

Dear Editor,

We thank the referee 2 for their comments on our manuscript “An integrable deformed Landau-Lifshitz model with particle production?”, with reference number sci-post.202507.00048v1.

- (1.a) It is true that we are not able to compute the second component of the Lax connection, as we comment in the conclusions. This is the reason why we pivot to the computation of the infinite tower of conserved charges via the boost method as a test for the integrability of the action.
- (1.b) We actually explored that possibility, see e.g. the comment below eq. (2.23). However, we had difficulties in finding the second component, preventing us from such analysis.
- (1.c) We are able to match the charges of the sigma model from the charges of the quantum model in the continuum limit. We added footnote 6 above eq. (2.36) to clarify this point.
- (2) Although it is true that we have the quantum spin chain, it is a spin chain with a non-diagonalisable structure. If we consider it from the Algebraic Bethe Ansatz perspective, the S-matrix is read off the Bethe equations. However, as the Algebraic Bethe Ansatz is incomplete for non-diagonalisable models, this may not be a reliable way of computing it. For that reason, we have computed it from this alternative method. We added footnote 12 to explain this.
- (2.a) Our argument is based on the fact that by undoing the Drinfeld twist, the R-matrix goes back to the XXX model, where there is no particle production. This means that the continuum limit of the Drinfeld twist would take us from the basis where there is particle production to the one where there is no particle production (i.e. FK states).
- (2.b) As eliminating the particle production using FK states takes us back to the XXX spin chain, the S-matrix factorisation has to follow the same construction as in undeformed LL. Thus, we have substituted the phrase “It would be interesting to check if factorisability holds for the Class 5 model” by “As we commented, using FK states takes us back to undeformed LL action, so the deformed action should inherit this factorisation” in the conclusions.
- (3.a) Although there are other models with non-diagonalisability and particle production, the one we have studied provides a nice toy model, as it is a deformation of XXX (which is a very well-studied model), it is based on the  $\mathfrak{su}(2)$  algebra (in contrast with the eclectic spin chain associated with the fishnet theory, which is

non-diagonalisable for an  $\mathfrak{su}(3)$  sector or larger) and it has finite-dimensional representations (in contrast to the  $\mathfrak{sl}(2)$  spin chain studied in [2503.24223]). Of course, we do not claim that this toy model would capture all the peculiarities of Jordanian deformations or non-relativistic limits, but it may be a good laboratory for new ideas.

- (3.b) We have rewritten the part of the introduction starting with “In addition to their importance by itself, ...” clarifying that we expect the deformed LL theory from Class 5 to play a similar role of LL in a Yang-Baxter deformed AdS/CFT, although still unexplored.
- (3.c) As we commented on point 1, we could not find the classical Lax connection for the deformed LL theory (but we believe it exists), and therefore we also could not analyse its monodromy matrix. However, from the quantum model, it is not unreasonable to think that the classical monodromy matrix would be non-diagonalisable. Then, if this analysis had been possible, this may be a parallel example to the monodromy matrix of non-relativistic strings, therefore supporting our motivation.
- (3.d) We feel this is a subjective interpretation, and we believe the amount of details is enough at this stage.
- (4) We have eliminated that phrase.
- (5) We believe that the referee here refers to the fact that the first term in (1.6) is  $2u \mathbb{I} \otimes \mathbb{I}$  while the first term in (1.7) is  $(\frac{1}{2} + 2u) \mathbb{I} \otimes \mathbb{I}$ . This “shift” is due to the factor  $\frac{1}{2} \mathbb{I} \otimes \mathbb{I}$  inside  $P_{1,2}$ , which has been fully spelled out in (1.7).
- (6) Often in the literature the LL model is presented in the spin variables, therefore we believe it is opportune to discuss them. The generalised derivative allows to write  $Q_3$  in a simpler compact form, and for this reason we decided to introduce it.
- (7) We have added several comments and equation (2.56) to link the boost operator to the discussion on recursion operators.
- (9) We implemented referee’s suggestion.
- (10) We have removed the repeated reference.
- (11) As the co-cycle condition that we write is the one that appears in online resources, like wikipedia and nlab, we consider that it is not necessary to provide a reference to it. See also equation 1.12 of V. G. Drinfeld, “Quasi-Hopf algebras”, *Algebra i Analiz*, 1:6 (1989), 114–148; *Leningrad Math. J.*, 1:6 (1990), 1419–1457.
- (12) We have rewritten parts of section 2.3 to clarify it.

We thank once again the referee for their useful comments, and we hope that with the current changes the manuscript will be deemed acceptable for publication in SciPost Physics.

Best regards,

The authors

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