Response to Referee 1

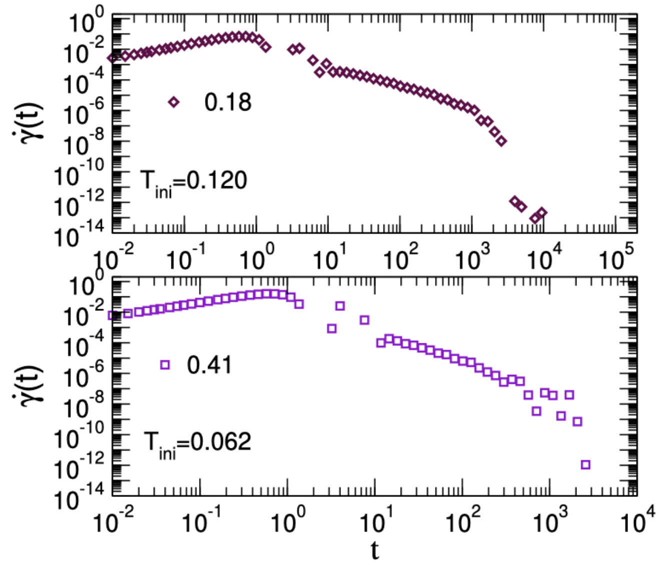
We thank the referee for the positive assessment of our work and for the insightful comments. Below, we respond to the received feedback and questions.

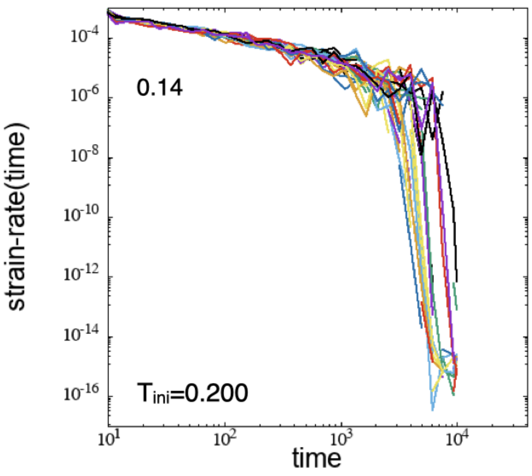
> *(A) Initial Growth of Strain Rate:*
> *The manuscript does not mention the initial growth of the strain rate. What is the origin of this*
> *growth phase? Is it related to an initial elastic response or transient effects?*

The initial growth of the strain rate at very short times is determined by the time evolution of the stress-control algorithm (barostat). In this paper, we use Eq. (1), which governs the second derivative of the strain (or the first derivative of the strain rate) under constant shear stress. Because the equation is second-order in time, the strain grows ballistically as t^2 at early times, so the strain rate itself increases linearly with t. Since typically the averaged shear stress is zero in the thermally quenched amorphous state for sufficiently large system sizes, the early time behaviour of the strain rate scales as $\sim B t \sigma$. Our simulations confirm this behavior. This aspect of the early time dynamics has been earlier discussed in Cabriolu et al, Soft Matter 2019, 15, 415, noting that the focus is on much longer timescales, namely, the creep and, later, fluidization regimes. Also, it has been earlier shown [Sentjabrskaja et al, Scientific Reports 5 11884 (2015)] that the microscopic dynamics relates to the macroscopic strain, in which case the initial growth of the strain rate is connected to the particles' exploration of the cage of neighbours.

> *(B) Power-Law Decay Below :*
> *The authors report that for , the strain rate follows a power-law decay with an exponent*
> *expected to be independent of annealing. However, the results indicate a very low exponent in*
> *poorly annealed samples, even below unity. This cannot be the long-time decay discussed in*
> *Popović et al., since the total strain is the time integral of the strain rate and would diverge for*
> *. From Figures 1a and 1b, it seems that at long times, the decay becomes faster. It would be*
> *interesting to check whether this corresponds to the expected value or if it transitions to an*
> *exponential decay, as mentioned for the short-time dynamics in the supplementary information*
> *of Popović et al.*

As the referee correctly comments, a strain-rate decay of the form $\sim t^{-\nu}$ with $\nu<1$ cannot persist forever, because integration would give an unbounded strain. Consequently, the observed power-law behaviour is necessarily transient and must cross over to a different regime at longer times, which is clearly visible in Fig.1a-c. Such sub-unity exponents are, however, often reported, both experimentally and numerically, across a broad range of amorphous materials, even when the applied stress lies below the macroscopic yield threshold (see, for example, Refs.[9–17] in Popović et al. PRL 2022). Ultimately, for athermal systems, the strain rate vanishes as the material jams and the strain reaches a plateau; see experimental data in Fall et al PRL 2009. In numerical simulations, it is very difficult to conclude on how the strain rate vanishes due to potential finite size effects whereby the shear-rate peels off from the power-law regime and suddenly gets arrested, as has also been observed in other contexts of onset of jamming. We illustrate this via the figure below, where we show the averaged data for the more annealed states and the trajectories for the least annealed states, corresponding to the cases where the system gets arrested at long times. In real materials, there could be subsequent evolution with processes involving structural aging and the relaxation of residual internal stresses (see, e.g., Lidon, Villa, and Manneville, Rheologica Acta 2017). We have added a brief comment on this point in the revised manuscript.

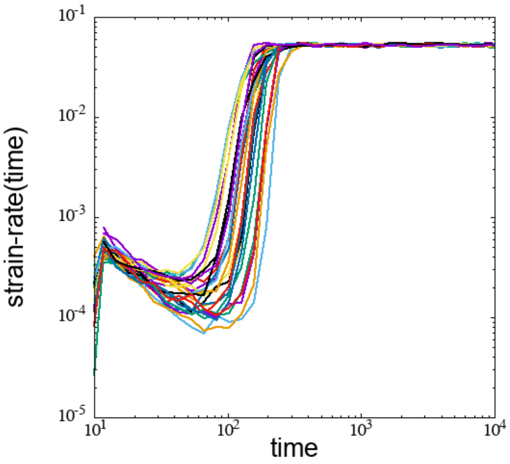


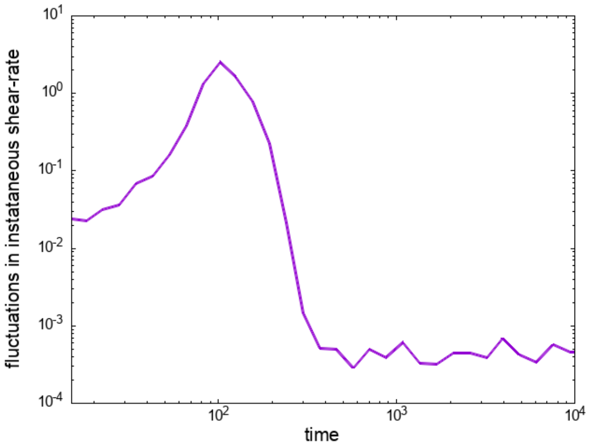


>(C) Divergence of Failure Time Near : The time to failure diverges as . I assume that
> determining the exponent is challenging due to sample-to-sample fluctuations in for a given
> annealing degree. Could the authors provide an estimate of these fluctuations in the case of
> the stable glass?

As the referee correctly points out, extracting the fluidization time and the corresponding exponent becomes increasingly challenging for highly annealed glasses. When the imposed stress is well above the threshold stress, the fluidization time can be determined relatively easily. As the stress approaches the threshold, however, sample–to–sample fluctuations grow; in fact, in the vicinity of the threshold, some samples eventually fluidize, whereas others get arrested. The magnitude of these fluctuations indeed increases with the degree of annealing, as the referee anticipated. To mitigate this issue, we estimate the fluidization time from the mean strain rate curves.

Below, to illustrate the heterogeneity, we show on the left the plot of the strain-rate curves from the most annealed ensemble (T_ini=0.062) that we have, for an imposed stress of 0.44 (which is the least value of stress for which all the initial states attain steady flow at long times). A measure of the heterogeneity within the ensemble can be obtained via the time-dependent fluctuations, viz. variance/(mean)^2, as is shown in the plot, on the right, below.


strain-rate(time)
10-1
10-2
10-3
10-4
10-5
10¹
10²
10³
10⁴
time


fluctuations in instataneous shear-rate
10¹
10⁰
10-1
10-2
10-3
10-4
10²
10³
10⁴
time