Experimental protocol for observing single quantum many-body scars with transmon qubits

Peter Græns Larsen[1⋆], Anne E. B. Nielsen[1†], André Eckardt[2‡], Francesco Petiziol[2∘]

1 Department of Physics and Astronomy, Aarhus University, DK-8000 Aarhus C, Denmark
2 Technische Universität Berlin, ~~Institut für Theoretische Physik~~Institut für Physik und Astronomie, Hardenbergstr. 36, 10623 Berlin, Germany

⋆ peter-26-03@live.dk † annebn@phys.au.dk ‡ eckardt@tu-berlin.de ∘ f.petiziol@tu-berlin.de

Abstract

Quantum many-body scars are energy eigenstates which fail to reproduce thermal expectation values of local observables, in systems where the rest of the many-body spectrum fulfils eigenstate thermalization. Experimental observation of quantum many-body scars has so far been limited to models with multiple scar states evenly spaced in energy. It is thus an interesting question whether even *single* isolated scars, which theoretically embody the weakest possibile violation of eigenstate thermalization and may be thought to have no detectable impact in experiments, can leave a trace in measurable quantities. Moreover, single scars offer an interesting scenario for exploring the connection between quantum many-body scars and the original notion of scarring in quantum dynamical systems theory. Here we propose protocols to observe *single* scars in architectures of fixed-frequency, fixed-coupling superconducting qubits. We first adapt known models possessing the desired features into a form particularly suited for the experimental platform. We develop protocols for the implementation of these models, through trotterized sequences of two-qubit cross-resonance interactions, and verify the existence of the approximate scar state in the stroboscopic effective Hamiltonian. Since a single scar cannot be detected from coherent revivals in the dynamics, differently from towers of scar states, we propose and numerically investigate alternative and experimentally-accessible signatures. These include the dynamical response of the scar to local state deformations, to controlled noise, and to the resolution of the Lie-Suzuki-Trotter digitization.

Contents

1 Introduction

The discovery of quantum many-body scars (QMBSs) has represented an important twist in the theory of thermalization of isolated quantum systems, with potential applications envisioned in areas like quantum metrology [1,2] and quantum information processing [3].

~~QMBSs are energy eigenstates that do not give rise to thermal expectation values in otherwise generic quantum many-body systems with almost all other energy eigenstates obeying eigenstate thermalization~~ [4–6]. ~~There, they weakly violate the eigenstate thermalization hypothesis~~ [7,8]. ~~They are typically highly excited energy eigenstates characterized by low bipartite entanglement entropy compared to other states at similar energies.~~

Generic quantum many-body systems are believed to fulfill the eigenstate thermalization hypothesis (ETH) [7,8], according to which any eigenstate with finite energy density should yield thermal expectation values of local observables. This behaviour would thus reconcile the reversibility of isolated quantum dynamics with the predictions of equilibrium statistical mechanics. Known exceptions are integrable and many-body localized [9–12] systems, with the latter being still under debate [12,13]: these systems strongly violate the ETH, in the sense that even all their eigenstates could avoid thermalization, due to the presence (or emergence, in the case of many-body localization) of an extensive number of conservation laws. As an intermediate scenario, weak forms of ETH-violation have been identified [4–6], where only a few eigenstates violate the ETH, in a number which is exponentially smaller than the Hilbert-space size. This is the case of QMBSs, which are thus excited eigenstates that do not give rise to thermal expectation values in otherwise generic quantum many-body systems. QMBSs are also related to the more general phenomenon of Hilbert-space fragmentation [14], where the Hilbert space dynamically decomposes into exponentially-many disconnected subspaces with the system size, which are not associated with trivial local symmetries.

~~Many scarred models feature a tower of scar states which have equidistant energies and can lead to revivals in the time evolution of certain initial states. Towers of scar states are, however, not necessarily present in all scarred models. Building on the tensor-network framework~~ [15]~~, we can construct models either with a tower of scar states or with a single scar. It has been shown that a scar-state phase transitions can take place in such a model with a single QMBS~~ [16]. ~~This phenomenon exhibits both similarities with standard ground-state phase transitions, despite taking place in a highly excited state, and novel features. QMBSs have been observed experimentally in systems with towers of scar states through the revivals of certain initial states~~ [17–19]. ~~However, so far there has not been an experimental observation of a single QMBS.~~

Different theoretical mechanisms have been identified, which lead to the emergence of quantum many-body scarring and which explain the rich landscape of QMBSs that has been discovered. The first phenomenology of QMBSs encountered is as a set of low-entropy eigenstates with equal energy spacing, dubbed towers of states. Towers of QMBSs have been understood to appear, for instance, in models possessing (potentially approximate) so-called spectrum generating algebras [20], where the tower can be constructed by the repeated action of a ladder-type eigenoperator of the Hamiltonian on a suitable eigenstate. The tower of states can in this case be interpreted as the creation of quasiparticles which are non-interacting (due to the equal energy spacing) on top of a given state [6,21]. A key feature of towers of QMBSs, which enabled their observation in experiments [17–19], is that they can produce recurrences in the dynamics from certain simple initial states, e.g., in the probability of return to the initial state (Loschimdt echo). This is a consequence of the homogeneous level spacing across the tower, which makes possible for the amplitudes of the initial state on the states of the tower to interfere constructively at certain times. Besides models with emergent towers of states, models with an arbitrary number of QMBSs can also be designed by “manually inserting” scars at the heart of the spectrum. This can be achieved, for example, by means of so-called projector-embedding methods [15,16,22], often combined with matrix-product-state (MPS) formulations [23]. This approach highlights that the phenomenon of quantum many-body scarring is not necessarily associated with towers of states, but isolated QMBSs can also exist. Examples include models where individual scars are annihilated by a set of quasilocal operators [16,22], similarly to the ground states of frustration-free models, as well as frustrated [24,25] and topologically ordered [26–28] systems.

While towers of scar states can be diagnosed from the coherent recurrences in the dynamics, as discussed above, individual QMBSs cannot give rise to such an effect and have so far not been observed. The possibility to observe a single scar is, however, an interesting challenge for several reasons. First of all, QMBSs are by definition hard to detect, being excited states immersed in a thermal crowd, whose existence often relies on fine-tuned conditions. A natural question is, then, how weak can the violation of ETH be for actually representing an experimentally relevant and detectable phenomenon: can the ETH be violated even at the single-state level? It has further been shown that individual QMBSs can exhibit novel phenomena, such as “scar-state phase transitions” [16], where a QMBS embedded in the middle of the spectrum undergoes a phase transition while remaining a QMBS as a parameter is changed.

Finally, at an even more fundamental level, single QMBSs could be seen as closer counterparts to the original idea of quantum scarring in quantum dynamical systems [29,30], as opposed to towers of scars. Indeed, from the perspective of quantum chaos theory, the presence of towers of equally-spaced and localized states and the related dynamical recurrences may emerge also from the quantization of locally stable orbits of the corresponding classical system associated with invariant Kolmogorov-Arnold-Moser tori [31]. It is an interesting observation that for a single quantum many-body scar this scenario can be excluded. The remarkable feature of quantum scarring in its original formulation is, actually, that a weak form of eigenstate localization is possible even in the absence of locally-stable orbits [29]. However, such a behavior is generally difficult to prove in quantum many-body systems, which usually do not have a well defined classical analog. Thus, the absence of a tower of states, which might be related simply also to analogs of locally stable KAM-tori, might provide an indication of behavior that is closer to the original idea of quantum scarring.

In this work, we put forward a protocol to experimentally observe the single scar state predicted in Ref. [16]. We first explore candidate models to identify Hamiltonians that are closer to state-of-the-art experimental capabilities. In the models selected, the scar state corresponds to certain known states whose preparation has been already achieved (but not as scars) in experiments, in particular a 1D cluster state [32] and an x-polarized state. The key interactions required to realize the corresponding parent Hamiltonians are two-qubit Pauli couplings of the form $\sigma_i^z\sigma_j^x$ between neighbouring qubits i and j. Interactions of this type appear “natively" in fixed-frequency superconducting qubits through the so-called cross-resonance effect [33–36]. They are indeed employed as the building block of entangling gates in such platforms, such as in IBM Quantum's devices [37]. Given the direct availability of this key element, we propose an experimental realization in an array of fixed-frequency, fixed-coupling transmon qubits. Nonetheless, the protocols developed remain applicable also in other qubit platforms. The different model Hamiltonians considered will then be realized through Lie-Suzuki-Trotter sequences of modified cross-resonance drives.

~~While towers of scar states can be diagnosed from the coherent revivals in the dynamics arising from their interference, this method cannot be used for a single scar,~~ As discussed above, the experimental observation of single scars cannot rely on coherent recurrences, differently from towers of states, and thus requires novel approaches. We propose different signatures which take advantage of the accurate single-site control and measurement available in the circuit QED toolbox. In particular, we combine controlled trotterized dynamics with the possibility to measure certain local observables as well as (Rényi) entanglement entropies. Moreover, we analyse the impact of (controlled) noise and digitization errors as a probe of the instability of the scar to thermalization.

The conceived signatures aim at probing general characteristics of a scar state, namely that (1) it is an energy eigenstate, (2) it has low entanglement entropy, and (3) most states near the same energy have entanglement entropies expected for thermal states. Showing these three characteristics experimentally provides convincing evidence of a single scar state. To do so, we propose to prepare the scar state as an initial state and time evolve it with the digital sequence implementing the corresponding parent Hamiltonian for some time. Then, one measures its entropy and the expectation value of a local operator, for which the scar state has some known non-zero value. If, after the time evolution, the expectation value is close to the known value and the entropy is still low, then (1) and (2) are satisfied. If the scar state is an exact eigenstate of the Hamiltonian implemented (more precisely, a Floquet mode of the Lie-Suzuki-Trotter sequence), it will remain stationary during the time evolution. To also show (3), we propose to repeat the time evolution and measurements on a locally deformed version of the scar state. A local deformation will not change the (quasi-)energy much, assuming a local Hamiltonian, but the deformed scar state will have support on many thermal states which are nearby in energy, leading to quick thermalization (high entropy and different expectation value). Comparing the time evolutions of the scar state and its deformed version can thus be used as an experimental signature of the scar state. This method also takes advantage of the fact that scar states are generally efficient to prepare in experiments, unlike generic thermal states.

In Sec. 2 we briefly review the model of Ref. [16] and write down the parent Hamiltonians of the x-polarized and cluster states. We propose a blueprint for an experimental implementation of these two parent Hamiltonians in Sec. 3. In Sec. 4 we numerically test if the scar state can be detected by our proposed method and we check how robust the results are to a random error as well as propose ways to probe the scar's sensitivity. Finally, in Sec. 5, we summarize our results and discuss potential further developments.

2 Parent Hamiltonians

In Ref. [16], a family of 1D spin-1/2 models has been proposed which features a single zero-energy scar state $|S\rangle$, based on a matrix-product-state construction. The Hamiltonian is built