Could the authors explain briefly why the projection operator keeps all but one tensor fixed?

We agree that it is not as trivial as most literature claims it to be. In short, the reason is that the projection operator itself is built up from the tensors of the TTN. These tensors are simply reproduced during an application of the projector and H onto a state and that is what keeps all other sites fixed.
Let's use a MPS with ortho center i as an example:

$$|\psi\rangle = \sum A^{s_i}_{\alpha\beta} |L_\alpha\rangle |s_i\rangle |R_\beta\rangle \tag{1}$$

$$P_i^{|\psi\rangle} = \sum_{\alpha\beta s_i} |L_\alpha\rangle\langle L_\alpha| \otimes |s_i\rangle\langle s_i| \otimes |R_\beta\rangle\langle R_\beta| \tag{2}$$

$$\to P_i^{|\psi\rangle} H|\psi\rangle = \sum_{\alpha'\beta's'_i} |L'_\alpha\rangle|s'_i\rangle|R'_\beta\rangle \underbrace{\sum_{\alpha\beta s_i} \langle L'_\alpha s'_i R'_\beta | H | L_\alpha s_i R_\beta\rangle A^{s_i}_{\alpha\beta}}_{\tilde{A}^{s'_i}_{\alpha'\beta'}}. \tag{3}$$

This shows that during a single application of PH, the left and right basis stay the same and only the center tensor changes. Since a time evolution is (according to its taylor series) just a sum of repeated applications of PH, this is also true for a full time evolution step.