Dear Editor,

Thank you for providing us the comments/suggestions of the reviewers on our manuscript entitled "Coupled dynamics of resource competition and constrained entrances in a multi-lane bidirectional exclusion process". We are extremely thankful to the reviewers for dedicating their time to thoroughly evaluate our work and for offering constructive comments that have undeniably contributed to the enhancement of both the presentation and the overall quality of our manuscript. Outlined below are our detailed responses to each comment provided by the reviewers. We have carefully addressed all comments and implemented the necessary revision accordingly. Response to Reviewer 2's comments are marked in blue throughout both the rebuttal report and the revised manuscript.

Response to Reviewer 2

I recommend to publish the paper after the revisions below have been considered. Although the work is not orginal, the research is well done and reported. Given that the results exist, I see no reason not to publish them.

We thank you for your positive assessment and recommendation to publish our work. We appreciate your encouraging remarks and are grateful that you found the research well executed and clearly presented. Below, we provide point-by-point response to the comments you have raised.

Comment 1: Please explain in more detail how the right-hand sides in (18) are obtained (i.e. the second equality signs)

Response: Thank you for your comment. In Eq. (18), the right-hand sides arise from the definition of the effective entrance rates, which account for the competition between the two oppositely directed particle species at the entry sites. The bulk and boundary currents are given by Eqs. (15) and (16), while the modified entrance rates for narrow entrances are introduced in Eq. (17) for each lane k. For illustration, consider lane A, where the effective entrance rate for + particles is defined as

$$\alpha^{A+}_{eff} = \frac{\alpha^{A+}_{mod}(1-\rho^A_1-\sigma^A_1)}{1-\rho^A_1} \tag{1}$$

From Eq. (17), the modified entrance rate is $\alpha^{A+}_{mod} = \alpha^+(1-\sigma^B_1)$. Substituting this expression, we obtain

$$\alpha^{A+}_{eff} = \frac{\alpha^+(1-\sigma^B_1)(1-\rho^A_1-\sigma^A_1)}{1-\rho^A_1} = \frac{\alpha^+(1-\sigma^B_1)(1-\rho^A_1-\sigma^A_1)}{\frac{\alpha^+(1-\sigma^B_1)(1-\rho^A_1-\sigma^A_1)}{\alpha^+(1-\sigma^B_1)}+\frac{\beta\sigma^A_1}{\beta}}. \tag{2}$$

Using Eq. (16), this expression can equivalently be written in terms of the entrance and exit currents as

$$\alpha^{A+}_{eff} = \frac{J^{A+}_{\text{enter}}}{\frac{J^{A+}_{\text{enter}}}{\alpha^{A+}_{mod}}+\frac{J^{A-}_{\text{exit}}}{\beta}}. \tag{3}$$

Due to current continuity between the bulk and boundary regions, we have $J^{A^+} = J^{A+}_{\text{enter}} = J^{A+}_{\text{exit}}$ and $J^{A^-} = J^{A-}_{\text{enter}} = J^{A-}_{\text{exit}}$, this simplifies the above equation to

$$\alpha^{A+}_{eff} = \frac{J^{A+}}{\frac{J^{A+}}{\alpha^{A+}_{mod}}+\frac{J^{A-}}{\beta}} \tag{4}$$

which corresponds to the second equality in Eq. (18) and consistent with the expression used in previous studies as reported in Refs. [28,29] of the manuscript.

Comment 2: Section 3.2: It is said "numerical investigations corroborated by Monte Carlo Simulations confirm the absence of several theoretical feasible phases".

Could you please elaborate on this? Does this imply that there parameter regimes for which multiple phases can exist, but only one is observed?

Response: Thank you for your careful reading and pointing out this mistake. In Section 3.2, we discuss the asymmetric phases that can arise in the system and identify the conditions under which certain theoretically possible phases are not physically realizable. There was an incorrect phrasing in the original sentence, "numerical investigations corroborated by Monte Carlo simulations confirm the absence of several theoretical feasible phases." We apologize for this mistake. The intended meaning is that the existence of these phases is theoretically intractable, and our numerical investigation, corroborated by Monte Carlo simulations, confirms that such phases do not exist in the system. This has been corrected in the revised manuscript at page 11 and highlighted in blue color.

Comment 3: At present figures captions say "Solid lines represent theoretical predictions". It would be helpful if this can be made more precise, and the captions refer to specific equations in the main text that have been used to plot the solid lines.

Response: Thank you for your helpful suggestion. In each phase diagram, the solid lines represent the theoretically obtained phase boundaries, while the symbols correspond to the Monte Carlo simulation results. To enhance clarity, we have now included references to the specific equations used to generate the theoretical lines in the first figure captions. For all subsequent figures derived from the same theoretical formulation, the captions explicitly mention that the phase boundaries are derived from the same set of equations as in the first figure. These changes have been incorporated in the revised manuscript (pages 15-18) and highlighted in blue color.

Comment 4: Figure 13 and 14: Figure 13 shows that for beta=0.25 and alpha=10 or beta=0.28 and alpha=4 the theory deviates significantly from the numerical simulations. Figure 14 seems to indicate that this is a finite size effect. However, Figure 14 is not showing resuls for the same parameters as Figure 13. Therefore, this is inconclusive. I think the authors should show a figure 14 for exactly the same parameters as figure 13, and overlay it with theoretical results, demonstrating that numerics converges towards theory. Otherwise, it remains inconclusive that the shock is localised as claimed in the paper.

Response: Thank you for your insightful comment. In Figs. 13(a) and 13(b), we present the density profiles for fixed entry rates while varying the exit rates under both symmetric and asymmetric filling factors. As noted, the theoretical predictions deviate from the Monte Carlo simulation results in the shock profile, which we attribute to finite-size effects. To clarify this point, we have revised Fig. 14(a) to correspond exactly to the same parameter set used in Fig. 13, namely $(\alpha, \beta) = (10, 0.25)$ for the symmetric filling case with $\mu^{\pm} = 0.94$. The updated Fig. 14(a), shown on page 29 demonstrates that as the system size increases, the simulation results systematically converge toward the theoretical predictions. This confirms that the deviations observed in Fig. 13 originate from finite-size effects and that the shock remains localized, consistent with our theoretical analysis.

Comment 5: In the legend of figure 14: it is not clear what is meant by MCS.

Response: Thank you for your comment. In Fig. 14(a), we present the density profiles for the shock case at fixed parameter values for different lane lengths L. The term MCS is used as an abbreviation for Monte Carlo Simulations. In the figure legend, MCS specifically denotes the simulation results corresponding to different system sizes. This clarification has now been explicitly included in the revised manuscript on page 14 and highlighted in blue color.