Dear Editors,

We would like to thank both referees for their careful examinations of the manuscript, their positive assessment of our article, and their suggestions which we could use to improve our manuscript. We have implemented all comments and suggested changes in the updated manuscript. Below we provide additional information explaining details of the changes made.

Response to Referee 1
We thank very much the Referee for the very detailed assessment of our manuscript and for their corrections of typos and suggestions. We agree with almost all of the comments made and have revised the manuscript accordingly. For most questions we fully agree with the Referee, we thank them, and we have revised the manuscript accordingly by precisely following the suggestion of the Referee or adding the definition directly below the mentioned equation. In what follows we discuss point by point:

1. **Referee:** In Eq, (3), does the τ_3 multiply ξ_a instead of ξ_s?

   Our response: At the level of BdG-like Hamiltonians time-reversal symmetric contributions come with τ_3, while time-reversal odd contributions come with τ_0, see Eq. (9). This reflects the usual sign change when when going form the electron to the hole representation of the Hamiltonian. In the Lagrangian Eq. (3), and therefore in the action of Eq. (1), the presence of an additional τ_3 reverses the appearance of τ_3 and τ_0 leading to the natural identification of Nambu subblocks with the time-reversed components.

2. **Referee:** Throughout the text, the authors refer to ϕ as the superconducting phase, but in the formulas it reads φ.

   Our response: We thank the Referee for pointing out this typo. We have replaced all instances of ϕ with φ throughout the text.

3. **Referee:** In Eq. (22), there is a matrix σ_a missing in the term parametrized by the tensor χ

   Our response: Thank you for pointing this out. We have added the matrix σ_a in the corresponding term in Eq. (22).

4. **Referee:** Why are Eqs. (23), (59) and (84) different? The superconducting term is either absent or have different forms in the first commutator of the right-hand side of the equation.

   Our response: The Referee is right. The superconducting term was missing. Now it appears in Eqs. (23), (59), and Eq. (84), which in the new version is renumbered to Eq. (85), in the same form, $\hat{\Delta}$.

5. **Referee:** Does the minus sign in Eq. (32) only affects the first term $g\partial_k g$?. If this equation is derived from Eq. (24), it should be like this. The same applies to Eqs. (60) and (85).

   Our response: The minus sign indeed affects only the $g\partial_k g$. We corrected this in all mentioned equations.

6. **Referee:**The authors should provide a reference (or a derivation in the appendices) for Eq. (34).

   Our response: We have added references to two seminal papers (Refs. [77,78] in the new version) before the equation.

7. **Referee:** I have found a discrepancy between Eqs. (42)-(43) and (45)-(46) if Eq. (44) is correct. Is there a factor 2 missing in the term γP of Eq. (44)? In addition, does P in this equation refer to P_z?

   Our response: We thank the referee for indicating this discrepancy. The factor 2 in Eqs. (42-43) was erroneous, we have corrected the equations and modified the text after Eq. (44) to properly define P.

8. **Referee:** The authors should provide a reference for Eq. (47).

   Our response: In the revised version, we refer to Refs. [77, 78] before this equation.

9. **Referee:**In Eq. (48), what is the definition of $\tilde{f}$? I think that it is provided in Eq. (F6), but it should be also given in the main text.

   Our response: We have written explicitly the structure of the matrix $\tilde{f}$ in this equation, in the new version Eq. (49).

10. **Referee:** a) In Eqs. (53) and (54), should $\tilde{f}$ be replaced by f in the first two terms of the right-hand side? b) Should it read $\partial_x \tilde{f}_{\downarrow\downarrow}$, in the second term of the right-hand side?. c) What is the value of expression for σ in these equations?

    Our response: Eqs. (53,54) indeed contained typos that we have corrected in the last version. Specifically, we have changed f into $\tilde{f}$ where necessary, changed $\tilde{f}_{\uparrow\uparrow}$ to $\tilde{f}_{\downarrow\downarrow}$ in the second term and added the subscript D on σ, which reads σ_D.

11. **Referee:** In Eq. (56), it should read $j(-h)^*$ instead of $J(-h)^*$

    Our response: We thank the Referee for pointing out this typo. It has been corrected.

12. **Referee:** According to Eqs. (28) and (29), D should not be a global prefactor in Eq. (60). The same applies to Eq. (61) for the torque and to Eq. (85) for the current.

    Our response: We thank the Referee for pointing out this discrepancy. To unify and simplify notation, after Eq. (22), we clarify that we will focus on the usual case of an isotropic diffusion tensor, i. e. $D_{ij} = D\delta_{ij}$. The generalization to anisotropic diffusion is straightforward. We also corrected the definition of the tensors in Eqs. (28-29) by explicitly extracting D, which ensures consistency with Eqs. (30)-(33) and (58)-(61).

13. **Referee:** In the subsection 'Superconductivity and altermagnets', how are f_s and f_t related to f_0 and f_z introduced after Eq. (49)?. If they are the same, I would recommend the authors to unify the notation.

    Our response: The Referee is right: there is no need to introduce new symbols. We have kept $f_{s,t}$ throughout the text.

14. **Referee:** In Eqs. (67) and (68), there is a $-D$ prefactor missing in front of the $\partial_x f$ terms (see comment 12). In addition, which component of the polarization axis does P_a refer to?

    Our response: After correcting the definitions in Eqs. (28)-(29), D becomes a common factor which cancels out in Eqs. (67) and (68). The polarization axis z is now indicated explicitly.

15. **Referee:** After Eq. (69), what does q_k refer to? Is it the superconducting phase gradient q ?

    Our response: The Referee is correct that q_k should read q here. We have modified the manuscript accordingly.

16. **Referee:** Again, which component does P^a refer to in Eq. (72)? The left-hand side of this equation represents a scalar, whereas the right-hand side represents a vector (indexed by a).

    Our response: All P_a in the mentioned section refer to P_z. We have corrected all of them.

17. **Referee:** In Eq, (74), what does γ_B mean? The Kuprianov-Lukichev boundary conditions are formally introduced in Appendix G and no discussion of them is found in the main text.

    Our response: After Eqs. (74,75) we have explicitly defined γ_B as the K-L boundary parameter, which is proportional to the inverse of the interface resistance. We added a reference to the original K-L paper.

18. **Referee:** In Eqs. (73)—(80) there are issues with the global prefactor D and the value of P^a along the lines of comments 12 and 16.

    Our response: As stated in the above item 16, all P_a's refer to P_z (now corrected). The issue with the prefactor D is resolved by modifying the definitions of Eqs. (28-29).

19. **Referee:** In Eq. (87), the prefactor of the second term should read $(1 - \sigma_z)$

    Our response: We thank the Referee for pointing out this typo. We have corrected it in the new version of the manuscript.

20. **Referee:** In Eq. A15, the third term should contain $\tau_3\sigma_y\sigma_a\sigma_y$ instead of $\tau_3\sigma_a\sigma_y\sigma_a\sigma_y$. The same applies to the third term of Eq. (A19).

    Our response: We thank the Referee for spotting these typos, which have now been corrected.

21. **Referee:** After Eq. (A20), I guess that the authors mean that the tensors are even/odd in spin indices.

    Our response: Yes. the Referee is right. We have correct the sentence after Eq. A20.

22. **Referee:**In Eq. (B7), there are subindices 'j' missing in the partial derivatives of Q appearing in the functional derivatives of the action.

    Our response: The Referee is correct. We have added the missing indices in Eq. (B7)

23. **Referee:** How are Eqs. (C3)-(C5) related to Eqs. (50)-(52)? For example, there is a prefactor 2 appearing on the right-hand side of Eqs. (C3)-(C5) that is not present in Eqs. (50)-(52).

    Our response: We have added the required factor of 2 in front of $|\omega_n|$ in Eqs. (50-52). Next to this, we noticed that a sign of ω_n was missing in the term that couples singlets and triplets in Eq. (50), we have added this factor as well.

24. **Referee:** What is the definition of h_m and $\tilde{h}$ in Eqs. (C3)-(C5)?

    Our response: To avoid confusion we have now decided to keep the strength of the exchange field the same in each region and correspondingly changed $\tilde{h}$ and h_m to h everywhere.

25. **Referee:** After Eq. (C9), is the expression for the superconducting coherence length ξ correct? (for example, $P \to \gamma P$?)

    Our response: The Referee is right. We corrected the coherence length to $\xi = \sqrt{\frac{D(1+\gamma P)}{2|\omega|}}$ and wrote explicitly that is the coherence length in the ferromagnet.

26. **Referee:** In Eq. (C10), which is the effect of the conductivities $\sigma_\uparrow$, $\sigma_\downarrow$ [see Eq. (53)]?

    Our response: We thank the referee for pointing out that the conductivity was missing in our expression. Since we only consider $f_{\uparrow\uparrow}$ the corresponding conductivity in this expression is $\sigma_\uparrow$. We have corrected the corresponding equation.

27. **Referee:** Similar to the comment 12, in Eqs. (E1)—(E4) there are issues with the global prefactor D and the use of the tensors T and K .

    Our response: The modification of Eqs. (28)-(29) in the main text resolve this issue as well.

28. **Referee:**Before Eq. (E20) and in Eq. (E21), the boundary condition should be applied at $y = \pm L_Y/2$, not at $z = \pm L_y/2$

    Our response: We agree. We have corrected the typo.

29. **Referee:** In Eq. (E20) should it read σ_D instead of D?

    Our response: . In Eq. (E20) the appearance of D in the equation $D\partial_{xx}\mu_z^s = 1/\tau^s \mu_z^s$ is correct. Having σ_D there would not be consistent regarding the units.

30. **Referee:** The right-hand side of Eq. (E24) should read $(1/l^s)^2\mu_s^z$

    Our response: The Referee is right. We have corrected the missing r.h.s.

31. **Referee:** Unify the subindex of λ in Eqs. (E38) and (E39).

    Our response: Thanks for spotting this. We have unified teh notation.

32. **Referee:** Check the global prefactor D in Eqs. (F10)—(F13)

    Our response: The modification of Eqs. (28)-(29) in the main text takes care of this issue as well.

33. **Referee:** Are there terms missing in Eq. (F27)?

    Our response: No, there are no terms missing in this equation. The expression for the pair amplitudes in this linearized case only contains a constant term and the hyperbolic sine. We note that here we are only keeping terms of linear order in the altermagnetic tensor.