(1) A quantitative benchmark would strengthen the manuscript—for instance, identifying the array size or regime at which the proposed method surpasses AOD-based rearrangement in transport speed. A direct performance comparison with the GS algorithm would also be valuable in demonstrating the method's practical benefits.

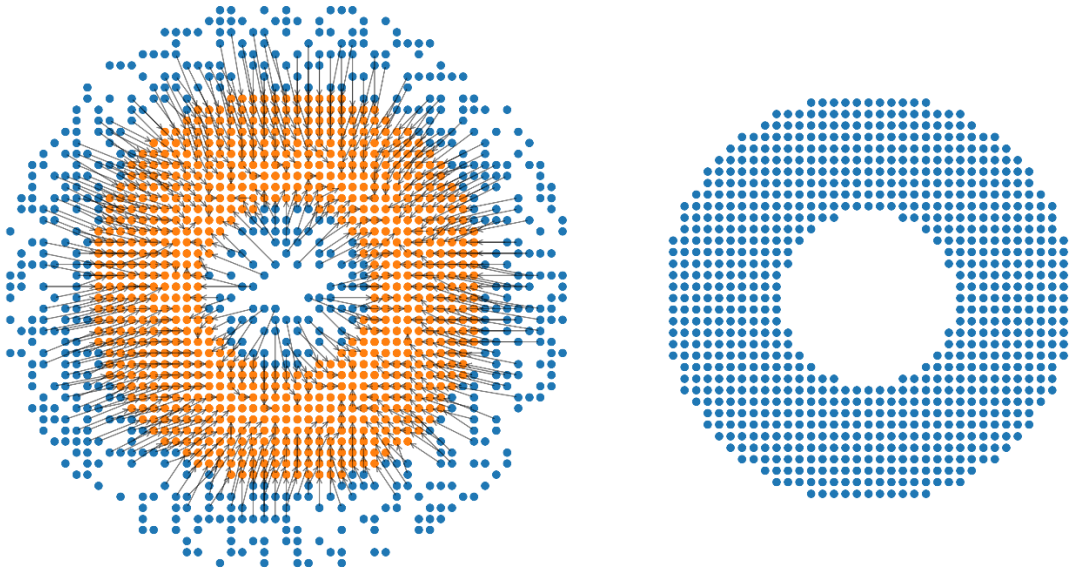
This is indeed a valid point worth discussing. Unfortunately, a detailed comparison of total sorting time between the LPI method and AOD-based methods depends on the total number of holograms and the number of AOD tweezers that are used in parallel. We have added a small paragraph in the "Discussion and outlook" where we compare our method with two explicit AOD examples:

- Prichard et al., Phys. Rev. Applied **22**, 024073 (2024) | This is to our knowledge the largest rearrangement to date. It took 1.65s to rearrange 848 atoms using a single AOD tweezer.
- Manetsch et al., ArXiv, e-print available at https://arxiv.org/pdf/2403.12021 (2024) | This is, to our knowledge, the most ambitious proposal that aims to rearrange 6100 atoms with 4 pairs of AODs steering up to 64 tweezers each.

Below we give more details on these comparisons, which we consider too much detail to include in the paper.

For comparison, we calculated the same initial and final geometries, and computed the maximal distance that an atom would need to travel, after obtaining a mapping with the Jonker-Volgenant algorithm. The maximal distance sets the number of holograms and therefore the total rearrangement duration in our method. The number of holograms per lattice period is varied between 6-10, depending on the reported tweezer spacing and waist.

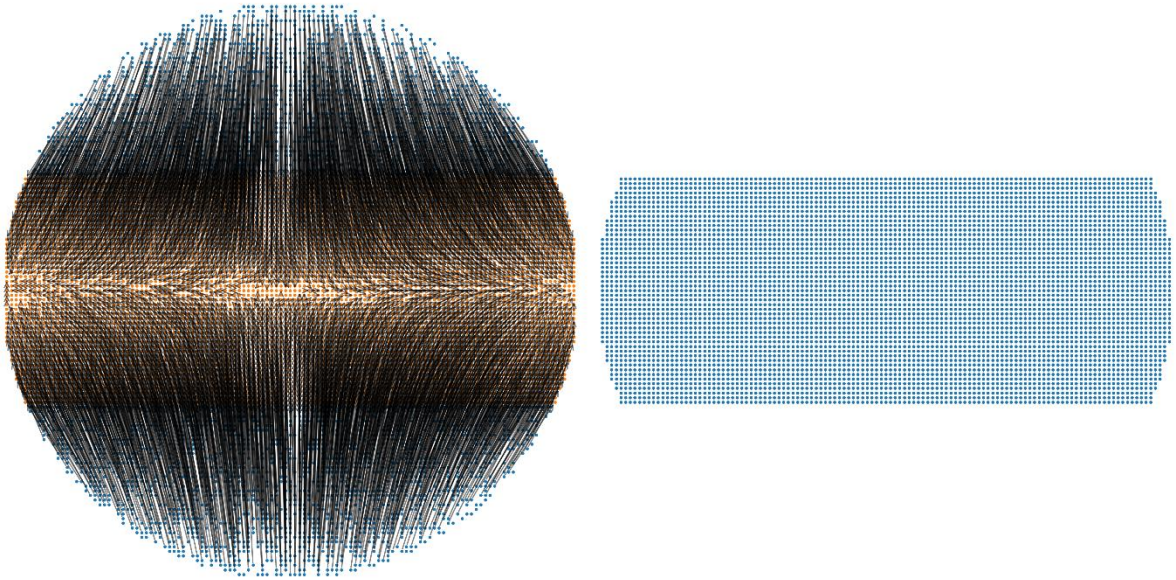
Prichard et al. (2024)



The maximal path length obtained in this geometry is around 6 lattice periods. The reported tweezer spacing is 3.7um. With our current Fourier unit size, this means travelling a lattice period takes 8 holograms. With our current speed of 2.7ms per pattern, that means 6*8*2.7ms = 130ms, which is an order of magnitude faster than the reported value.

Manetsch et al. (2024)

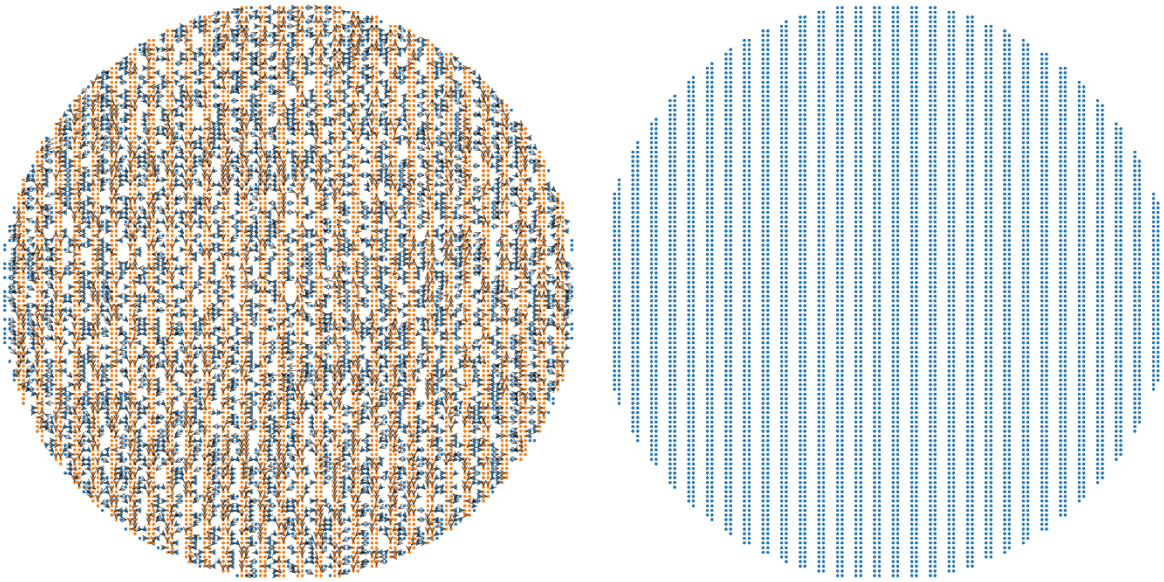
In this work, 6100 atoms are proposed to be rearranged. The proposed method of many AODs could be done per quadrant (520ms) or in parallel (140ms).



Copying the same target and end geometry, we obtain a longest move of around 37 lattice periods. Assuming that we could move as much as a Gaussian waist per

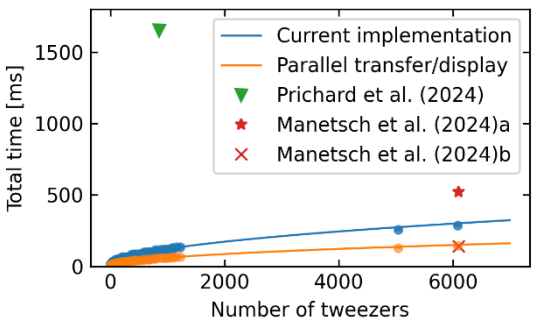
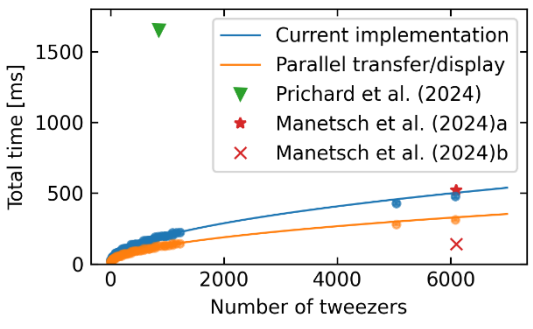
hologram, we need 6 holograms per period, leading to 37*6*2.7ms = 600ms for the full rearrangement. This is similar to the 'per quadrant' case.

At this point, it is worth mentioning that the geometries here are very unfavorable for the proposed SLM rearrangement method, because of the long path lengths. For example, having several reservoir sites around each target site could drastically shorten the longest move. Placing an extra reservoir site diagonally next to every target site, we observe only ~1.5 lattice period maximal length. Note that the 0.5 is due to the reservoir being in between two target sites. Alternatively, one could sort into columns pairs:



This idea reaches 3 moves, maximally, which is only about 48ms.

Having concluded that the geometry is very important for the scaling of SLM rearrangement, we consider sorting square tightly-packed arrays of NxN into MxM. As stated in the main text, the number of holograms scales proportional to M. See the figures below. The dots are averages of 100 realizations and the solid lines are fits of an sqrt(MxM) dependency.

Distance 6 Fourier units per lattice site	Distance 10 Fourier units per lattice site
	

For the blue data and lines, we use a 2.7ms per hologram timing as we report in the paper, while for the orange data, we use 1.77ms, which is to be expected if the memory transfer and display are parallelized in separate threads. For the left plot, we use a distance of 6 Fourier units per lattice site and for the right plot, we use a distance of 10 Fourier units. In both figures, we also included the experimental results from Prichard et al. (2024) (in green triangle) and the predictions from Manetsch et al. (2024) (red cross and star).

The proposed method outperforms the current experimental work by an order of magnitude, and is similar to the theoretical predictions in Manetsch et al. (2024). We note that our setup is significantly simpler. It is a single SLM, while to obtain the fully parallel speed of 140ms in Manetsch et al. (2024) one needs an SLM, four pairs of AODs with in total 4x64 tweezers sorting at a time.

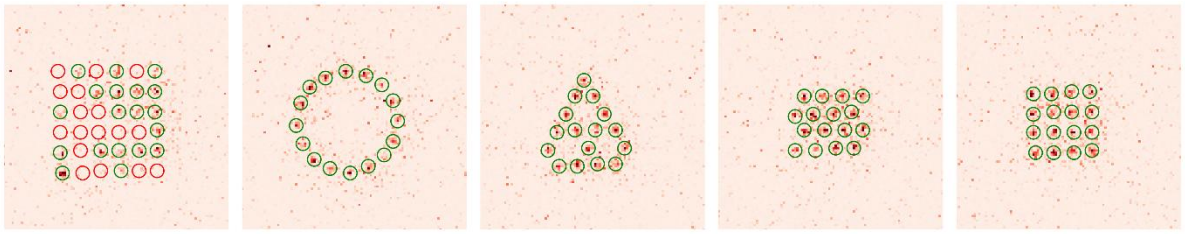
Concluding, it is hard to give a definitive answer to this question because it depends on several choices. We think a careful discussion of all these choices takes considerable time and wording and is beyond the scope of the current work.

We have tried to reflect these deliberations in the first paragraph of the Discussion and Outlook by rephrasing it as:

“Looking forward, the LPI method presented here shows promise for rearranging large tweezer arrays into arbitrary geometries. The reported success probability of rearrangement and imaging of >0.99 in this work is competitive with AOD sorting methods [1, 10, 14]. Under the assumption that fewer than a few tens of holograms are needed, the LPI method can rearrange atoms in tens of milliseconds. This is comparable to the total rearrangement duration with existing AOD sorting methods in arrays of a few hundred atoms [1, 2, 16]. A detailed comparison of total sorting time between the LPI method and AOD-based methods depends on the total number of holograms and the number of AOD tweezers that are used in parallel. For example, using eight holograms per lattice period, the current LPI implementation could rearrange the same arrays as [8] in $\sim$ 130 ms, which is an order of magnitude faster than their single-AOD demonstration. Assuming six holograms per lattice period and parallelizing the display of each hologram with the calculation and memory transfer of the next, the LPI method could assemble a $78 \times 78 \approx 6100$ square array in a similar time. In this case, the LPI method with a single SLM would match the performance of a technically much more complicated solution using multiple pairs of AODs with many tweezers per pair [10].”

(2) In Figure 3, it would be helpful to include visual markers for trap and atom positions, similar to those in Figure 2, to enhance clarity and consistency.

We have added the same type of indicators. See the new Figure 2.



(3) The argument in Chapter 3 and Figure 4(b) regarding atom loss due to phase differences could be further substantiated by including plots of the inter-frame phase differences. Correlating these with the intensity flicker observed in Figure 4(a) would reinforce the proposed interpretation.

It is a good point that a correlation between the two would strengthen the argument. However, not having a proper synchronization at the time we could use the highspeed camera makes it hard to perform a quantitative correlation. We have no access anymore to the highspeed camera and thus cannot remake the plot with proper synchronization. We therefore cannot create a correlation in the main text.

The point of Figure 4a is mostly to show the large difference in noise between both methods, highlighting that the WGS power goes close to zero multiple times. It is important that the choice of integrating counts within an ROI is not undebatable, as for example intensity noise in the laser or background lighting could influence the counts. We attribute a part of the noise in the LPI method trace in Figure 4a to this choice. At the same time fitting a Gaussian to each tweezer is complicated due to the changes in spot shape in between frames.

Furthermore, we comment that for many tweezers, the LPI method does not guarantee that the uniformity of the pattern remains constant. An extreme case of this is visible in the holograms where tweezers are turned off. A potential way of improving the method could be to iterate a WGS-like algorithm several times, leaving the optical phases rather constrained but optimizing the amplitudes to generate more uniform patterns.