We thank the referee for their report and constructive comments. The referee raised four issues to which we now reply in turn.

(1) We have followed the referee's suggestion to add explanations on section 2.1.

(2) Scaling of the Liouvillian gap. We made no claims that the states we constructed have the largest non-zero eigenvalues, we merely stated that the Liouvillian gap vanishes in the thermodynamic limit. The latter indeed follows from our construction of particular eigenstates with a gap the scales as L^{-2}. In order to avoid misunderstandings we have added a sentence stating that there well could be states with smaller Liouvillian gaps.

(3) Twisting the boundary conditions. It is well known that twisting the boundary conditions for the $GL(N^2)$ models is compatible with integrability. We have added some references on different ways (quantum inverse scattering method, co-ordinate Bethe Ansatz) of showing this.

(4) Continuum limit. The purpose of this section is to point out that the standard way of deriving integrable quantum field theories from integrable lattice models by taking appropriate scaling limits does not work as we are dealing with a master equation rather than a Schrödinger equation. We have added a sentence to make this more clear.

We disagree with the referee's statement that "the continuum limit Eq. (149) is valid only at the zero-density limit in which the divergence of the coefficient in the first term would not be a problem." The offending operator is the particle number, not the particle density. Hence adding even a single particle would generate a divergent contribution once we take $\gamma \to \infty$. This can of course also be seen directly from the Bethe Ansatz solution of the model. Linearizing around the Fermi points at finite density leads to the same problem. The reference quoted by the referee, Rep. Prog. Phys. 79, 096001 (2016), considers a model of interacting bosons rather than fermions, and applies bosonisation techniques without taking a scaling limit as we do here, i.e. it keeps γ finite. As we understand it the aim of this approach is to obtain an approximate description of the original lattice model, applicable for an appropriate class of initial density matrices and sufficiently short time scales. For our purposes the problem with not taking a scaling limit is that all higher derivative terms a priori have to be retained in the Liouvillian and they break integrability. We have changed "continnum limts" to "scaling limits" in the title of the section in order to stress that we are interested in the latter rather than the former.