Reply to Referee Report 1 on "Connecting quasinormal modes and heat kernels in 1-loop determinants"

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We thank the referee for their constructive and thought-provoking feedback. Below we will address their two editorial suggestions.

We agree with the author's comment that a higher spin generalization would be interesting. However, it turned out there were some interesting subtleties that arose and we saved this generalization for another work, which we recently posted (1910.07607). To briefly summarize, relabeling the Selberg integers k_1 and k_2 needs to be carefully generalized for higher spin fields. Specifically, we must ensure quasinormal modes Wick rotate to square-integrable Euclidean zero modes, a necessary condition in building the 1-loop partition function for higher spin fields. We have added a paragraph below Eqn. (69) to provide some additional details about the subtleties of extending to higher spin, as well as a footnote in the conclusion to briefly mention the higher spin generalization.

The referee also suggested we comment on whether the "poles of the Selberg zeta function correspond to 'quasi-normal modes'" of spacetimes whose analytical continuations are quotients by Schottky groups, such as the black hole solutions discussed in Krasnov. Indeed, we believe the quasinormal mode method is sufficiently general that a quasinormal mode analysis of fields propagating on these g-handled black holes would allow us to build the 1-loop partition function of these more general hyperbolic quotients. To our knowledge, however, the Lorentzian quasinormal modes for such handle-bodied solutions are not known. In fact, while the Euclidean zero mode analysis on $\mathbb{H}^3/\Gamma$ is well known, it is ambiguous as to what the Euclidean zero modes Wick rotate to because it is unclear which of the cycles of the handle-body is to be identified as the thermal circle. It is natural to expect that when the arguments of the Selberg zeta function in Eq. (71) are tuned to its zeros, we may find a condition analogous to $\omega_{\text{Matsubara}} = \omega_{\text{quasi}}$. As such, our observations might be used to predict the quasinormal mode frequencies of the g-handled black holes discussed in Krasnov. To do this we would need to know the Matsubara frequencies and the Selberg zeta function of the hyperbolic quotient in question. It may be possible to make some headway when Γ is a Schottky group. For example, in the case of computing the holographic entanglement entropy of two intervals on a line, the q_γ in Eq. (71) are known explicitly to some order in an expansion in small cross ratio (as shown in Eq. (58) of (1306.4682). We have added a paragraph at the end of section 3 with these comments.

We hope that these clarifications will adequately satisfy the referee's request.