Response to the Report of Referee-1

Dear Editor,

First, we would like to thank the referee for carefully reading our manuscript and giving valuable feedback. Please find our reply to the comments and the details of the revisions that we intend to make upon resubmission.

1. "*In the third paragraph of the introduction, Ref.[30] was not the first example of computing shadow correlators for celestial amplitude. Actually Ref.[20] provided the first example for four-point. I invite the authors revise the references for that sentence.*"

   Response: We will change the reference as suggested by the referee upon resubmission.

2. *"Above Eq.(2.3), the expression for the holomorphic conformal weight holds for the scalar case but not for the spinning case as the authors explain in Eq.(2.6). I would recommend the authors to revise the sentences above Eq.(2.3).*"

   Response: We will also change it upon resubmission.

3. *"The quadratic gravity is introduced from section 3. It might be useful to discuss a bit more about the basic properties of the quadratic gravity. Including some useful references might be helpful. For example, it might be helpful to explain how the mass of the massive mode of it is related to the parameters in Eq.(3.1). And is it clear that the amplitudes computed in quadratic gravity are unitary?"*

   Response: We thank the refree for rasing the question. We intend to add this discussion after Eq. (3.4): "Before proceeding to the main discussion, we begin by outlining several foundational aspects of quadratic effective field theories (EFTs) of gravity. It is well understood that quantum corrections to Einstein gravity generically induce higher-derivative terms, including those quadratic in the curvature, such as R^2. This naturally motivates the study of such terms at the level of the microscopic (bare) action, providing a framework amenable to consistent perturbative renormalization. The inclusion of curvature-squared contributions in classical gravity was first proposed in [1], and their role in rendering gravity power-counting renormalizable was subsequently demonstrated in [2], with full renormalizability to all orders in perturbation theory established in [3].

   Despite these notable achievements, higher-derivative gravitational theories are often regarded as problematic due to issues related to unitarity. This concern becomes manifest at the level of the tree-level propagator (cf. Eq. (3.2)), where the ultraviolet behavior improves from a $1/q^2$ to a $1/q^4$ fall-off, yet this improvement introduces a massive spin-2 ghost with mass $m = \sqrt{\kappa/\alpha}$, signaling a breakdown of perturbative unitarity. More specifically, while it is possible to quantize the theory such that all excitations have positive-definite energy, this necessitates the inclusion of negative-norm (ghost) states in the Hilbert space. As shown in [4], such states inevitably violate unitarity and cannot be consistently projected out without sacrificing the unitarity of the S-matrix. Therefore, despite the theory's favorable UV behavior, its physical consistency remains fuzzy in the absence of a resolution to the unitarity problem (see the Appendix of [3] for a detailed discussion).

However, as emphasized in [5], unitarity is fundamentally a dynamical property, and cannot be definitively assessed within the confines of a purely perturbative treatment or at the level of the tree-level propagator alone. A comprehensive analysis requires accounting for loop corrections and potentially non-perturbative effects at fixed energy. As we will demonstrate in the following sections, the eikonal scattering amplitude constructed within a bottom-up approach—non-perturbative in the coupling—is manifestly unitary by construction.

Nevertheless, more recent analyses [6–8] have shown that Einstein gravity, when treated as a (complete) quantum theory, violates tree-level unitarity in the high-energy limit, failing to satisfy the unitarity bound $|M(s,t)| < C$, where C is a finite constant. In particular, in the Regge limit, the amplitude in general relativity exhibits unbounded growth, scaling as $|M(s,t)|_{\mathrm{GR}} \sim O(s^1) \to \infty$. In contrast, the same studies demonstrate that scalar scattering mediated by graviton exchange in quadratic gravity satisfies unitarity bound at high energies, despite the presence of negative-norm (ghost) states. This result is in accord with the expectations of the Llewellyn Smith conjecture [9], which heuristically suggests that renormalizable quantum field theories should also exhibit unitary behavior.

In addition, our explicit computation of the tree-level scattering amplitude in quadratic gravity yields the following structure:

$$M(s,t) \sim \frac{s^2}{t} + O(s^0), \tag{1}$$

where the former term arises from the Einstein-Hilbert (GR) sector, while the later one originates from the quadratic curvature corrections in the effective action. When the theory is treated as an effective field theory, the eikonal limit $t/s \to 0$ remains well-defined by taking the momentum transfer t to be sufficiently small and keeping the center-of-mass energy s bounded by the EFT cutoff Λ. Moreover, it is straightforward to observe that the quadratic curvature contributions exhibit improved high-energy behavior compared to the GR term. Consequently, there is no apparent violation of the unitarity bound—at least within the regime where the theory is interpreted as a low-energy effective description of some UV-complete quantum gravity theory.

Under this interpretation, Einstein gravity treated as an effective field theory (EFT) remains consistent with unitarity, as the amplitude can not grow arbitrarily. Importantly, the constant contribution from the higher-derivative terms does not introduce any violation of the unitarity bound; that is, one finds

$$|M(s,t)| \leq f(\Lambda) + O(s^0), \tag{2}$$

for some finite function $f(\Lambda)$.

In summary, when quadratic gravity (or Einstein gravity) is treated as a low-energy EFT, unitarity of the scattering amplitude is not much problematic (e.g in the S-matrix bootstrap program the goal is to put bound on the EFT coupling (α, β) assuming the existence of high energy UV completion which is causal and unitary). This contrasts with the situation where one attempts to promote the theory to a full UV-complete quantum field theory, where issues such as ghost modes and associated violations of unitarity may become significant. "

4. *"The sentence below Eq.(3.11) is confusing. From Eq.(3.10), the Einstein gravity term is still present with the distributional nature. The EFT correction term behaves better but the entire celestial amplitude is not improved by that."*

Response: We thank the referee for pointing out the unintentional misleading statement. In perturbative approach the distributional nature coming from GR part is inevitable. We will change the line appropriately.

5. *"In Eq.(3.15), the authors computed the eikonal phase in quadratic gravity by perform Fourier transform on the modified Born amplitudes, following the same procedure in GR. It might be useful to explain why in the presence of the EFT corrections, the eikonal phase can still be computed in the same way as GR. Or are there other similar examples that existed in the literature can justify this point. "*

Response: We appreciate the referee's insightful question. This question is partially addressed in point 3, but we offer a more intuitive explanation here. The explanation goes as follows: The validity of eikonal exponentiation in quadratic gravity can be addressed by considering the structure of gravitational scattering amplitudes in the high-energy, small-angle (eikonal) regime. In this limit, the dominant contributions arise from the exchange of soft, long-wavelength gravitons. Crucially, this infrared (IR) behavior is governed by the long-distance propagation of the gravitational field, which remains dictated by the Einstein-Hilbert term—even in the presence of higher-curvature corrections such as R^2, $R^2_{\mu\nu}$, or R^3. These UV modifications primarily affect the short-distance (non-eikonal) part of the interaction and do not interfere with the leading IR dynamics responsible for exponentiation.

Within the effective field theory framework, and under the assumption that the higher-derivative couplings (e.g., α, β) are large compared to the inverse energy scale of interest (which is m_p), the eikonal exponentiation is expected to persist. This expectation is supported by the IR universality of the gravitational interaction, and it follows the logic that the eikonal resummation is dominated by ladder diagrams with soft graviton exchanges.

We acknowledge, however, that a full demonstration of exponentiation in quadratic gravity would require explicit computation of loop-level diagrams, particularly the sum of multi-graviton ladder diagrams. We intend to explore this in future work. This was first shown for Einstein gravity in the seminal work of 't Hooft [10] and then by others (see, e.g., [11]). Apart from gravitational field theories, for QFTs involving *massive* (relevant for our case) meson exchange the ladder structure can been nicely be identified and the S-matrix exponentiates [12]. There, the resummation of eikonal ladder diagrams leads to the well-known exponential form of the amplitude:

$$A_{\text{eik}}(s, q_\perp) \sim 2s \int d^2b\, e^{iq_\perp \cdot b} \left(e^{i\chi(s,b)} - 1\right), \tag{3}$$

where $\delta(s,b)$ is the eikonal phase and is given by,

$$\chi(s,b) \sim \int d^D q\, \delta(2p_1 \cdot b)\delta(2p_2 \cdot b)A_{\text{tree}}(s,-q^2), \text{ with, } q^\mu = p_1^\mu - p_1^{\mu\prime}, b^\mu \text{ is the impact parameter.} \tag{4}$$

Recent review [13] discussed that this structure holds not only in General Relativity but also in UV-complete frameworks such as string theory. Given this, it is natural to expect that intermediate theories like quadratic gravity—lying between Einstein gravity and UV completions—should also admit eikonal exponentiation, at least in the IR regime. We refer specifically to Section 3.1.5 of [13], which supports this viewpoint.

While a dedicated study of loop-level exponentiation in quadratic gravity is indeed warranted, we believe the arguments above (as well as supported by our bottom-up computation presented in the manuscript) justify the assumption of exponentiation within the eikonal limit of the effective theory.

We intend to add these remarks in the beginning of Section (3.2) as well as mention this as one of the future directions in the discussion section upon resubmission.

6. *"Although the authors presented a nice discussion on the delta function on page 11, it is not very clear how the author obtained the (e-1) factor in Eq.(3.22). I hope the authors could explain a bit more below Eq.(3.22). "*

   Response: As explained in the manuscript, the analytic linearization suggested that,

$$f(\delta_a) \rightsquigarrow \mathbb{I} + \sum_{n=1}^{\infty} \frac{f^{(n)}(a)}{n!}\, \delta_a. \tag{5}$$

   Now for the exponential map,

$$e^{\delta_0} \rightsquigarrow \mathbb{I} + \sum_{n=1}^{\infty} \frac{f^{(n)}(\delta_0 = 0)}{n!}\, \delta_0 = \mathbb{I} + (e-1)\delta_0, \tag{6}$$

   where, we have used the fact that, for $f(\delta_0) = e^{\delta_0}$, $\sum_{n=1}^{\infty} \frac{f^{(n)}(0)}{n!} = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} - 1 = e - 1$. Please see Example 3.2.1.5 of [14].

   We will add the discussion below (3.22) as suggested by the referee.

7. *"In Eq.(4.11), the authors did not thow the contribution from GR. I am wondering if the authors had tried to compute it. If it is not doable for general conformal dimensions, one might try to take some specific limit of the conformal dimension. See, e.g. 2501.05805. "*

   Response: The GR contribution can indeed be done exactly. However we did not show it explicitly as our primary focus is on the higher curvature corrections to it. We briefly show the derivation here.

$$
\begin{aligned}
&\text{(contribution from GR)}\\
&= \delta_2(\gamma)\int_1^\infty \frac{dz}{(z-w)^{2-\Delta_2}(z-\bar{w})^{2-\Delta_2}}\frac{(z-1)^{\frac{\Delta_1-\Delta_2-\Delta_3+\Delta_4}{2}-1}}{|z|^{\Delta_1+\Delta_2-3}}\\
&\xrightarrow{z\to 1/z} \delta_2(\gamma)\int_0^1 \frac{dz}{(1-z\,w)^{2-\Delta_2}(1-z\,\bar{w})^{2-\Delta_2}}(1-z)^{\frac{\Delta_1-\Delta_2-\Delta_3+\Delta_4}{2}-1}z^{\frac{1}{2}(\Delta_1-\Delta_2+\Delta_3-\Delta_4)}\\
&= \delta_2(\gamma) B\left(\frac{1}{2}\left(\Delta_1-\Delta_2+\Delta_3-\Delta_4+2\right),\frac{1}{2}\left(\Delta_1-\Delta_2-\Delta_3+\Delta_4\right)\right)\\
&\quad\times \mathrm{F}_1\left(\frac{1}{2}\left(\Delta_1-\Delta_2+\Delta_3-\Delta_4+2\right),2-\Delta_2,2-\Delta_2,\Delta_1-\Delta_2+1,w,\bar{w}\right)
\end{aligned}
\tag{7}
$$

We will add the derivation appropriately in the revised version.

8. *"In Eq.(5.1), the physical meaning of n in the second line seems to be related to the number of particles. It might be helpful to clarify it. "*

Response: We changed and removed the typo 'n'. Though it means the number of particle participated in the scattering event, it should be scaled out according to Eq. (2.8) in the manuscript. We will correct it accordingly in the manuscript.

We hope that our response has answered the queries raised by the referee.

Best,
Authors

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