Response to Report 1 on "Four no-go theorems on the existence of spin and orbital angular momentum of massless bosons" (scipost_202412_00011v2)

Eric Palmerduca[*] and Hong Qin[†]

Department of Astrophysical Sciences, Princeton University, Princeton, New Jersey 08544, USA and

Princeton Plasma Physics Laboratory, Princeton, NJ 08540, USA

Dear Referee,

We thank you for the additional comments and have responded to them below. Your first comment in particular brought our attention to some notational improvements we could make which clarify our discussion of the massive spin operator. We were instructed to postpone posting a revised manuscript until all referee reports are submitted, but the minor modifications we have made to the manuscript are described below.

Referee comment:

> 1) Are the eigenvalues of the $\boldsymbol{S}$ operators in Eq. (7) integers, discrete real values, or continuous real values? I think that the corresponding section in the article would benefit from explicitly answering this question.

Reply:

To avoid confusion we will use the notation used in the paper, so we will write the spin operator as $\boldsymbol{S}^m$ for a mass m particle with spin s. The eigenvalues of any component of $\boldsymbol{S}^m$ are integers between $-s$ and s, where s is the spin of the particle. We have rewritten the discussion around that equation and made notational adjustments to make this clearer. If we restrict to the fiber at $\boldsymbol{k} = 0$, denoted $E_m(\mathbf{0})$, we can let $v_{-s}, ..., v_s$ be the eigenstates a single component of $\boldsymbol{S}^m|_{\boldsymbol{k}=0} = \boldsymbol{J}|_{\boldsymbol{k}=\mathbf{0}} \doteq \boldsymbol{J}_0$, say $\boldsymbol{J}_{0,3}$, where the integer subscripts on the v_a denote the eigenvalues (these were previously labeled as $(v_1, ..., v_{2s+1})$ which does not make it clear these are eigenvectors of a single component of $\boldsymbol{J}_0$). Thus on $E_m(\mathbf{0})$, $\boldsymbol{J}_0$ is given by the spin s matrices $\boldsymbol{S}_s$, so that

$$\boldsymbol{S}^m(\mathbf{0}, v) = \boldsymbol{J}_0(\mathbf{0}, v) = (\mathbf{0}, \boldsymbol{S}_s v) = \boldsymbol{S}_s(\mathbf{0}, v) \tag{1}$$

where the last two equalities assumes $v \in E_m(\mathbf{0})$ is expressed in coordinates with respect to the eigenbasis. Thus, for arbitrary $(\boldsymbol{k}, v)$,

$$\boldsymbol{S}^m(\boldsymbol{k}, v) = \Sigma(\Lambda_{\boldsymbol{k}})\boldsymbol{S}_s\Sigma(\Lambda_{-\boldsymbol{k}})(\boldsymbol{k}, v) \tag{2}$$

where we have again used the coordinate representation at $E_m(\mathbf{0})$. We can then label all polarization states by their rest frame polarizations, defining a basis of $E_m(\boldsymbol{k})$ at arbitrary $\boldsymbol{k}$ by

$$(\boldsymbol{k}, v_a) \doteq \Sigma(\Lambda_k)(\mathbf{0}, v_a). \tag{3}$$

* ep11@princeton.edu

† hongqin@princeton.edu

so that if arbitrary $(\boldsymbol{k}, v)$ is expressed in coordinates with respect to this basis, then

$$\boldsymbol{S}^m(\boldsymbol{k}, v) = \Sigma(\Lambda_{\boldsymbol{k}})(\boldsymbol{0}, \boldsymbol{S}_s v) = (\boldsymbol{k}, \boldsymbol{S}_s v). \tag{4}$$

so that in this basis we can write

$$\boldsymbol{S}^m = \boldsymbol{S}_s. \tag{5}$$

Note in particular that for any $\boldsymbol{k}$, $(\boldsymbol{k}, v_a)$ is an eigenvector of S_3^m with eigenvalue a, where a is an integer between $-s$ and s. Since we could chosen a basis with respect to any other axis, we see that the eigenvectors of any component of $\boldsymbol{S}^m$ are likewise integers between $-s$ and s.

Referee comment:

> 2) When the single particle state is a superposition of different momenta, each such momenta will require a different boost to bring it to $(M, 0, 0, 0)$ before applying the rotation. This plurality of rest frames complicates the physical understanding of the proposed transformation for massive particles. I think that this is important enough to merit its comment in the article.

Reply:

As we see from Eq. (5) above, it is possible to choose coordinates such that the action of the spin operator simple applies an internal action via the spin s matrices, without the need to apply any boosts. In this case one is choosing to label the internal coordinates by their value in the rest frame. To us, this makes a lot of physical sense. If one wishes to think about the internal coordinates (the spin states) as entirely decoupled from the external coordinates $(\boldsymbol{k})$, as we would always like to do if possible, we need to find some way of identifying internal states at different $\boldsymbol{k}$. A natural way to do this is to compare the internal coordinates at some fixed $\boldsymbol{k}_0$ and the only distinguished momentum is $\boldsymbol{k}_0 = 0$, so it is a natural choice. Thus, we do not view the fact that particles with different $\boldsymbol{k}$ have different rest frames as a significant complication to the physical picture.

Referee comment:

3) I do not think that the transformation in Eq. (5) is generated by the operators in Eq. (7). If one takes one of the three components in Eq. (7), say S_j, and uses the usual recipe to go from the generators to the generated transformations

$$\exp(-i\theta S_j) = \sum_{l=0}^{\infty} \frac{(-i\theta S_j)^l}{l!} = \sum_{l=0}^{\infty} \frac{(-i\theta\Sigma(\Lambda_{-k})S_{sj}\Sigma(\Lambda_k))^l}{l!} \tag{6}$$

the result seems to be different than the transformation in Eq. (5):

$$\Sigma(\Lambda_{-k})\exp(-i\theta S_j)\Sigma(\Lambda_{-k}). \tag{7}$$

Reply:

Thanks for catching this, this issue was due to a typo in Eq. (7) in our reply to your previous comments. Λ_{-k} and Λ_k should have been switched, the correct version of Eq. (7) reads

$$\boldsymbol{S} = \Sigma(\Lambda_k)\boldsymbol{S}_s\Sigma(\Lambda_{-k}). \tag{8}$$

We note that this equation appears correctly in Eq. (20) in the manuscript. Using the correct equation and the fact that $\Sigma(\Lambda_k)\Sigma(\Lambda_{-k}) = 1$, we obtain

$$\begin{aligned} e^{-i\theta S_j} &= \sum_{l=0}^{\infty} \frac{(-i\theta S_j)^l}{l!} = \sum_{l=0}^{\infty} \frac{(-i\theta\Sigma(\Lambda_k)S_{sj}\Sigma(\Lambda_{-k}))^l}{l!} & (9)\\ &= \Sigma(\Lambda_k)\left(\sum_{l=0}^{\infty} \frac{(-i\theta S_{sj})^l}{l!}\right)\Sigma(\Lambda_{-k}) & (10)\\ &= \Sigma(\Lambda_k)e^{-i\theta S_{sj}}\Sigma(\Lambda_{-k}) & (11)\\ &= \Sigma(\Lambda_k)e^{-iJ_{0j}}\Sigma(\Lambda_{-k}) = \Sigma(\Lambda_k)e^{-iJ_j}\Sigma(\Lambda_{-k}) & (12)\\ &= \Sigma(\Lambda_k)\Sigma(R_j)\Sigma(\Lambda_{-k}). & (13)\\ &= \Sigma^S(R_j) & (14) \end{aligned}$$

Note that $\boldsymbol{S}_s$ is the coordinate representation of $\boldsymbol{J}_0$ with respect to the eigenbasis of $J_{0,3}$. In writing down Eq. (9) we have assumed that in the rest frame fiber at $\boldsymbol{k} = 0$ we are using such coordinates, so $S_{sj} = J_{0j} = J_j$ at $\boldsymbol{k} = 0$. We use this fact to go from Eq. (11) to Eq. (12).