Reply to the report of Referee #1.

Strengths

- studies an interesting problem in various ways and settings
- provides analytical results for unusual quenches, both chaotic and integrable
- is nicely written

Weaknesses

- Methods are standard
- Some relevant calculations are left for further work

Report: I believe the journal expectation "Open a new pathway in an existing or a new research direction, with clear potential for multi-pronged follow-up work" is easily met, as the work opens new ways to study quantum quenches from entangled initial states, both for integrable and chaotic models.

The paper provides results in three quite different settings for the same nontrivial initial states and yields insights into differences in integrable and chaotic behaviour. It also discusses how the membrane picture needs to be modified for correlated initial states. I would like to see the following changes, but I don't insist that all need to be addressed in this paper.

Our reply: We would like to thank the referee for carefully considering and reading our manuscript and asking important clarifying questions and moreover giving very insightful comments on how to improve the manuscript. The replies to the different questions/comments made by the referee are found below and answered in a point-by-point manner.

Answer to the points made in the report:

Comment 1: The discussion about how the membrane picture needs to be modified for correlated initial states is a bit vague and imprecise. Extending it, generalizing, and providing more details would help the paper, in my opinion.

Our reply: For the single interval case the entanglement membrane picture is actually the same as originally proposed in references [33-35]. Already in that case it was anticipated that one should incorporate the effects of a highly entangled initial state. Typically, however only product initial states are considered. For the multiple interval case once again only the initial state contribution needs to be modified. This is done by accounting for the non-local nature of the entanglement. We have modified the discussion to clarify the modifications of the membrane picture.

Comment 2: I believe the paper would be quite a bit stronger if the recursion equation 2.35 were solved for finite q. I don't see a reason why authors would need to delegate this to further work (if it is doable). Very similar recurrences have been solved in quite a few recent works, so I expect the authors can modify the procedure, for example, arxiv 2004.13697. If this does not work, authors can at least comment about why it is more difficult.

Our reply: Unfortunately the techniques introduced in arXiv:2004.13697 cannot be straightforwardly applied in our case. The main reason for this is that if one includes terms which are subleading in $1/q$, the shape of the resulting diagram can change leading to complications. In the process of solving the recursion relation fully, the appearance of diagrams that are different in shapes for each choice of parameters (subsystem length and time) do not allow to simply apply the aforementioned techniques. Hence we restrict our analysis to leading order in q in the large q limit. We have added a comment on this to the draft.

Comment 3: Authors can provide a short explanation of why and when Eq. 2.44 holds (apart from citing 54). This approximation gives zero correlations, so in some sense loses all microscopic.

Our reply: The relation 2.44 arises from standard transfer matrix type arguments as follows. Let us denote the object on the left hand side by $T(z,p)$. From its definition one can see that $T(z,p) = [T(1,p)]^z$ and so we can expand $T(z,p)$ in terms of projectors onto the eigenvectors of $T(1,p)$. Equation 2.44 approximates this by retaining only the leading eigenvector, which can be shown to be a product over all circle states. We have added a comment to this effect in the manuscript

Comment 4: Label under Figure 7. There is probably a typo of $\rightarrow$ or. They mention random unitaries, but do they mean their $q \rightarrow \infty$ infinity limit?

Our reply: We have corrected the caption to remove the typo and clarify the meaning. Indeed, we mean the large q limit.