

Dear Editor,

Thank you for providing us the comments/suggestions of the reviewers on our manuscript entitled “Coupled dynamics of resource competition and constrained entrances in a multi-lane bidirectional exclusion process”. We are extremely thankful to the reviewers for dedicating their time to thoroughly evaluate our work and for offering constructive comments that have undeniably contributed to the enhancement of both the presentation and the overall quality of our manuscript. Outlined below are our detailed responses to each comment provided by the reviewers. We have carefully addressed all comments and implemented the necessary revision accordingly. Response to **Reviewer 1**’s comments are highlighted in **red** throughout both the rebuttal report and the revised manuscript.

## Response to Reviewer 1

This is an outstanding theoretical work that combines analytical calculations and computer simulations to investigate a complex bidirectional transport system stimulated by biological observations. The paper is well-written, and the results are clearly explained. The authors did an excellent job of explaining the physical meaning of the results. It is definitely an important advancement in the field.

We sincerely thank you for the positive and encouraging assessment of our work. We appreciate your thorough evaluation and detailed feedback, which has been invaluable in improving our manuscript's clarity, depth and overall quality. Below, we have provided point-by-point response to the comments you have raised.

**Comment 1:** Eq. (3) - It is the simplest assumption about the entrance rates, but it is not the only one. Will the results change if other choices are made? Some brief discussions might be useful here.

**Response:** Thank you for your valuable comment. In Eq. (3), we define the effective entrance rates of  $+$  and  $-$  particle species as

$$\alpha^+ = \alpha f(N^{r+}(t)), \quad \alpha^- = \alpha g(N^{r-}(t)), \quad (1)$$

where  $f(N^{r+}(t)) = \frac{N^{r+}(t)}{N^{\text{tot}+}}$ , and  $g(N^{r-}(t)) = \frac{N^{r-}(t)}{N^{\text{tot}-}}$ . These functions are monotonically increasing and bounded by 0 and 1 which results in the effective rates to be bounded by 0 and the innate rate  $\alpha$ .

Alternative functional forms have also been considered in the literature as reported in Refs. [43,46] of the manuscript. However, such modifications would primarily rescale the rate of particle exchange between the reservoir and the system, affecting only the quantitative aspects of the density profiles and currents. The overall qualitative behavior such as the nature of the steady-state phases and transitions would remain unchanged. To clarify this, we have updated the text on page 6 and highlighted the changes in red in the revised manuscript.

**Comment 2:** I would add more discussion on the expectation of what might happen when  $q < 1$ . Specifically, if mean-field theory could capture the processes well.

**Response:** Thank you for your valuable comment. We have added Appendix C which details the case for  $q < 1$ , on page 33 in the revised manuscript and highlighted with red color.

**Comment 3:** I would mention that the success of mean-field theory for this model is because the coupling (correlations) are very local (only at the entrances).

**Response:** Thank you for your insightful comment. We agree with you that the success of mean-field approximation in this model, particularly for the case  $q = 1$ , arises from the fact that correlations are absent in the bulk and remain localized near the entrances as a consequence of the constrained entry condition imposed at the boundaries. To clarify this

we have revised the text in conclusion section on page 30 and highlighted in red color in the revised manuscript.