**Response to Ref 1** :

- 1 Referee: Is there any scaling behavior in the time evolution in Fig.2? Since this is essentially a free-fermion calculation, I think the scaling behavior can be obtained if there is.

  Response: We thank the referee for this question. In our manuscript, the inset of Fig.2(b) shows the time evolution of entanglement asymmetry of the second Renyi entropy. The different curves indicate different values of drive amplitude and frequency, while keeping the ratio between drive amplitude and frequency fixed. This fixed ratio is chosen such that the first order effective Floquet Hamiltonian has total magnetization as an emergent conserved quantity. The finite value of entanglement asymmetry in this inset signifies the breakdown of perturbation theory and the first order effective Floquet Hamiltonian would not be useful to understand the physical findings in these regimes. The exact Floquet Hamiltonian in this regime is long-ranged in real space (in contrast to the short range interactions of the Ising/XY Hamiltonian). Therefore we think, there is no simple scaling behavior that could be extracted by simple analytical arguments here as the form of the quasi-particle velocities becomes quite complex in these regimes. We have also checked numerically that this is indeed the case.

- 2: Referee: The results in driven spin chains and in driven CFTs are qualitatively different. Is there any physics to understand this difference? Apparently, the driving protocols are very different in these two cases. But it is not clear to me what essential factors result in this qualitative difference in the entanglement asymmetry evolution.

  Response: The results in driven spin chains are qualitatively different compared to that obtained in the heating phase of the driven CFT. This difference is not due to the difference in the drive protocol. It is, to the best of our knowledge, due to the fact the evolution operator for the driven CFT is described by a non-compact $SU(1,1)$ matrix which has a heating phase where the Casimir has a negative sign leading to monotonic growth of entanglement. This is to be contrasted with the non-heating phase of driven CFTs where the Casimir is positive and entanglement is oscillatory. For spin (XY) chains, the associated group is compact and the Casimir is always positive; there is no analogue of a heating phase for driven spins. We have now added a short discussion clarifying this point in the Section 5 of the paper.

3 The analytical results of entanglement asymmetry in driven CFTs are very interesting. I have a technical question here. In Fig.4, the parameters are chosen as l=100 and L=1000. It is known that in driven CFTs, whether one includes the energy density peak(s) in the subsystem or not will give different features in the von Neumann entropy evolution. For the entanglement asymmetry studied here, I wonder if this choice (with energy peak(s) included in the subsystem or not) is still important.

Response: This feature is connected to the nature of the drive protocol. Such a feature indeed appears if the off-diagonal component of the $2 \times 2$ evolution matrix $U(T, 0)$ is real as shown, for example, in Ref 70. In contrast, for our case, where the off-diagonal elements of $U(T, 0)$ are purely imaginary (or in cases where they are complex numbers), this feature does not arise. Technically, this happens due to the fact that in the latter case both the branches of $(\sqrt{\zeta_m} - \sqrt{\zeta_m^*})^{2h_j}$ yield identical result in the large $m$ limit. We have already discussed this point in the paper following Eq 43.

4 Since the driven CFT can also be analytically studied when the initial state is a thermal state, as has been recently studied in literature, I think it may be interesting to check how the finite temperature affects the entanglement asymmetry in a driven CFT. This may be an interesting future work to study.

Response: This is indeed an interesting topic which, in our opinion, requires a separate study. We thank the referee for this suggestion.