Question 2

Eq. (22) of the main text is

$$P_+ = \frac{\pi}{8}\exp\left(\frac{-\pi\omega^2}{4}\right)\left|\frac{2}{\Gamma\left(\frac{1}{2}-\frac{i\omega^2}{4}\right)} + \frac{(-1+i)}{\sqrt{2}}\frac{\omega}{\Gamma\left(1-\frac{i\omega^2}{4}\right)}\right|^2 ,$$

where $\omega^2 = E_{th}/E$. The expansion shown in Eq. (23) is

$$\ln\left(P_+(1-P_+)\right) \approx -\ln(4) - \frac{\pi\omega^2}{2} + \frac{\pi\omega^4}{8}(\pi - 4\ln(2)) + \mathcal{O}(\omega^8) .$$

As the amplitude of the oscillations squared is proportional to $P_+(1-P_+)$, we can write

$$-\ln(A^2) \approx \frac{\pi}{2}\left(\frac{E_{th}}{E}\right) - \frac{\pi}{8}(\pi - 4\ln(2))\left(\frac{E_{th}}{E}\right)^2 + C ,$$

which is Eq. (24) in the main text.

In the following plot we can see the relative error (in %) between the expansion and the exact value of $\ln\left(P_+(1-P_+)\right)$. The relative error is smaller than 0.2% in the range of $0 < E_{th}/E < 3$.

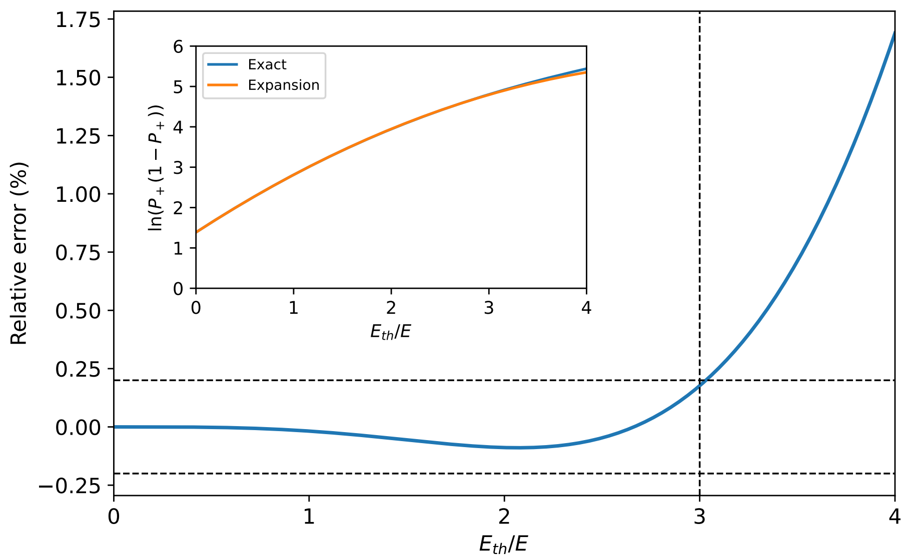

Figure 1: Relative error between the exact expression and the expansion of $\ln\left(P_+(1-P_+)\right)$. Dashed horizontal lines correspond to a $\pm 0.2\%$ relative error. Dashed vertical line is $E_{th}/E = 3$. Inset: Exact values and expansion of $\ln\left(P_+(1-P_+)\right)$.