Dear Referees,

We would like to thank you for your report, which will undoubtedly help improve the manuscript. Below, we answer to each of your points specifying, if necessary, the changes that we could include in the main work.

Referee 1

1. *While gluons are not asymptotically free and one cannot really construct Bell-type relations for gluon scattering (at least if the goal is to test them experimentally) the LHC is a prodigious source of gluon collisions. In particular, multijet production receives large contributions from gluon-only scattering. At the light of the findings of the paper, the authors should comment on whether there are observables at the LHC, such as multijet production, which can be used as experimental probes of entanglement in gluon scattering (in the same way as it has been done in the top quark sectors in recent measurements from ATLAS and CMS).*

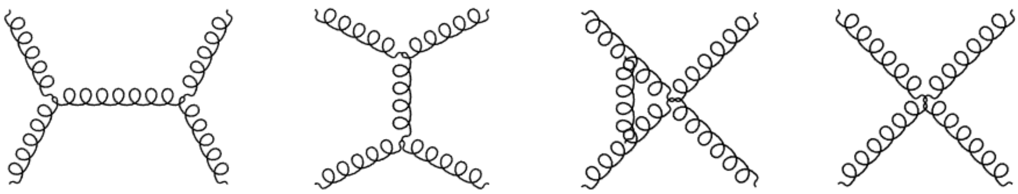
   Even if our work was only theoretical (we did not intend to provide an experimentally feasible path to explore entanglement in QCD), you correctly point out the relation between gluon production and quantum observables that will be relevant in our discussion, such as the top-quark production from gluon interaction that can lead to experimental entanglement witnesses in accelerators such as LHC. We cited those works in the introduction but without further clarification. We could modify the introduction paragraph to include its connection with gluon scattering, in particular (line 52):

   "For top quarks, there have also been studies in quantum tomography techniques [24] and, recently, entanglement between a top quark pair has been experimentally detected by the ATLAS and CMS collaborations at the LHC [25–27]. [NEW] The production of these top quarks comes primarily from gluon interactions in these collisions. Then, since top quarks decay faster than their hadronization, they transfer their spin properties to their decay products, which allows the estimation of their entanglement properties from the measurement of the angular dependence of the detected jets. Therefore, the phenomenology surrounding gluon scattering is of special interest for high-energy physics experimentalists."

2. *One of the Feynman diagrams in Figure 1 does not seem to be correct. The u-channel diagram shown in actually part of the one-loop correction to the tree*

level amplitude.

Thanks for pointing it out. We will correct it in a new version of the manuscript as



and adding a clarification in the caption, since even with the correction it could seem a loop diagram.

3. *When computing the scattering amplitudes, do the author assume gauge symmetry when deriving the Feynman rules? And if the calculation carried out analytically or with the help of numerical tools?*

   All the computations are carried out analytically, as they are tree-level. We use Mathematica to handle the expressions and simplifications and generating the plots, also to check that the total amplitudes match the theoretical ones (where one can use gamma matrices properties when computing the total cross sections). Regarding the first part of the question, we break gauge invariance only in the interaction term of the Lagrangian, leaving the kinetic term untouched. This way the Feynman rules stay the same except for the 4-vertex interaction which becomes proportional to $-ikg^2$ instead of $-ig^2$. To clarify this point, we could add the following modification to the text in Section 5 (line 196):

   "As we shall discuss shortly, the outcomes for $k \neq 1$ correspond to interactions that are not gauge invariant. [NEW] To be precise, we break gauge invariance in the interaction term of the QCD lagrangian only. Therefore, other Feynman rules such as the gluon propagators, or the gluons degrees of freedom (they are massless bosons) are not affected by this modification. Although there are other ways to break gauge invariance, we chose this one as we consider it a minimal gauge symmetry braking that allow us to explore the power of imposing MaxEnt in a more general theory."

4. *I am intrigued by the statement from the authors that "As far as a would be*

violation of a Bell inequality is concerned, color degrees of freedom are neutral witnesses". This is a bit unclear since without colour there are no gluon-only scattering amplitudes right? Maybe it would be good to rewrite this to avoid possible confusions.

We can clarify this statement in the following way (line 151):

"A relevant feature in the above result is the cancellation of color degrees of freedom for all non-zero amplitudes. To be precise, for those color amplitudes which are non-vanishing, the balance between LR and RL states is not affected by gauge indices. [NEW] In other words, the final state generated is the same independently of the color of the gluons involved in the interaction, which implies that the color degrees of freedom are neutral witnesses for any quantum information quantity computed from the final state wavefunction, including the entanglement measured with the concurrence or the violation of a Bell inequality. [/NEW] This is in no contradiction with the fact that output colors have different probabilities, as shown in Eq. (6) and dictated by the color structure functions."

Of course, the gluons have color (which dependence is explicit in Eq. (6)) , but the contribution of the structure constants is factorized when you normalize the state. Therefore, if you want to measure any quantity from the final state generated, the result will be the same for any color combination.

5. *The introduction of the 4-gluon weight k of course violates gauge symmetry. There are other ways in which the authors could implement a violation of gauge symmetry in pure YM scattering amplitudes, such as via a modification in the SU(N) matrices or structure constants. Can the author discuss how robust their results are if one chooses some other way to violate gauge symmetry in their analysis (to then check that it is recovered by the MaxEnt principle?)*

   Indeed, there are other ways to break gauge invariance. We chose the modification of the four vertex coupling as we considered it a "minimal" or "simple" gauge symmetry breaking, since only one diagram is affected and only one Fenyman rule is affected. Also, it does not modify the gluons propagator or their degrees of freedom (we keep them massless), otherwise we would need to use other figures of merit to quantify high-dimensional entanglement (we will deal with qutrits instead of qubits).
   It will be very interesting to check what happens when we use other mechanism

to break gauge invariance and we will probably pursue this direction in future works. A modification of SU(N) matrices could be the generalization of gamma matrices in the vertex, in a similar fashion as what some of the authors did in the work SciPost Phys. 3, 036 (2017). There, the gauge-invariant solution is almost recovered (up to relative signs between gamma matrices) for the case of QED. We would expect that a similar result can be found for gluon scattering. However, a modification of the structure constants will not introduce any change in the results, since, as shown in Eqs. (9) and (10), the structure constants dependence disappear when we consider the normalization of the state (this is related with the previous question).

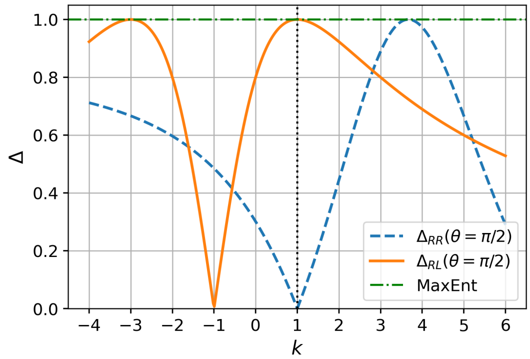
We stressed out that we only consider one way of breaking gauge invariance in the modification we mentioned in point 3 of this response. We could also extend the conclusions to include this point (line 248):

"The production of entanglement in gluon scattering is independent of the gauge group. However, a small departure of the gauge-tuned relation between the three- and four-gluon vertices would entail a reduction of entanglement. [NEW] We analyze this possibility by breaking gauge invariance in the interaction term by modifying the balance between the 3- and 4-gluon vertex. Other ways of exploring this gauge symmetry emergence from MaxEnt can be explored, but what we can observe is that those gauge symmetry breaking choices related with the color degrees of freedom will be blind to such modifications. [/NEW] Therefore, even if a possible Principle of MaxEnt does not select a particular gauge group as preferred by Nature, Nature fulfills such a Principle so that universality of entanglement on gauge theories emerges."

6. *As a related point, Figure 2 shows that there are unphysical solutions for k that lead to MaxEnt. These solutions violate gauge invariance. I wonder to which extent these unphysical solutions go away if one chooses a different way to parametrise the deviations from gauge invariance? I would also in this plot add a vertical line to indicate the physical k=1 solution.*

   As mentioned in the previous question, not all ways to break gauge invariance will lead to meaningful results (e.g. modifying the structure constants will not introduce any change in the results). Therefore, there is indeed a possibiliy that other ways to break gauge invariance and the later imposition of MaxEnt, generate other un-physical solutions or cancel out those obtained in this work.

We have improved Figure 2 and added the vertical line indicating the gauge-invariant solution.



7. *A recent related study https://arxiv.org/abs/2410.23343 proposes instead a Minimum Entanglement principle, and shows that it can be used to approximately reproduce numerical values of some of the SM parameters in the neutrino sector. What makes, conceptually, a MinEnt principle better (or worse) than a MaxEnt principle? Maybe it depends on the process, or it is part of a bigger, more fundamental principle?*

   This is indeed a great question that we also share with you. Several works (cited in the introduction of our manuscript), including the one you mentioned, impose Minimal Entanglement instead of Maximal Entanglement (as in this work and the previous from some of the authors) to predict or find some bounds on free SM parameters. Our motivation to impose MaxEnt was the fact that Nature is quantum as violation of Bell inequalities exist, so there must be some fundamental mechanism to generate the necessary "quantumness" to obtain such violations. For pure systems, such as the one analyzed in this work, violation of Bell inequalities implies generation of entanglement. Also, Bell inequalities are violated maximally, so MaxEnt must be achievable by such interactions. On the other hand, the imposition of MinEnt implies that, for some reason, Nature favors classical simulability in such sectors. Moreover, other recent works (published after the preprint of this manuscript) that analyze the generation of magic states in fundamental interactions (e.g. arXiv:2508.14967 [hep-th] that share an author with the one cited in your question) also seem to favor the hypothesis that Nature seeks as much classical simulability as possible (since low magic or zero magic states can be simulated efficiently with classical resources, even if they are entangled).
   We believe that more analysis is required to draw any conclusions on these

MaxEnt vs MinEnt perspective, including a comparison of imposing MaxEnt and MinEnt in the same processes and more elaborated ones (maybe beyond tree-level). But as a general conclusion, it is becoming clearer that quantum information principles may lay behind the structure in fundamental interactions, which makes this growing subfield exciting and with many further directions to explore.

Referee 2

1. *The u-channel in the figure has one loop and it doesn't look like a u-channel to me.*

   Thanks for pointing it out. We will correct it in a new version of the manuscript as shown in the response to point 2 from Referee 1.

2. *Statement of the principle and discussion as to why it works at tree-level and for helicity (one may beat classical correlation thresholds even with other degrees of freedom).*

   We do not robustly define the principle of MaxEnt, nor is our main goal in this work. Precisely, we share the same point of view as this Referee (and also the other one, in line with their points), that more work has to be done to completely define such a principle in a robust way. The potential principle is defined in the first paragraph of section 5 "The laws of Nature...". We try to reduce any strong claim in that matter (the existence of a fundamental principle) in all the work, using words such as "...result suggests the idea of..." or "exploring". However, we identify this bold statement that we decided to modify slightly:

   "It is tantalizing to investigate whether the relation between the three- and four-gluon vertices, as dictated by gauge symmetry, can be imposed from a MaxEnt Principle. ~~This, we show, is indeed the case.~~ [NEW]While this is not the case in full generality, we demonstrate that a clear and robust relationship does emerge. [/NEW] This result suggests a deep relation between local symmetries and entanglement."

   In the present work, we focus on a single scattering process (gluon–gluon scattering) and consider entanglement only in the helicity degree of freedom and our findings contribute to the accumulated evidence supporting a possible MaxEnt principle. Indeed, extending the analysis to other degrees of freedom will be necessary to define such a principle in a more robust way. The choice of helicity is motivated by two reasons: i) it is Lorentz invariant for massless particles and ii) can be linked to observables in colliders, as explained in point 1 from the other Referee report, therefore it has a potential experimental link.