

Response to Referee Report (SciPost Physics)

Manuscript: *Single-Wall Torsion-Flux Realisation of Duality between M-Theory and Type I String Theory*

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Dear Editor and Referee,

We thank Referee 1 for a careful and detailed assessment. We agree that the original submission contained serious errors in (i) the global geometry and fixed-locus analysis, (ii) dimensional bookkeeping and normal-bundle statements, and (iii) the algebraic-topology/charge-classification presentation (including incorrect attributions and ill-typed AHSS expressions).

Summary of major revisions.

- **Geometry and dimensional bookkeeping:** We corrected the fixed-point analysis of the diagonal involution on $S^1 \times S^1$, corrected the codimension/normal-bundle rank discussion, and removed unsupported claims about an \mathbb{RP}^4 link in the stated geometry.
- **Novelty/scope:** We removed the overstated “hitherto unexplored” phrasing and precisely delimited what is (and is not) new relative to the established Hořava-Witten framework.
- **Algebraic topology:** We removed ill-typed AHSS differentials that summed operators of incompatible degrees, and we separated the standard H_3 -twisting of K -theory from the degree-four flux G_4 discussion.
- **Flux quantization:** We replaced the incorrectly attributed “Witten formula” by the standard shifted quantization condition, and we removed the internal contradiction involving $\lambda = 0$ and $w_4 \neq 0$ by stating precisely the hypotheses under which mod-2 reductions apply.
- **Brane charges and literature attribution:** We corrected the blanket statement that M-brane charges are classified by K -theory, replacing it by a correct discussion of Page charges/shifted cohomology in 11D and K/KO classification after dimensional reduction, and corrected the attribution to Diaconescu-Moore-Witten.
- **Cycle dimensions and the “instanton” check:** We corrected the erroneous identification of a 4-manifold as a “three-cycle” and removed the corresponding M2-wrapping claim (or, where retained as context, we explicitly mark it as incomplete pending a correct 3-cycle specification).
- **Exposition/typography:** We reorganized sections to introduce definitions before use, removed duplicated action definitions, and fixed LaTeX/typographical artifacts.

Below we respond point-by-point, quoting each criticism verbatim and explicitly indicating the corresponding manuscript changes.

Point-by-point response

Weakness 1: Geometry (fixed loci, dimensional checks, normal bundle, \mathbb{RP}^4)

Referee comment

“1. Fundamentally Flawed Geometry.” The central premise of a “single-wall” background is topologically incoherent. The proposed diagonal involution on a torus possesses four fixed points, not one, and does not topologically identify the two distinct Hořava-Witten boundaries. Furthermore, the geometric construction fails basic dimensional checks. The authors assign a rank-5 normal bundle to a codimension-1 hypersurface, implying a 15-dimensional spacetime rather than 11. The manuscript never explains the origin of the claimed \mathbb{RP}^4 geometry.

Throughout the manuscript, the M-theory spacetime is described as $X^{9,1} \times S^1 \times S^1$, which is a 12-dimensional manifold, yet it is repeatedly referred to as eleven-dimensional.

Response

Response (Referee is correct). We agree with the referee on each of these points.

- **Fixed points of the diagonal involution.** For $\omega : (y, z) \mapsto (-y, -z)$ on $S_y^1 \times S_z^1$, the fixed-point condition $y \equiv -y \pmod{2\pi R_y}$ and $z \equiv -z \pmod{2\pi R_z}$ has two solutions on each circle (0 and πR), hence *four* fixed points on the torus. The revised manuscript corrects the fixed-set description and removes the incorrect claim of a single fixed locus for this action.
- **“Single-wall” wording and identification of Hořava-Witten boundaries.** Given the corrected fixed-point structure, the original “single-wall” phrasing (and any implication that the diagonal involution topologically identifies the two Hořava-Witten boundaries) was overstated. The revised manuscript removes the claim as originally stated and tightens the scope accordingly.
- **Dimensional bookkeeping.** We agree that writing $X^{9,1} \times S^1 \times S^1$ and calling it “eleven-dimensional” is inconsistent. The revised manuscript corrects the dimensional bookkeeping and ensures that every occurrence of “11D” is consistent with the actual manifold under discussion.
- **Normal-bundle rank/codimension.** A ten-plane in an eleven-manifold is codimension one, hence has a rank-one normal bundle. The earlier rank-five normal-bundle statement was incorrect and has been removed.
- **Origin of the claimed \mathbb{RP}^4 link.** In the originally stated geometry, the transverse directions to a fixed ten-plane at a point in $S^1 \times S^1$ are two-dimensional, so the local quotient has the form $\mathbb{R}^2/\{\pm 1\}$ and its link is $S^1/\{\pm 1\} \cong S^1$, not \mathbb{RP}^4 . The revised manuscript removes the \mathbb{RP}^4 -as-link claim (and any subsequent steps that relied on it) unless and until a correct geometric construction is provided.

Weakness 2: Novelty

Referee comment

“2. False Claims of Novelty.” The manuscript presents the M-theory / Type I correspondence as hitherto unexplored, contradicting the original Hořava-Witten literature.

Response

Response (Referee is correct). We agree. The earlier phrasing overstated novelty. The existence of strong/weak relations within the Hořava-Witten / heterotic / Type I duality web is standard and must not be framed as newly discovered. In the revised manuscript we remove “hitherto unexplored” wording and replace it with a precise, limited statement describing what is being tested (namely, the internal consistency of a specific torsion/flux ansatz), without claiming historical precedence over Hořava-Witten.

Weakness 3: Mathematical inconsistency (AHSS degree, cycle dimension, twists, flux quantization contradiction)

Referee comment

“3. Mathematical Inconsistency.” The Atiyah-Hirzebruch Spectral Sequence differential attempts to sum operators of different cohomological degrees, which is mathematically impossible.

The manuscript further misidentifies products of two-manifolds as three-cycles, conflates the M-theory four-form with the K-theory twist, and contains a direct logical contradiction in the flux quantization argument.

Response

Response (Referee is correct). We agree, and we have corrected each of the listed issues:

3.1 Ill-typed AHSS differential. The original manuscript wrote AHSS expressions that mix degree-3 and degree-4 operations inside a single differential (e.g. schematically $d_3 = Sq^3 + (\text{degree-4 class}) \cup (\cdot)$), which is ill-typed. In the revised manuscript we remove these expressions and state the correct grading: for (complex) twisted K-theory with twist $H_3 \in H^3(M; \mathbb{Z})$, one has

$$d_3 = Sq^3 + H_3 \cup (\cdot), \quad d_3 : H^p(M; \mathbb{Z}) \rightarrow H^{p+3}(M; \mathbb{Z}),$$

and we do *not* present a “degree-four K-theory twist” via the same d_3 formula.

3.2 3-form vs 4-form “twist.” We explicitly separate: (i) the standard H_3 twist of (complex) twisted K-theory, versus (ii) degree-four M-theory flux quantization and Page-charge constraints. The revised manuscript no longer conflates $[G_4/2\pi] \in H^4(\cdot)$ with the standard Dixmier-Douady twist in H^3 .

3.3 Cycle-dimension mismatch. We agree that the originally defined “three-cycle” $RP^2 \times K$ (with K the Klein bottle) is four-dimensional, hence cannot be wrapped

by an M2-brane. The revised manuscript corrects this by removing the erroneous M2-wrapping claim (and any downstream inference) unless a correct three-cycle is specified and analysed.

3.4 Flux-quantization contradiction. We agree that the earlier discussion mixed hypotheses (spin vs non-spin) and led to an apparent contradiction. The revised manuscript replaces the incorrect attribution and removes the contradictory statements by formulating flux quantization using the standard shifted condition on the appropriate ambient manifold, with clear hypotheses and consistent mod-2 reductions.

Weakness 4: Exposition (tools used before introduction; repeated definitions)

Referee comment

“4. Erratic Exposition.” Advanced tools are used before being introduced, and key concepts are redefined multiple times, indicating a lack of structural coherence.

Response

Response (Referee is correct). We agree. The revised manuscript has been reorganized so that: (i) the geometric setup and group actions are defined once, early, with explicit conventions; (ii) the flux-quantization conventions and hypotheses are stated before use; and (iii) the topology/charge discussion is introduced only after the relevant geometric and physical structures are fixed.

Weakness 5: Misrepresentation of literature (Witten formula; DMW scope)

Referee comment

“5. Misrepresentation of Literature.” The manuscript attributes formulas to Witten that do not exist and misrepresents the work of Diaconescu, Moore, and Witten.

Response

Response (Referee is correct). We agree.

- **Witten flux quantization.** The earlier version presented an incorrectly attributed “Witten formula” including an added w_4 term. The revised manuscript replaces this with the standard shifted quantization condition (on a spin eleven-manifold X)

$$\left[\frac{G_4}{2\pi} \right] - \frac{1}{4}p_1(TX) \in H^4(X; \mathbb{Z}), \quad \text{equivalently} \quad \left[\frac{G_4}{2\pi} \right] - \frac{\lambda}{2} \in H^4(X; \mathbb{Z}), \quad \lambda := \frac{p_1}{2}.$$

We remove any spurious additive terms and ensure that mod-2 reduction statements are used only under the appropriate hypotheses.

- **Diaconescu-Moore-Witten (DMW).** The original phrasing overstated DMW as directly providing an eleven-dimensional K -theory classification of M2/M5 charges. The revised manuscript corrects this: it treats eleven-dimensional conserved quantities as Page charges with the appropriate quantization constraints, and it discusses how K/KO structures arise after reduction to ten dimensions, with DMW cited in its proper scope (global C -field/ E_8 -bundle description and related quantization statements).

Weakness 6: Reliability (arithmetic errors; LaTeX artifacts)

Referee comment

“6. Reliability of Content.” Numerous arithmetic errors and LaTeX artifacts undermine confidence in the manuscript’s validity.

Response

Response (Referee is correct). We agree, and we have performed a full consistency pass to remove LaTeX artifacts, correct typographical errors, and ensure that every displayed equation is well-typed and consistent with the surrounding text. Where a claim depended on an incorrect intermediate step (e.g. ill-defined differentials or mis-dimensioned cycles), the claim was removed or rewritten with correct hypotheses.

Evaluation sections, conclusion, and requested changes

Referee comment

Evaluation Against Acceptance Criteria. The manuscript fails to meet general acceptance criteria regarding clarity, reproducibility, and proper citation.

Evaluation Against Expectations. The manuscript fails to demonstrate novelty or physical coherence.

Conclusion. This manuscript is scientifically unsound and contains fatal geometric and mathematical errors.

Recommendation. I do not recommend publication in SciPost Physics or any other scientific journal.

Requested Changes (in the Event of Revision). (1) Clarify and properly motivate the RP^4 geometry. (2) Correct the classification of M-brane charges and remove false attributions.

Response

Response. We acknowledge that the referee’s assessment correctly identifies serious problems in the original submission. The revision directly addresses the acceptance-criteria concerns as follows:

- **Clarity and reproducibility:** The manuscript has been reorganized so that the

geometric setup, group actions, and conventions are defined once and used consistently. Tools (AHSS/twisted K -theory, flux quantization, anomaly inflow) are introduced before use, and ill-typed or unsupported derivations have been removed.

- **Proper citation and attribution:** We corrected the flux-quantization statement to match Witten’s standard formula and corrected the scope/attribution of Diaconescu-Moore-Witten, as detailed above.
- **Physical coherence:** The geometric inconsistencies (fixed loci, codimension/normal bundle, and the unsupported \mathbb{RP}^4 link claim) have been corrected by removing incorrect statements and tightening the scope to what can be defended under standard hypotheses.

Requested changes.

- (1) *\mathbb{RP}^4 geometry.* In the originally stated geometry, an \mathbb{RP}^4 link does not arise from the fixed-locus neighbourhood; the revised manuscript removes the \mathbb{RP}^4 -as-link claim and any downstream conclusions that depended on it. If an \mathbb{RP}^4 structure is to be retained in future work, it must come from a geometrically correct construction with explicit dimensional and bundle-theoretic bookkeeping.
- (2) *M-brane charges and attributions.* The revised manuscript corrects the classification statements (11D Page charges/shifted cohomology; 10D K/KO after reduction) and removes false attributions, with DMW cited only in its proper scope.

Finally, regarding the referee’s recommendation and conclusion: we respect the severity of the concerns for the original submission, and we agree that the original version contained errors that needed correction. We nevertheless hope that the substantially revised manuscript-in which the incorrect geometric and mathematical claims have been removed or corrected, novelty claims have been tightened, and citations/attributions have been fixed-can be reassessed on its corrected content.