Response to Referee 2:

Referee 2:

> **Report**: The paper studies the possibility of disentangling the ground state of one dimensional systems that are conformal invariant through local Clifford disentangler. The question is timely and interesting; however, I am not sure to follow exactly what the authors want to claim. I understand that the disentangling power is related to the amount of magic, but then I don't really see a clearly presented relation between short range magic and long-range one. The authors introduce $m_{\chi=2}$ which it is really difficult to understand what its meaning. Why $\chi = 2$ and not $\chi = 1$ or 6? Disentangling should be about decomposing log scaling entanglement to area law. Moreover, given that these are CFTs, one should also study how critical proprieties change by increasing chi and disentangling power. I find the paper quite drafty, with no clear sense of what are the main statements and physical implications.

Response: We thank the Referee for their careful evaluation of the manuscript, and for recognizing the potential impact of our work on the study of disentangling. From the overall comment, it is clear that there has been a lack of clarity on our side on the main message of the work, and, at the more technical level, on the quantity $m_{\chi=2}$. Below we address the referee's constructive comments.

Referee 2:

> **Requested changes**:
>
> - Clarify the main messages regarding magic and entangling power

Response: We have modified a paragraph in the introduction to more clearly summarize our findings. We emphasize that for us, disentangling is not about reducing a logarithmic scaling of entanglement to an area-law, but rather about achieving a gain in entanglement reduction that itself grows logarithmically with system size. Now, the paragraph reads:

"A key result of our work is the close connection between magic and the efficiency of stabilizer disentangling. We find that the total magic content strongly correlates with, and in some cases is directly proportional to, the ability of local stabilizer operations to reduce entanglement. We argue how such - in principle, very unexpected - finding can be justified based on the very special correlation structure of ground states, that is at odds with that of random Haar states. Secondly, we show how the efficacy of local stabilizer cooling with respect to size is dictated by the mutual stabilizer Renyi entropy (mSRE) Leone et al. *(2022); Haug and Piroli (2023); Tarabunga* et al. *(2023). This quantity is akin to mutual information in the context of Renyi entropies and describes how spread in stabilizer space a given partition is. Our results show that when mSRE is negative, stabilizer disentangling improves with system size, as seen in LCD and fLCD states. Conversely, when mSRE is positive, disentangling ceases to improve with size, which is characteristic of nLCD states (see Fig. 1f)."*

Referee 2:

> - Clarify the role of $m_{\chi=2}$

Response: We appreciate the Referee's suggestion regarding the clarification of the section on $m_{\chi=2}$ and thank them for bringing this to our attention. We have correspondingly modified the text, and presented an extended analysis to justify the relevance of this quantity.

We have added a paragraph in the introduction where we summarize the role of $m_2^{\chi=2}$, namely:

"Lastly, we show that the efficiency of disentangling is also related with a more refined measure of magic constrained by bond dimension. To probe this connection, we introduce an additional observable, $m_2^{\chi=2}$, which quantifies the magic of a state when restricted to a low-entanglement, limited bond dimension description. This quantity provides insight into the portion of magic that can be removed solely through local operations. Numerically, we establish a direct relationship between $m_2^{\chi=2}$ and the classification of states in terms of disentangling power. We find that when $m_2^{\chi=2}$ is much smaller than the magic of the full representation of the state, the state belongs to the non-local-Clifford disentanglable (nLCD) class. On the other hand, when the two magic measures are comparable, the state is local-Clifford disentanglable (LCD)."

Moreover, we added another paragraph in Sec 2.5 to justify the choice of setting $\chi = 2$ for this measure, that is:

"We choose $\chi = 2$ because it is the smallest bond dimension that exhibits a non-pathological and non-trivial behavior. In fact, the choice $\chi = 1$ corresponds to product states, and the projection onto this manifold is too abrupt and leads to an unpredictable behavior of magic. Furthermore, some of the phases present in the model we are considering have a fixed point that can be exactly captured with a bond dimension of 2, making it crucial to retain those correlations in our analysis."

In response to the Referee's comments, we performed a numerical analysis comparing m_χ for different bond dimensions $\chi = 1, 2, 4, 8$, and the maximum value χ_{max} computed as an output of DMRG. Our results show that in both cases, shown in Figs. 1 and 2, $\chi = 1$ is not well-behaved, as it significantly deviates from the other curves. For non-local-Clifford disentanglable (nLCD) states, represented in Fig. 1, we find that $m_{\chi=2}$ is consistently smaller than all other values, indicating that a substantial portion of magic is tied to entanglement correlations. Conversely, for local-Clifford disentanglable (LCD) states, $m_{\chi=2}$ closely follows the higher χ values, as shown in Fig. 2, suggesting that most of the magic can be removed through local operations. These observations validate the performances observed in the entropy decrease.

Referee 2:

- study critical proprieties (for example the spectral gap) as function of bond dimension and disentangling power

Response:

We are not sure we understood the Referee's comment on this point. In the spirit of our work, we are not immediately interested in analyzing the performance of an algorithm working with finite resources, and make a (rigorous) statement on its working (such as, e.g., the convergence of certain critical properties such as central charge as a function of the bond dimension). We decided to focus solely on working with numerically exact representations (converged MPS), and then disentangle those.

Still, our approach can provide a rough resource budget on a concrete computation, such as a ground state search with CAMPS. For instance, given an initial value of S_A, a conventional

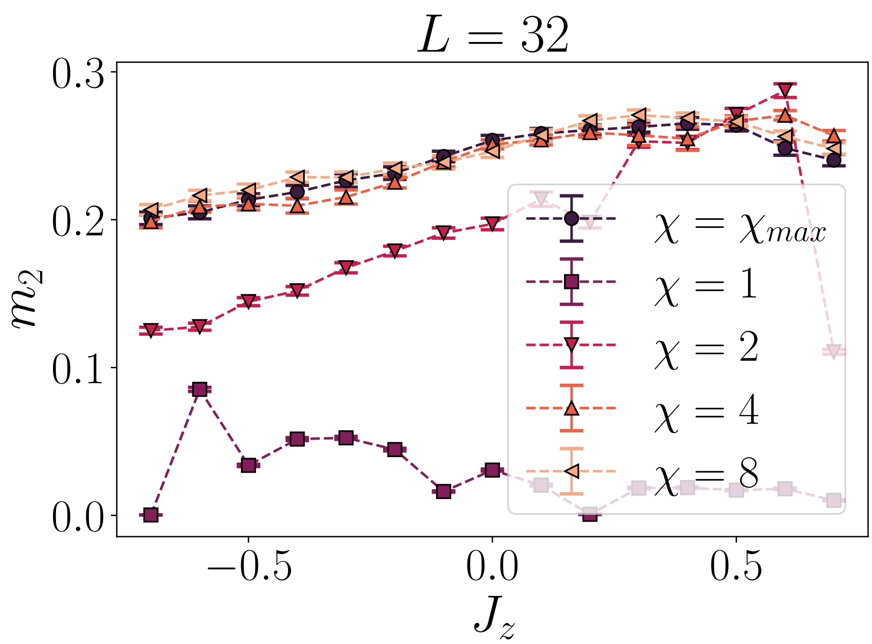

Figure 1: m_2^χ for different values of $\chi = 1, 2, 4, 8$ inside the critical phase of XXZ model, varying the value of the parameter J_z. The system size is $L = 32$ and $m2^\chi$ are computed with $N_s = 1000$ samples.

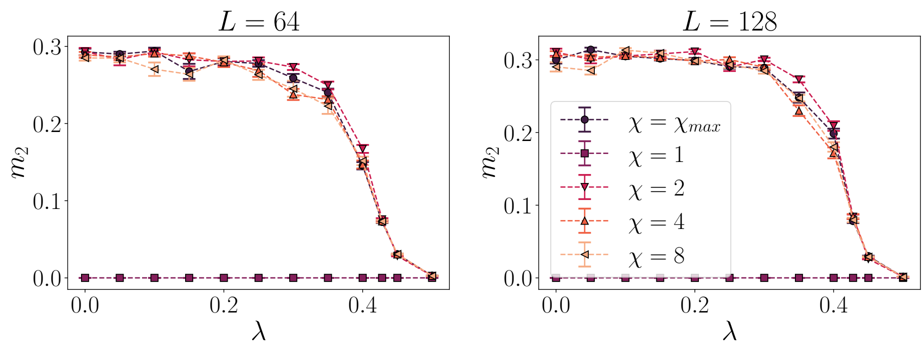

Figure 2: m_2^χ for different values of $\chi = 1, 2, 4, 8$ in the OF model, varying the value of the parameter λ. The system sizes are $L = 64, 128$ and m_2^χ are computed with $N_s = 1000$ samples.

MPS simulation would require approximately $\chi_{\text{MPS}} \simeq e^{EE}$ states. For an algorithm working directly within the CAMPS manifold (such as those proposed in Refs. 16, 18), 22-23, the corresponding bond dimension needed would instead be $\chi_{\text{CAMPS}} \simeq e^{SMEE}$. This implies that, to obtain converged critical properties, the bond dimension relative to an MPS simulation shall scale as:

$$\frac{\chi_{\text{CAMPS}}}{\chi_{\text{MPS}}} \simeq e^{SMEE-EE} = e^{\Delta} \tag{1}$$

which, for the different classes of dynamics we report, corresponds to:

- nLCD: $\frac{\chi_{\text{CAMPS}}}{\chi_{\text{MPS}}} \simeq \text{const};$

- LCD and fLCD: $\frac{\chi_{\text{CAMPS}}}{\chi_{\text{MPS}}} \simeq 1/L^{a_1}, a_1 > 0$, where a_1 depends on the difference between the real central charge, and the effective CAMPS central charge.

We remark that these statements shall be taken as guiding principles rather than statistically exact budget resource estimates.

There is of course another viewpoint that can be taken from the Referee's remark, that is: to which extent are other critical properties affected by the disentangling procedure? This is definitely informative in fLCD cases: there, quantum correlation functions will exactly factorize at the size where entropy vanishes. However, for generic LCD, it is not clear what effect disentangling might have. We thus decided to look at this in the context of two body correlations in all the models considered.

In Fig. 3 here, we consider the XXZ (nLCD) and Tricritical Ising Model (LCD) at their respective critical points. In both cases, the expected power-law decay of two-body correlation functions is observed before the disentangling process begins (denoted as sweep0 in the figure).

We apply Stabilizer Disentangling to these two critical ground states for a system size of $L = 96$ performing a total of $N_s = 20$ sweeps. Interestingly, although the SMEE converges immediately after the first sweep and remains unchanged afterward, correlations exhibit a different behavior: they retain their power-law decay (as expected for cases where the disentangling maintains locality in some form, given that the SMEE still exhibits logarithmic scaling), but the exponent extrapolated from the fit oscillates between three different values across the sweeps, even after convergence of the entanglement entropy.

By comparing the decay of the two-body connected correlation function $\langle\sigma^z_{L/4}\sigma^z_{L/4+j}\rangle_C$ with that of $\langle\sigma^x_{L/4}\sigma^x_{L/4+j}\rangle_C$ we observe that within each model - comparing Fig. 3a with Fig. 3c and Fig. 3b with Fig. 3d)- the three exponents of the power-law decays remain the same, but they appear permuted in a different order. This suggests the following interpretation: once the Stabilizer Disentangling algorithm has converged in terms of entanglement entropy, the exponents of different two-body Pauli operator correlations become fixed. The application of Clifford operations on top of the state can still change the Pauli string $P = \sigma^a_i\sigma^a_j$ with $a = x, y, z$ into $P' = \sigma^b_i\sigma^b_j$ with $b = x, y, z$ and $b \neq a$, leading to the observed oscillations between the three exponent values. Thus, for a fixed amount of entanglement, a faster decay of the $\sigma^x_i\sigma^x_j$ correlations correlations comes at the cost of a slower decay in $\sigma^z_i\sigma^z_j$ or $\sigma^y_i\sigma^y_j$. This is also confirmed by the shape of the stabilizer operators applied after the first sweep, that roughly correspond to local change of basis.

In Fig. 4 we present the same analysis applied to the critical Cluster Ising model (fLCD) for a system size of $L = 96$. Both correlations functions considered, $\langle\sigma^z_{L/4}\sigma^z_{L/4+j}\rangle_C$ in Fig. 4a and $\langle\sigma^x_{L/4}\sigma^x_{L/4+j}\rangle_C$ in Fig. 4b, exhibit a critical power-law decay during the initial sweeps. However, unlike the previous case, the exponent does not oscillate between three fixed values but instead varies at each sweep. This behavior is expected, as the entanglement entropy in this case shows a sweep-dependent evolution and converges only at sweep $N^c_s = 24$. At sweep 12, we observe a sudden drop to zero in the correlations, as expected in the fLCD case, where the Stabilizer Disentangling Algorithm successfully splits the chain into two separable sub-chains.

We are not immediately sure this piece of information fits within the scope of our work, but if found informative by the referee, we could include a summary of these findings in the manuscript.

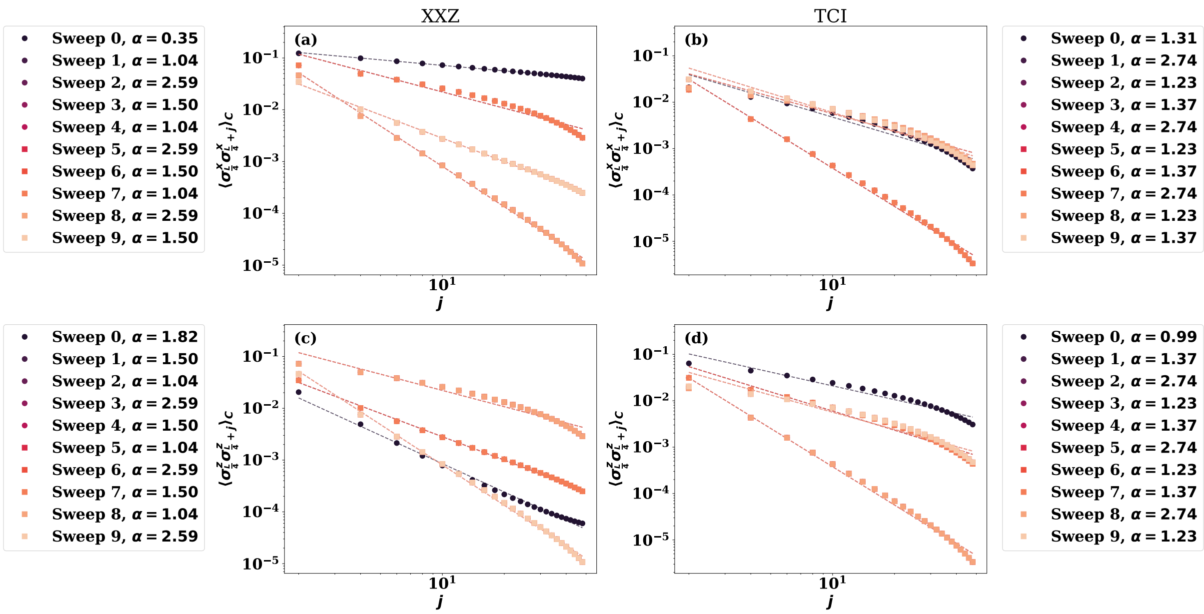

Figure 3: Two-point correlation functions for different models, in log-log scale. Each line corresponds to a sweep of the Disentangling algorithm. The system size is $L = 96$. The lines are fitted with a power-law decay, with the resulting exponent α shown in the labels. The dashed lines correspond to the fits. **(a)**: Decay of the $\sigma^x_{L/4}\sigma^x_{L/4+j}$ correlation function with distance j inside the critical phase of XXZ model with $J_z = 0.5$. **(b)**: Decay of the $\sigma^x_{L/4}\sigma^x_{L/4+j}$ correlation function with distance j at the critical point of the tricritical Ising model, $\lambda_c = 0.428$. **(c)**: Decay of the $\sigma^z_{L/4}\sigma^z_{L/4+j}$ correlation function with distance j inside the critical phase of XXZ model with $J_z = 0.5$. **(d)**: Decay of the $\sigma^z_{L/4}\sigma^z_{L/4+j}$ correlation function with distance j at the critical point of the tricritical Ising model, $\lambda_c = 0.428$.

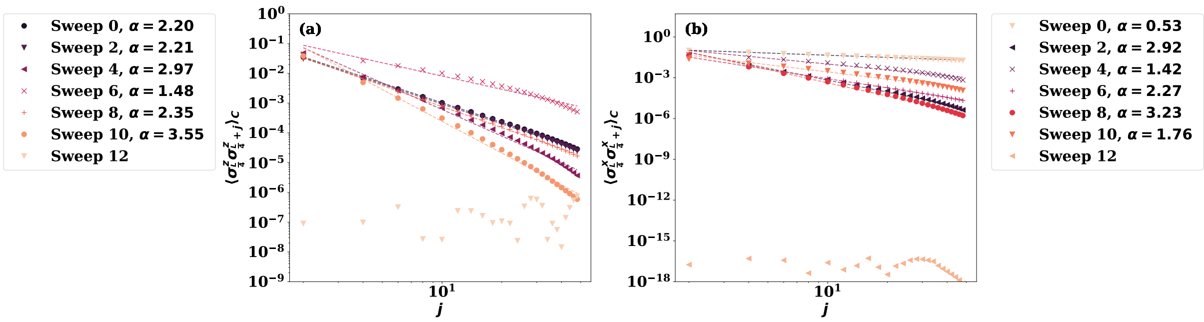

Figure 4: Two-point correlation functions in the critical point of Cluster Ising model, $h = 1$, in log-log scale. Each line corresponds to a sweep of the Disentangling algorithm. The system size is $L = 96$. The lines are fitted with a power-law decay, with the resulting exponent α shown in the labels. The dashed lines correspond to the fits. **(a)**: Decay of the $\sigma^x_{L/4}\sigma^x_{L/4+j}$ correlation function with distance j. **(b)**: Decay of the $\sigma^z_{L/4}\sigma^z_{L/4+j}$ correlation function with distance j.

List of changes

- We double-checked Arxiv references for updates, in particular, we updated Refs.[16,17,48,50,52] and the *Note added* paragraph
- We added a DOI in Refs. [1-4,7-10,12,13,18-23,25,36,40,41,49,58]

- We changed a paragraph in the introduction to clarify the summary of our main results, including a description of the connection between $m_2^{\chi=2}$ and the disentangling power.
- We added a paragraph in Section 2.5 to motivate the choice of employing $\chi = 2$ to measure the magic that cannot be removed by local operation.

References

L. Leone, S. F. E. Oliviero, and A. Hamma, Phys. Rev. Lett. **128**, 050402 (2022).

T. Haug and L. Piroli, Quantum **7**, 1092 (2023).

P. S. Tarabunga, E. Tirrito, T. Chanda, and M. Dalmonte, PRX Quantum **4**, 040317 (2023).