SHiP will be sensitive to the electroweak-loop dipole, at least for mixing angles close to their maximum allowed values, and for masses between 0.5 and a few GeV. This causes a major change in the figures with respect to the first version, since now there are regions in parameter space in which the SHiP sensitivity extends to arbitrarily small values of the dimension-five dipole d, simply because here the photon signal is driven by the electroweak dipole instead. To include the discussion of this effect in our paper, we have made a number of changes with respect to the first version. The most relevant ones will be shown in purple in the paper resubmission: they include, in particular, a new section 2.3, an updated discussion around Eq. (3.1) and around Figs. 3–5, as well as a new appendix B.

We thank the Referee once again for the valuable feedback. We hope that, with the clarifications and modifications made to the manuscript, the document would be suitable for publication in SciPost.

Yours faithfully,
The Authors