Reply to the report of Referee #2.

Strengths

- it performs a systematic study of quantum quenches from initial states with a large amount of long-range entanglement.
- both brickwork quantum circuits and Hamiltonian dynamics have been studied; both integrable and chaotic systems have been studied; analytical and numerical results are compared when possible.
- it proposes modifications to the quasiparticle and membrane picture that were previously established for studying entanglement dynamics.

Weaknesses
no obvious weakness

Report
The paper by Chalas et al. studies quench dynamics in several cases starting from crosscap states. These states have special structure in that lattice sites that are separated by a long distance are entangled. In most previous studies about quantum quenches, the initial states are chosen to be simple ones with low entanglement (product states or eigenstates of certain Hamiltonians). Going beyond this paradigm can potentially reveal some other interesting phenomena. This paper makes a good step along this direction. The investigations are quite comprehsive and sufficient details are given. Physical implications have been unveiled based on suitable generalization of the quasiparticle and membrane pictures.

Our reply: We would like to thank the referee for carefully considering and evaluating our manuscript. The questions and remarks were particularly helpful to make the manuscript better and clear. Moreover, we are particular indebted for suggesting the references. We proceed by replying to the following questions made by the referee in a point-by-point fashion.

Comments of the referee:
It would be helpful to clarify a few minor issues.

Comments 1-2: It would be good to emphasize that Eq. (1.1) is a rather general definition. It includes not only the crosscap states defined in conformal field theory (CFT), but also many other possibilities that most likely do not correspond to crosscap states in CFT. It should be mentioned that states like Eq. (1.1) have also been termed "entangled antipodal pair" states in some papers [Phys. Rev. Lett. 133, 170404 (2024); Phys. Rev. Res. 6, L042062 (2024); 2412.18610]

Our reply: We have added this to the draft and the suggested references.

Comment 3: It seems that all calculations are performed using periodic boundary conditions. Is there any special reason? When studying crosscap states in CFT, this is necessary because they are not defined for open boundary conditions. For the current setting, especially considering the possibility of experimental investigations using quantum simulators, open boundary conditions may be useful to explore.

Our reply: We chose to explore only systems with periodic (or anti periodic in the case of fermions) boundary conditions as it was the most straightforward to analyse using the techniques that we have used. As the referee points out, it would be possible to examine systems with different boundary condition. Given the amount of material present in the current draft, we will leave examination of this interesting point to future work.

Comment 4: In the case of Hamiltonian dynamics, if one turns to models that cannot be solved exactly by Bethe ansatz, will the quasiparticle and/or membrane pictures survive? The authors may comment on this issue briefly.

Our reply: This is an interesting question and worthy of further consideration. Typically, in the presence of weak integrability breaking one might expect features of the quasi-particle picture to persist at least on time scales shorter than the quasiparticle lifetime. In the present case, however, features of the quasiparticle picture appear on time scales which scale with system size. The competition between the two effects would be interesting to explore. We have added a comment in the conclusions to reflect this.