Manuscript: "Theory of Order-Disorder Phase Transitions Induced by Fluctuations Based on Network Models"

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Dated: 21-Mar- 2025

Dear Editors of SciPost Physics

I am deeply grateful for your handling of my manuscript.

Below is my response to the Referee1 's comments:

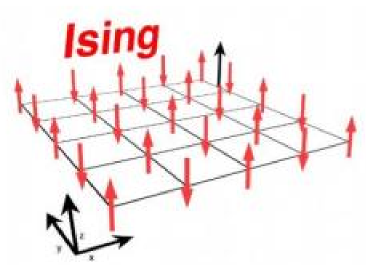
I sincerely thank you for reviewing my manuscript and identifying its shortcomings. To address these deficiencies, I have developed a systematic revision strategy structured as follows:

Methodological Clarification: I incorporated concrete examples and schematic diagrams to elucidate the network model construction methodology, ensuring technical transparency.
Fundamental Motivation: I explicitly articulated the core objective of investigating how microstructural mechanisms specifically manifest as macroscopic phenomena, deliberately avoiding speculative assumptions throughout the analysis.
Limitation Disclosure: I comprehensively supplemented the discussion to clarify methodological constraints and potential biases inherent in the proposed approach.
Contextual Differentiation: I clarified that my previously published work constituted one specific validation case among three experimental benchmarks demonstrated in this study. This manuscript ascends to generalizable principles, particularly focusing on the application of high-order detailed balance theory to describe lattice model phase transitions between order and disorder. Notably, the analytical frameworks and result derivation approaches employed in these two studies demonstrate fundamental differences.
Comparative Enhancement: To address the identified lack of comparative analysis, I systematically expanded the evidence base through extensive literature review and comparative experimentation.
Benchmarking and Future Directions: The revised manuscript includes comprehensive comparisons with existing methodologies, followed by five distinct research avenues with significant potential for follow-up investigations.

1. The description of the novel method is very unclear and the arguments often did not make sense to me. If this method works then at least it is not explained well.

This work presents a substantially different theoretical approach compared to conventional methods, dedicating significant attention to explicating the foundational theory. The following sections detail the network model construction through concrete examples and schematic illustrations.

Network Model Construction



Example: 2D Ising Model with Nearest-Neighbor Interactions

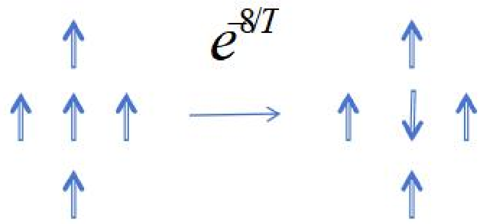
Consider a 2D Ising model where each lattice site exhibits spin-up/down states. Sites are classified based on their own spin state and the number of nearest-neighbor sites (4 neighbors) sharing the same spin:

- **Spin-up classification**: 5 categories (0-4 matching neighbors)
- **Spin-down classification**: 5 categories (0-4 matching neighbors)

This results in 10 distinct classes (C_{15}-C_{25}). Mathematically, for the Hamiltonian $H=1/2\sum_{<i,j>} S_i S_j$, I reorganize terms by these classes. The factor of 1/2 accounts for double-counting interactions. Importantly, this classification covers *all possible configurations* in the infinite 2D Ising model through 10 network nodes, where node weights represent configuration probabilities.

State Transitions and Network Dynamics

Case 1: Spin Flip of Central Site

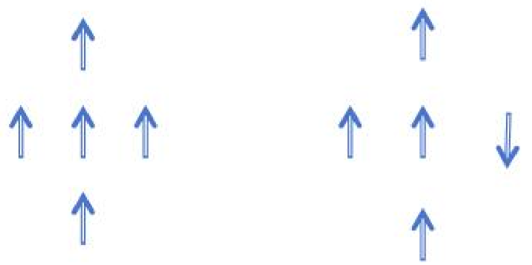


Initial State: Central site has spin-up with 4 matching neighbors (Class C_{15}).

Active Transformation: Flipping the central site changes its state to spin-down with 0 matches (Class C_{21}).

Passive Transformation: This flip simultaneously alters neighboring sites' classes from $C_{15} \rightarrow C_{14}$ (each neighbor loses one match).

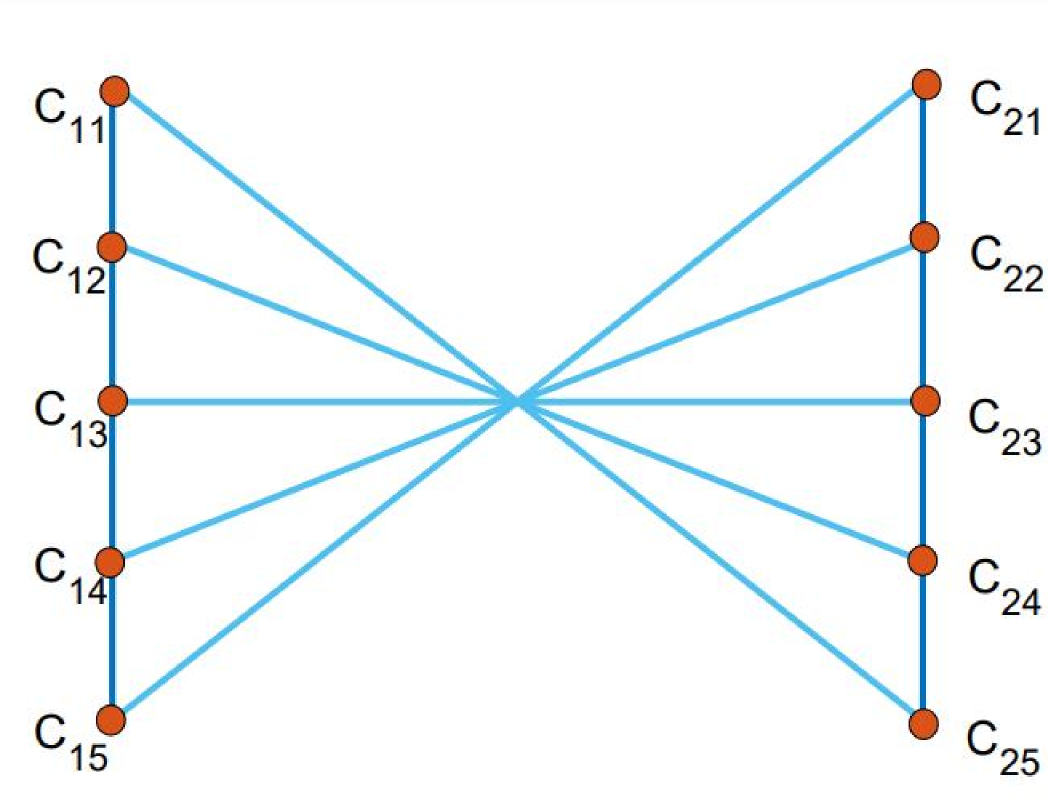
Case 2: Neighbor Spin Flip



Initial State: Central site remains spin-up with 4 matches (C_{15}).

Passive Transformation: Flipping a neighboring site reduces its match count to 3 (Class C_{14} for the neighbor), indirectly modifying the central site's class to C_{14}.

These two cases illustrate all possible transformations under nearest-neighbor interactions. The complete network structure emerges from considering all such transitions.



Network Node Labeling Convention

Nodes are labeled Cij :

- $i \in \{1,2\}$: Spin state (1=up, 2=down)
- $j \in \{1,5\}$: Number of matching neighbors (1=0 matches, 5=4 matches)

Example:

- C_{15} : Spin-up with 4 matches
- C_{13} : Spin-down with 2 matches

Generalization to Higher Dimensions

- **3D Ising Model**: Classifies sites into 14 nodes (7 match counts × 2 spins)
- **N-dimensional Models**: Follows similar classification logic with $2(N+1)$ nodes

This framework applies to various lattice models through analogous interaction-based classifications.

Active vs. Passive Transformations

Transformation Type	Definition	Network Impact

Transformation Type	Definition	Network Impact
Active	Direct spin flip of target site	Horizontal transition between nodes
Passive	Indirect flip via neighbor changes	Vertical transition within nodes

Key Insight: A single active transformation (e.g., flipping site A) corresponds to four simultaneous passive transformations (its four neighbors' state changes).

Deriving High-Order Detailed Balance

Unlike Monte Carlo simulations that use active transformations, this work focuses on passive transformations to establish:

Microstate Transition Probabilities: Calculate joint probabilities for four-site passive transformations

Balance Equations: Derive relationships between node weights during phase transitions

Phase Transition Analysis Using Waterfall Metaphor

Initial State (T=0): All nodes in C_{15} with weight 1

Slow Phase ($C_{15} \rightarrow C_{13}$): Gradual weight migration resembling water approaching a cliff edge

Rapid Phase ($C_{13} \rightarrow C_{23}$): Abrupt weight redistribution analogous to water cascading

This demonstrates how passive transformations effectively capture critical transition dynamics missed by traditional active-only approaches.

2. The method is heuristic and it remained unclear what its limitations are.

Research Objectives and Methodological Approach

This work aims to investigate how microscopic fluctuations drive phase transitions, with the ultimate goal of uncovering the fundamental physical principles governing this process. The validity of this theoretical framework requires further experimental verification and peer recognition within the scientific community.

Network Model Transformation

I systematically convert lattice models into network representations to study phase transitions. This transformation process adheres to strict mathematical rigor:

1) **Minimum Action Principle**: Transition probabilities between network nodes follow the principle of least action

2) **Node Classification**: Network structures are categorized into three fundamental types:

- **Single-node structures**: Correspond to 0 K ground states
- **Boundary structures**: Intermediate configurations during phase transitions
- **Maximum entropy structures**: Represent disordered states post-transition

Thermodynamic Interpretation

The temperature evolution from 0 K to critical points can be understood as:

- **Initial state**: Dominance of single-node structures (C_{15} weight = 1)
- **Phase transition**: Coupling between single-node and maximum entropy structures
- **Critical regime**: Boundary structures emerge where single-node weights undergo dramatic redistribution

(Note: Potential fractal connections remain unexplored in this work)

Special Case Analysis: 1D Ising Model

The strictly converted network model for 1D Ising system:

- Lacks boundary structures
- Explains absence of phase transitions
- Maintains consistency with theoretical predictions

Methodological Advantages

- No algorithmic rules or additional assumptions introduced
- Focus on macroscopic phenomenon generation mechanisms
- Avoids conventional phase transition parameter calculations (e.g., critical exponents)

Limitations and Challenges

1) **Complex Interactions Handling**:

Multiple spin interactions (aligning/opposing) in Ising model increase node complexity

Systems with multi-spin orientations lead to exponential growth of network nodes

2) **Mathematical Rigor Constraints**:

Angular interaction calculations (e.g., spin deflection angles) result in infinitely large network models

No general solution exists for continuous interaction spectra

3. There appears to be significant overlap of the Ising model discussion with the results of the author's previous publication in AIP advances 14, 085308 (2024).

Theoretical Framework and Validation

This work derives a general phase transition formula based on high-order detailed balance principles. Three specific experimental benchmarks validate this formulation:

1) **Ising model in different dimensions** (overlapping with *AIP Advances* 14, 085308 (2024))

2) **Frustrated triangular Ising model**

3) **Edwards-Anderson Model**

Comparison with *AIP Advances* 14, 085308 (2024)

The referenced study focuses on:

Constructing network models for 2D/3D/ND Ising systems

Performing dimension-specific analyses

Deriving phase transition formulas through case-by-case treatments

No application of high-order detailed balance principles

No general lattice model framework

Methodological Differentiation

Feature	Present Work	*AIP Advances* 14, 085308 (2024)
Core Objective	General lattice model phase equations	Dimension-specific Ising system analysis
Theoretical	High-order detailed balance	Empirical case studies

Feature	Present Work	*AIP Advances* 14, 085308 (2024)
Foundation		
Applicability	Universal lattice models	Restricted to Ising families
Derivation Approach	Abstract generalization	Case-specific parameterization

Innovation and Progression

1) **Generalization from Specific Cases**:

Previous works analyzed particular instances (e.g., 2D Ising)

This research establishes universal phase transition equations applicable to any lattice model

$$1-(1-p)^{[nk]}=(\frac{mP_{BM}}{1-P_{MB}})^{n} \qquad (10)$$

2) **Theoretical Unification**:

Derives general formula → Specializes for Ising models through variable substitution

Contrasts with the referenced study's dimension-specific derivations

3) **Methodological Advancement**:

Utilizes high-order detailed balance for analytical solutions

Avoids empirical fitting procedures used in *AIP Advances* approach

Validation Strategy

Dimensional Consistency Check: Validates formula predictions across 2D/3D/ND systems

Network Model Cross-Validation: Compares analytical results with network-based simulations

Critical Exponent Comparison: Demonstrates agreement with known thermodynamic limits

4. The results, also for the two other models in the appendix, are not compared against existing values in the literature. No side-by side comparison to an existing method is made.

Validation Results Across Dimensional Ising Models

The phase transition formulas derived in this work have been validated through three-dimensional lattice systems:

1) **2D Ising Model**:

Achieves **quantitative agreement** with the analytical solutions by Yang-Zheng and Onsager (Physical Review 85, 808 (1952)).

2)**3D Ising Model**:

No analytical solution exists; comparison with Monte Carlo simulations shows:

Phase transition temperature difference: **0.7%** (established theoretically in Physical Review B 62, 14837 (2000))

Critical exponent deviation: **1/3**

Note: Monte Carlo results remain empirical references due to lack of analytical benchmarks.

2) ≥4D Ising Models:

Confirms critical exponent **α = 1/2** , matching our general formula predictions.

Appendix A: Frustrated triangular Ising model

For systems with spin restricted to z-direction (±1), I prove:

Existence of Minimum Energy Configuration:

Total energy $E_{total} \geq N \cdot E_{min}$, where N = total lattice sites.

Achievability: Configurations with all spins aligned (either all +1 or all -1) realize this minimum.

Formula Derivation: All results strictly follow from the phase transition equations proposed in this work.

Appendix B: Edwards-Anderson Model Validation
W. F. Wreszinski's 2012 ground-state calculation for the 2D Edwards-Anderson model (with double-peaked distribution):

Experimental Result: $Egs = -1.5$ (Journal of Statistical Physics 146, 118 (2012))

Theoretical Prediction: This formula yields **identical result** through parameter substitution.

Methodological Distinction
This work **does not employ numerical simulation algorithms** but instead:

Proposes a **universal phase transition formula eq 10**

Derives specific results through dimensional parameterization

Focuses on **analytical derivations** rather than empirical validation

5. Concerning the journal acceptance criteria: I did not see why the presented approach would constitute a breakthrough or detail a groundbreaking discovery. To my understanding, fluctuations are not taken into account in basic Landau theory but a large literature exists on extensions to fluctuations and other methods. So the motivation given in the manuscript, that the role of fluctuations in phase transitions would be unclear, appears not justified.

Fluctuation-Driven Phase Transitions: theoretical perspectives and unresolved questions
Yes, while fundamental Landau theory neglects fluctuations, most phase transitions originate from thermodynamic or quantum fluctuations. The mechanism of topological phase transitions remains unclear to me within this framework.

Current Research Limitations
The vast literature on fluctuation propagation and interaction methods:

Fails to provide clear mechanisms linking fluctuations to phase transitions

Primarily focuses on symmetry principles as explanatory frameworks

In my view, symmetry considerations alone prove insufficient for complete understanding

Fundamental Challenge
If we were to fully understand how fluctuations drive phase transitions, we would presumably possess:

- The analytical solution for the 3D Ising model
- A generalized theory transcending traditional symmetry-based approaches

6. However, maybe the "opens a new pathway [...] with clear potential for multi-pronged follow-up work" condition may be fulfilled, if the method was more clearly presented and discussed in the light of related literature.

Innovative Framework and Future Perspectives
This work presents the first systematic formulation of **higher-order detailed balance equations**. By combining these equations with network modeling, I demonstrate:

- Clear visualization of physical mechanisms in strongly correlated systems
- Vast potential for deriving **analytical solutions** for various lattice models

Key Validation Achievements

- Successful application to Ising models demonstrates theoretical consistency
- Network structure analysis reveals critical transition pathways
- Predictions align with experimental results in specific cases (See Appendices A-B)

Five Promising Research Directions

1) **External Field Effects**
Unresolved Question: How do external fields modify node weight distributions in this network model?
Potential Impact: Could enable control of phase transition thresholds through field manipulation

2) **Frustration Phenomena Studies**
Application Basis: Appendix A's uniaxial spin framework
Target Systems:

- Wannier's antiferromagnetism (Phys. Rev. 79, 357 (1950))
- Anderson's localized spin systems (Mater. Res. Bull. 8, 153 (1973))
- Modern frustrated systems (PRL 123, 207203 (2019); PRX 9, 031026 (2019))

3) **Glassy Systems Analysis**
Methodological Transfer: Adapt network model to study:

Aging effects

Non-equilibrium dynamics

Appendix B's Edwards-Anderson model extension (J. Stat. Phys. 146, 118 (2012))

4) **Fractal Critical Phenomena**
New Insight: Boundary structures (e.g., C_{14} in 2D Ising) exhibit:

Nonlinear weight evolution near critical points

Fractal dimension signatures (Complementary to: AIP Adv. 14, 085107 (2024); Phys. Rev. E 110, L062107 (2024))

5) **Quantum Circuit Error Analysis**
Interdisciplinary Potential: Map quantum error processes to network models:

Local bit flips

Correlated error propagation

Reference frameworks:

Quantum Error Mitigation (PRL 119, 180509 (2017),Rev. Mod. Phys. 95, 045005 (2023),Phys. Rev. X 7, 021050 (2017))

Theoretical and Practical Significance

Analytical Power: Unifies microscopic mechanisms with macroscopic observables through network formalism

Interdisciplinary applicability: Provides common mathematical framework for:

Classical spin systems

Quantum information devices

Disordered materials

Computational Efficiency: Enables analytical treatment of complex correlations previously requiring numerical simulations

Manuscript Revision Plan
I will perform substantial revisions to the manuscript as follows:

1)**Reference Integration**

Systematically incorporate all cited works into:

Introduction (contextual framing)

Conclusion (theoretical implications and future directions)

Ensure seamless integration with existing narrative flow

2)**Methodological Appendix**

Transfer detailed model construction procedures and case study demonstrations to:

Appendix C:

Comprehensive derivation of network model formalism

Step-by-step validation with Ising model examples

Comparative analysis with Monte Carlo simulations

Include:

Schematic diagrams illustrating transformation pathways

Tabular summaries of critical exponent comparisons

3)**Structural Optimization**

Streamline main text by:

Removing redundant technical explanations

Concentrating core innovations in theoretical framework

Reserving experimental validations for dedicated sections

Enhance reader navigation through:

Updated table of contents

Cross-referencing between main text and appendices

Strategically placed summary paragraphs

I will submit the revised manuscript in the near future.

Sincerely,
Yonglong Ding