Dear editor,

The referee's main complain is that equation (5) of our manuscript

$$\mathbf{E}_a \wedge \delta_\xi e^a + \mathbf{E}_\varphi \wedge \delta_\xi \varphi = dN\,,$$

does not follow from (2), (3) and (4). He/she acknowledges that (5) is correct, but states that it follows from an entirely independent calculation that involves integration by parts of the products of the equations of motion and the gauge transformations of the fields and the use of the Noether identities.

Indeed, if one wants to find the explicit form of N, one has to follow exactly that procedure: take the l.h.s. of that equation and integrate by parts all the terms involving derivatives of the gauge parameters until one obtains an expression that involves terms linear in the gauge parameters (and not involving their derivatives) plus a total derivative, dN. The term linear in the gauge parameters vanishes because of the Noether identities and only the total derivative dN survives.

While in simple cases the Noether identities can be easy to check by hand, the question is why there is always a Nother identity that cancels the term linear in the gauge parameters in any gauge-invariant theory. Our derivation, which we are going to explain in detail, is basically the derivation of the second Noether theorem that proves (5) in general, although, indeed, one has to do the calculation outlined by the referee in order to finf the actual value of N.

First of all, (2) is a general expression valid for general, arbitrary variations of the fields and it should be clear that (4) follows from it by using the particular transformations δ_ξ in it. (3) is the starting assumption that we have a g.c.t.-invariant action.

Now, the main observation is that the r.h.s.s of (3) and (4) are equal, because they are both equal to $\delta_\xi S[e,\varphi]$. Thus, combining (3) and (4) we arrive at the identity

$$\int \left\{\mathbf{E}_a \wedge \delta_\xi e^a + \mathbf{E}_\varphi \wedge \delta_\xi \varphi + d\left[\mathbf{\Theta}(e,\varphi,\delta_\xi e,\delta_\xi\varphi) + \imath_\xi \mathbf{L}\right]\right\} = 0\,,$$

which is satisfied off-shell for any ξ and, more importantly, independently of the integration domain, which leads to

$$\mathbf{E}_a \wedge \delta_\xi e^a + \mathbf{E}_\varphi \wedge \delta_\xi \varphi = d\left[-\mathbf{\Theta}(e,\varphi,\delta_\xi e,\delta_\xi\varphi) - \imath_\xi \mathbf{L}\right]\,.$$

This identity implies that, if we integrate by parts the l.h.s. as discussed before and we rewrite it in the form

$$\mathbf{E}_a \wedge \delta_\xi e^a + \mathbf{E}_\varphi \wedge \delta_\xi \varphi = \mathcal{N}_a \xi^a + dN\,,$$

the term linear in ξ must vanish on its own, off-shell, for any ξ because the whole expression must only be a total derivative, as the r.h.s. indicates. This leads to the ξ-independent Noether identities

$$\mathcal{N}_a = 0\,,$$

and to (5) in general.

Then one gets

$$dN = d\left[-\boldsymbol{\Theta}(e, \varphi, \delta_\xi e, \delta_\xi \varphi) - \imath_\xi \mathbf{L}\right] ,$$

which implies that N and $-\boldsymbol{\Theta}(e, \varphi, \delta_\xi e, \delta_\xi \varphi) - \imath_\xi \mathbf{L}$ differ by a total derivative which is, precisely $d\mathbf{Q}[\xi]$. The difference between these two objects is what we call, by definition, Noether current $\mathbf{J}[\xi]$ and we have

$$\mathbf{J}[\xi] \equiv N + \boldsymbol{\Theta}(e, \varphi, \delta_\xi e, \delta_\xi \varphi) + \imath_\xi \mathbf{L} = d\mathbf{Q}[\xi] \,.$$

Thus, (5) follows from (2), (3) and (4) in general, guaranteeing the existence of N. The procedure mentioned by the referee is necessary to obtain an explicit expression for N.

We intend to explain better this point in the new version of the manuscript extending the discussion on the lines of this answer.

Yours, sincerely,

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