Reply to the report of Referee #3

Strengths

- Investigates a new type of quench
- Examines multiple cases (quantum circuits and spin chains, both ergodic and integrable)
- Includes both analytical and numerical analyses
- Provides physical interpretations
- Well-written

Weaknesses

The models under investigation are introduced rather briefly.

Report

The article investigates quantum quenches of crosscap states. Unlike the initial states typically considered in the literature, which exhibit short range entanglement, crosscap states are characterized by long-range entanglement. The authors explore crosscap quenches in both quantum circuits and spin chains, employing analytical and numerical methods. The primary quantities analyzed are entanglement entropy and mutual information. The results are accompanied by physical interpretations and are found to be consistent with both quasiparticle and membrane-based descriptions.

Based on the specific models studied, the authors observe that in chaotic systems, entanglement entropy remains constant, whereas in integrable systems, it initially stays constant but eventually begins to decrease. For chaotic systems, mutual information decreases until it vanishes and remains zero, while in integrable systems, it decreases initially but later increases again.

Our Reply: We would like to thank the referee for carefully reading and reviewing our manuscript. The questions are indeed particularly useful to clarify certain subtle points regarding the analysis.

Comment 1: The discussion of Random Unitary Circuits could be elaborated further, particularly regarding the origin of equation (2.34).

Our reply: We have added a comment regarding this below the equation.

Comment 2: What is meant by "a specific class of dual unitary gates" prior to equation (2.44)? Does this refer to all chaotic gates? Additionally, what does "for large enough z" signify in equation (2.44)?

Our reply: This sentence is unintentionally vague and we have amended these statements. Dual unitary circuits have only been completely classified for $q = 2$. This parametrization can be extended to arbitrary values of q, however, for $q > 3$ there exist other classes of dual unitary gates. Equation (2.44) has been proven for this, special class of dual unitary circuits which encompasses all $q = 2$ cases. The proof essentially entails showing that the transfer matrix constructed from the dual unitary circuits has a unique largest eigenvector with eigenvalue 1. So, when it is taken to a large power it becomes a projector onto this leading eigenvalue as is the case on the right hand side. This drops terms which are of the order of the λ^z, λ is the next largest eigenvalue. We have added some additonal remarks on this to the text.

Comment 3: In equation (3.8), why is N_A conserved? The commutation $[N, H] = 0$ alone is insufficient; some condition on the initial state must also be specified.

Our reply: The subsystem fermion number, N_A is not conserved. However, the expectation value, $\langle N_A(t) \rangle$ remains constant as a consequence of the translation invariance of the system.

Comment 4: There appears to be a parenthesis mismatch in equation (3.9).
Our reply: This has been corrected.

Comment 5: In the case of the XXZ spin chain, the description of bound states and the TBA are valid in the infinite L limit. However, the crosscap state is ill-defined in this limit. Does this not lead to a contradiction?

Our reply: It is correct that the string structure of the Bethe roots is only valid in the thermodynamic limit. However, it has now been well established that the quasiparticle picture can be used in TBA integrable models, wherein the different string types correspond to different quasiparticle species. Moreover it is known that the quasiparticle picture compares excellently to exact numerics even for finite systems, both free and interacting. We have studied free fermions exactly in our scenario and shown that a quasiparticle picture emerges in that case despite the finiteness of the system. This provides the justification for the use of the quasiparticle picture in the interacting case also.