Tau-lepton Physics at the FCC-ee circular e^+e^- Collider

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Abstract

The future FCC-ee collider is designed to deliver e^+e^- collisions to study with ultimate precision the Z, W, and Higgs bosons, and the top quark. In a high-statistics scan around the Z pole, 1.3×10^{11} events $Z \to \tau\tau$ will be produced, the largest sample of $\tau\tau$ events foreseen at any lepton collider. With their large boost, τ leptons from Z decays are particularly well suited for precision measurements. The focus of this report is on tests of lepton universality from precision measurement of τ properties and on tests of charged lepton flavour violation in Z decays and in τ decays. In both of these areas, FCC-ee promises sensitivities well beyond experimental limits.

Contents

1 Introduction

The future 100-km circular collider FCC at CERN is planned to operate, in its first mode, as an electron-positron machine, FCC-ee [1]. The FCC-ee is designed to deliver e^+e^- collisions to study with the highest possible statistics the Z, W, and Higgs bosons, and the top quark. The run plan, spanning 15 years including commissioning, is shown in Table 1.

In its first phase of operation, FCC-ee is planned to produce 3×10^{12} visible Z decays in a scan around the Z pole, more than five orders of magnitude more than at LEP. This will allow an extreme precision on the determination of the Z-boson parameters that are important inputs to precision tests of the Standard Model (SM). As an important example, the large data sample, combined with the exquisite determination of the centre-of-mass energy by resonant depolarization, will allow measurements of the Z mass and width both to precisions of about 100 keV. Another example, where the enormous statistics comes in with full power, is the determination of $\sin^2\theta_W^{\mathrm{eff}}$, where, from only the Z-peak measurement of the forward-backward asymmetry of muon pairs, a factor $\mathcal{O}(25)$ improvement is expected with respect to the current precision from all available data.

The enormous Z-boson statistics, implying 1.3×10^{11} decays $Z \to \tau^+\tau^-$, also opens unique opportunities for precise studies of the τ lepton. The τ lepton is a convenient probe in the search for Beyond Standard Model (BSM) physics because of the well-understood mechanisms that govern its production and decay. In the SM, it is assumed that the electroweak couplings between the three generations of leptons are universal, and that the three lepton family numbers are individually conserved. The latter assumption is violated in the neutral sector by the observation of neutrino oscillations. Via loop diagrams, this induces also lepton flavour violation among charged leptons (cLFV). The rates of such processes are however negligible, so that any observation of cLFV would be an unambiguous signal for BSM physics (see *e.g.* the recent review in Ref. [2]).

This report focuses on two main topics: *i*) test of lepton universality via precision measurements of τ-lepton properties, and *ii*) tests of cLFV in the decay of Z bosons and in the decay of τ leptons. Within both areas, FCC-ee promises sensitivities far beyond current experimental bounds.

To illustrate the potential of FCC-ee for precise τ-physics measurements it is useful to take a look back at LEP. Based on rather modest samples of $\mathcal{O}(10^5)$ $\tau\tau$ events, the LEP experiments were able to take large steps forward in the measurements of τ-lepton properties, in particular of the lifetime and branching fractions. Since then, the $\mathcal{O}(10^3)$ times larger statistics at the *b*-factories has allowed an improvement in the lifetime measurement by a relatively modest factor of three, whereas most LEP branching fraction measurements, in particular those for the leptonic final states, still stand unchallenged. The advantage of a Z factory relative to a *b* factory lies in the nine times higher boost of the τs. With the increased flight distance the lifetime measurement becomes easier, but the higher boost also has strong positive effects on the quality of the particle identification

Table 1: Run plan for FCC-ee in its baseline configuration with two experiments

Phase	Run duration (years)	Center-of-mass Energies (GeV)	Integrated Luminosity (ab^{-1})	Event Statistics
FCC-ee-Z	4	88–95	150	3×10^{12} visible Z decays
FCC-ee-W	2	158–162	12	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events
FCC-ee-tt	5	345–365	1.5	10^6 $t\bar{t}$ events

of the final state particles.

Also for the tests of cLFV in Z decays, LEP measurements still stand largely unchallenged.

2 FCC-ee and Detectors

The unrivalled luminosity performance of FCC-ee is achieved via the use of techniques inspired from b-factories: strong focussing of very low emittance beams combined with full-energy top-up injection into separate e^+ and e^- rings. Circular colliders have the advantage of delivering collisions to multiple interaction points, which allows different detector designs to be studies and optimised. In the current design, FCC-ee has two interaction points, and two complementary detector concepts have been studied [1]: *i*) CLD, a consolidated option based on the detector design developed for CLIC, with a silicon tracker and a 3D-imaging highly-granular calorimeter system; and *ii*) IDEA, a bolder, possibly more cost-effective, design, with a short-drift wire chamber and a dual-readout calorimeter. Cross-sectional views of the two detector concepts are shown in Figure 1.

Focus for the detector-design effort is an extreme control of systematic effects matching, as far as possible, the supreme statistical precision of the physics samples, in particular of the very large Z-boson sample. Both detector concepts feature a 2 Tesla solenoidal magnetic field (limited in strength by the 30 mrad beam crossing angle and the requirement of keeping the beam emittance very low), a small pitch, thin layers vertex detector providing an excellent impact parameter resolution for lifetime measurements, a highly transparent tracking system providing a superior momentum resolution, a finely segmented calorimeter system with excellent energy resolution for e/γ and hadrons, and a very efficient muon system. Important figures for the detector performance are summarized in Table 2, where they are compared to those of a typical LEP detector. In particular for the vertexing and tracking performance, large improvements are observed compared to LEP. The development of ultra-thin, fine-pitch silicon sensors plays an important role for this development, as does the application of a smaller beam pipe with a radius of only 15 mm, about a quarter of that at LEP, allowing for a first measurement of charged particles very close to the primary vertex. For the calorimeter system, the much finer granularity is important for the delicate analysis of the collimated topologies of multibody hadronic tau decays.

Table 2: Important performance figures for a typical detector at FCC-ee and at LEP

Quantity			LEP	FCC-ee
Impact parameter resolution	$\sigma_d = a \oplus \dfrac{b \cdot \mathrm{GeV}}{p_\mathrm{T} \sin^{2/3}\theta}$	a	20 µm	3 µm
		b	65 µm	15 µm
Momentum resolution	$\dfrac{\sigma(p_\mathrm{T})}{p_\mathrm{T}} = \dfrac{a \cdot p_\mathrm{T}}{\mathrm{GeV}} \oplus b$	a	6×10^{-4}	2×10^{-5}
		b	5×10^{-3}	1×10^{-3}
ECAL energy resolution	$\dfrac{\sigma(E)}{E} = \dfrac{a}{\sqrt{E/\mathrm{GeV}}} \oplus b$	a	0.2	0.15
		b	0.01	0.01
ECAL transverse granularity			15×15 mrad^2	3×3 mrad^2

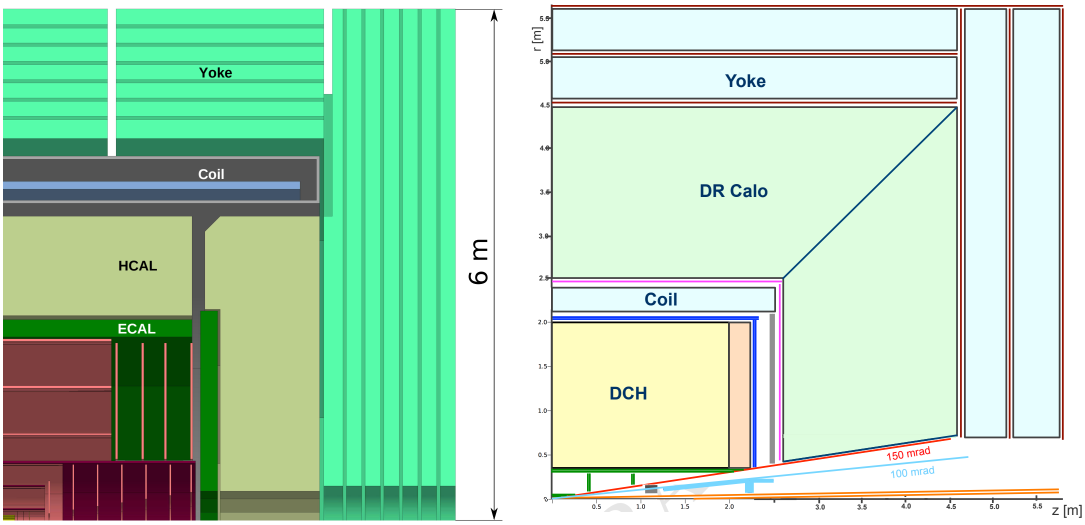

Figure 1: Vertical cross sections showing the top right quadrants of CLD (left) and IDEA (right)

3 Tau Lepton Properties and Lepton Universality

High precision measurements of the mass, lifetime, and leptonic branching fractions of the τ lepton can be used to test lepton universality.

Firstly, the ratio of the weak-charged-current couplings between muons and electrons, can be derived from the relation

$$\left(\frac{g_\mu}{g_\mathrm{e}}\right)^2 = \frac{\mathcal{B}(\tau \to \mu\bar{\nu}\nu)}{\mathcal{B}(\tau \to \mathrm{e}\bar{\nu}\nu)} \cdot \frac{f_{\tau\mathrm{e}}}{f_{\tau\mu}}, \tag{1}$$

where the phase-space factors are $f_{\tau\mathrm{e}} = 1$ and $f_{\tau\mu} = 0.97254$. Current data support universality to the precision $\delta(g_\mu/g_\mathrm{e}) = 0.14\%$ [3].

Secondly, the ratio of the weak-charged-current couplings between τ and electron and between τ and muon can be derived from the relation

$$\left(\frac{g_\tau}{g_\ell}\right)^2 = \frac{\mathcal{B}(\tau \to \ell\bar{\nu}\nu)}{\mathcal{B}(\mu \to \ell\bar{\nu}\nu)} \cdot \frac{\tau_\mu m_\mu^5}{\tau_\tau m_\tau^5} \cdot \frac{f_{\mu\mathrm{e}}}{f_{\tau\ell}} \cdot \frac{R_\gamma^\mu R_\mathrm{W}^\mu}{R_\gamma^\tau R_\mathrm{W}^\tau}, \tag{2}$$

with $\ell = \mathrm{e}, \mu$, and $f_{\mu\mathrm{e}} = 0.99981$, and where the last factor represents small radiative and electroweak corrections [4]. Current data support universality to the precision $\delta(g_\tau/g_\mathrm{e}) \simeq \delta(g_\tau/\mu) = 0.15\%$ [3], with the uncertainty dominated by the measurement of the τ leptonic branching fractions and lifetime. Figure 2 shows the current world-average situation for the $\tau \to \mathrm{e}\bar{\nu}\nu$ universality test. Also shown is the situation after a suggested one order of magnitude improvement in the τ branching fraction and lifetime measurements which is within reach at FCC-ee, as discussed below. At this level of precision, the universality test would be limited by the mass measurement, if no new measurements would be available. While FCC-ee may possibly be able to improve the m_τ measurement by a small factor, substantial improvements are more likely to come from a next generation of τ-factory experiments at the production threshold.

3.1 Lifetime

The world-average value of the tau lifetime is $\tau_\tau = 290.3 \pm 0.5$ fs [5]. Precision measurements were pioneered by the LEP experiments in the early 1990'ies following the

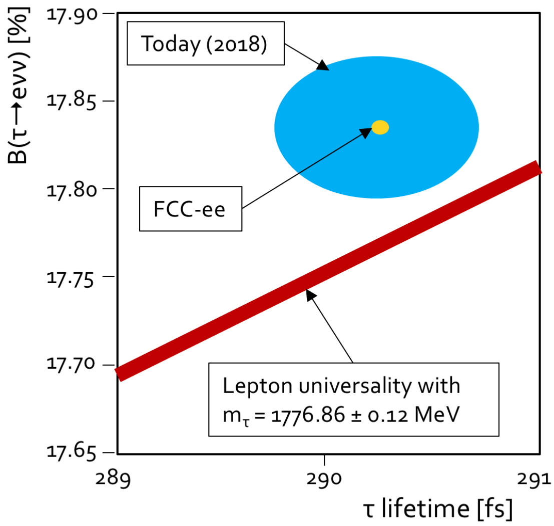

Figure 2: Branching fraction of $\tau \to e\bar{\nu}\nu$ versus τ lifetime. The current world averages of the direct measurements are indicated with the blue ellipse. Suggested FCC-ee precisions are provided with the small yellow ellipse (central values have been arbitratily set to todays values). The Standard Model functional dependence of the two quantities, depending on the τ mass, is displayed by the red band.

deployment of their precise silicon vertex detectors [6–9]. More recently, Belle, with its $\mathcal{O}(10^3)$ times larger statistics, has improved on these measurements [10].

The single most precise measurement from LEP, $\tau_\tau = 290.0 \pm 1.4$ (stat.) ± 1.0 (syst.) fs, was provided by DELPHI [7]. The analysis employed several complementary methods. The method with the smallest systematic uncertainty (1.3 fs) was the so-called decay vertex method, where the flight-distance was measured for τ decays to three charged particles. Here, the largest systematic uncertainty (1.0 fs) came from the 7.5 μm accuracy of the vertex detector alignment. This was estimated from samples of hadronic Z decays with three tracks in one hemisphere, and its value resulted from the (limited) statistical power of the test samples.

The Belle measurement, $\tau_\tau = 290.17 \pm 0.53$ (stat.) $\pm$ 0.33 (syst.) fs, was based on events in which both τs decayed to three charged particles. In these events, the constrained kinematics combined with the longitudinal boost of the $\tau\tau$ system provided by the asymmentric KEKB collider allowed Belle to reconstruct the two secondary vertices as well as the primary vertex and this way to extract the flight distances. As for DELPHI, the dominant systematic uncertainty was the accuracy of the vertex detector alignment. The assigned value of 0.3 fs corresponds to a vertex detector alignment accuracy, defined as it was done by DELPHI, of 0.25 μm, *i.e.* a factor 30 better than DELPHI.

The prospects for significantly improved τ lifetime measurements at FCC-ee are very good. Several factors contribute to this: *i*) Like Belle, the FCC-ee detectors have a 15-mm radius beam pipe allowing the first layer of the vertex detector to go four times closer to the beam line than at LEP; *ii*) Based on modern small pitch, thin layer technologies, the FCC-ee vertex detectors have a space point resolution of only 3 μm, four times better than at Belle (and LEP); *iii*) The much improved statistics compared to LEP (and even Belle) will allow very precise cross checks and studies of systematic effects to be carried out. For the systematic uncertainty, taking, perhaps conservatively, the 0.25 μm alignment uncertainty from Belle as an indication of the achievable precision, this translates immediately, with the higher boost, into a systematic precison at the level of 0.04 fs. Relative to LEP, the statistical precison will improve not only because of the much larger event sample but also due to the higher sensitivities of the vertex detectors. Hence, a factor $\mathcal{O}(10^3)$ improvement relative to LEP can be expected resulting in a statistical uncertainty of 0.001 fs.

Finally, it can be noted that the enormous FCC-ee statistics will allow extracting the lifetime from numerous complementary methods with partly different systematics. An obvious option would be, like Belle, to use events where both τs decay to three charged particles. At FCC-ee, nearly 3×10^9 events of this topology will be avaliable.

3.2 Leptonic Branching Fractions

Our knowledge about the τ leptonic branching fractions [5], $\mathcal{B}(\tau \to \mathrm{e}\bar{\nu}\nu) = 17.82 \pm 0.04\%$ and $\mathcal{B}(\tau \to \mu\bar{\nu}\nu) = 17.33 \pm 0.04\%$, is comletely dominated by results from 1.3×10^5 $\mathrm{Z} \to \tau\tau$ events collected at LEP [11–15]. Notwithstanding the $\mathcal{O}(10^3)$ times larger $\tau\tau$ samples at the b-factories, no results on this subject have appeared from there. Here, therefore, FCC-ee seems unrivalled in the pursuit of improvements.

Looking back at LEP, ALEPH provided the single most precise measurement of the leptonic branching fractions with statistical and systematic uncertainties of 0.070% and 0.032%, respectively [11]. With the FCC-ee data sample, the statistical uncertainty will reach a negligible 0.0001% level. Even if no improvements in detector performance relative to ALEPH would be realized, one would expect, with the much larger data sample, also a substantial reduction of the systematic uncertainties. In fact, for the ALEPH analysis, all major systematics contributions were estimated from limited-size test samples, and they were therefore essentially statistical in nature. Adding to this the substantially improved performance of the FCC-ee detectors, in particular the finer granularity of the calorimeters, a reduction of the systemtic uncertainty by an order of magnitude therefore seems within reach, reducing it to the level of 0.003%.

3.3 Mass

The world-average value of the τ mass is $m_\tau = 1776.86 \pm 0.12$ MeV [5], with the most precise single measurement provided by BESIII from an energy scan around the τ-pair production threshold [16]. Alternatively, m_τ can be extracted from the endpoint of the pseudomass distribution from $\tau \to 3\pi\nu$ decays, where the pseudomass is derived from the four-momentum of the 3π system together with the beam energy. This method, originally pioneered by ARGUS [17], has been pursued by OPAL [18] at LEP and by BaBar [19] and Belle [20] at the b factories. OPAL reached a systematic precision of 1.0 MeV. With their much larger statistics, Belle and BaBar were able to improve on this by nearly a factor of three. The dominant source of systematic uncertainty for all of these measurements came from the uncertainty on the calibration of the momentum scale. To derive this, OPAL compared 45.6 GeV tracks from $\mathrm{Z} \to \mathrm{ee}$, $\mu\mu$ events between real and simulated data and extrapolated the results down to momenta relevant for $\tau \to 3\pi\nu$ decays. With the much larger samples and the improved momentum resolution at FCC-ee, improvements relative OPAL can be certainly envisioned. The question is whether these are large enough to provide a competitive measurement. The statistical uncertainty will be at a negligible 0.004 MeV level. For the control of the mass scale one can envisaged to use the decay $D^+ \to K^-2\pi^+$ where the D^+ mass is known to 0.050 MeV [5]. Hence, a systematic uncertainty somewhat lower than the current precision, *e.g.* 0.1 MeV, may be within reach. With more than 2×10^8 decays $\tau \to 5\pi\nu$ avalilable, this mode can be likely used for a complementary measurement.

In summary, FCC-ee may be able to improve somewhat on the *current* world-average precision of the τ mass. In all likelihood, however, at the time of the FCC-ee, larger improvements have been already established from new experiments at the production threshold.

4 Charged Lepton Flavour Violation Z Decays

Searches for flavour violating Z decays into μe, $\tau\mu$, and τe final states have been performed at LEP and, more recently, at LHC. The current best bounds are summarized in Table 3. The LHC results, which so far all come from ATLAS, are based on about 10^9 Z decays, corresponding to around one percent of the total luminosity expected with the forthcoming high-luminosity upgrade of the LHC. Hence, future improvements can be expected both from ATLAS and from CMS, if/when they decide to enter the scene.

Table 3: Branching fraction limits (95% CL) on charged lepton flavour violation from Z decays

Mode	LEP		LHC	
$Z \to \mu e$	OPAL [21]	1.7×10^{-6}	ATLAS [22]	7.5×10^{-7}
$Z \to \tau\mu$	DELPHI [23]	1.2×10^{-5}	ATLAS [24]	1.3×10^{-5}
$Z \to \tau e$	OPAL [21]	9.8×10^{-6}	ATLAS [24]	5.8×10^{-5}

4.1 Z → μe

The decay $Z \to \mu e$ has the particularly clean signature of a final state with an electron and a muon whose invariant mass equals the Z mass. At LEP, the search for this mode was *background free*, *i.e.* there were no background events in the "signal box". In ATLAS, the search had several background sources the most important being $Z \to \tau\tau$ events, with the two τs decaying as $\tau \to e\bar{\nu}\nu$ and $\tau \to \mu\bar{\nu}\nu$, and multijet events where the two final state leptons were either genuine or fake. Since the ATLAS search has backgrounds, its sensitivity can be expected to scale as $1/\sqrt{\mathcal{L}}$, where $\mathcal{L}$ is the collected luminosity, implying ultimately an order of magnitude improvement of the sensitivity down to branching fractions somewhat below 10^{-7}.

With 3×10^{12} visible Z decays, FCC-ee will be able to improve the LEP sensitivity considerably for this mode. At some level, as at LHC, a background from $Z \to \tau\tau$ with the two τs decaying as $\tau \to e\nu\nu$ and $\tau \to \mu\nu\nu$, and where the final state e and μ are produced at the end-point of their momentum spectra, will eventually show up.[1] For the excellent momentum resolution of the FCC-ee detectors, corresponding to $\sigma_p/p \simeq 1.5 \times 10^{-3}$ for $p = 45.6$ GeV, this background is very small corresponding to a Z-boson branching fraction of about 10^{-11} (with a factor of nearly two included due ot the longitudunal spin correlation of the two τs). Here FCC-ee has a large advantage over LHC. Not only do the FCC-ee

[1] In principle there is also a background source from $Z \to \mu\mu$, with one of the muons decaying inside the tracking volume. The probability of a 45.6 GeV muon decaying over a distance of 2 m is 5.6×10^{-6}. However, only a very small fraction of the produced electrons come out at the end-point of the momentum spectrum, so this background is negligible. It is interesting to notice, that in more than one million $Z \to \mu\mu$ events at FCC-ee one of the muons will decay before reaching the calorimeter. This potentially opens the possiblity of measuring the polarisation of the muons from Z decays in the same way as the tau polarisation was measured at LEP in the decays $\tau \to \ell\nu\nu$, $\ell = e, \mu$.

detectors have a significant better momentum resolution, but the Z-mass constraint is also much more powerful. Whereas at LHC, the mass constraint is limited by the 2.5 GeV natural width of the Z boson, at FCC-ee the mass is known from the collision energy which has a dispersion of 85 MeV (0.9×10^{-3}) from the beam energy spread of 1.3×10^{-3}.

A potentially more serious background arises due to so-called catastrophic bremsstrahlung of muons in the material of the electromagnetic calorimeter (ECAL) by which a muon radiates off a significant fraction of its energy, with the subsequent risk of being mis-identified as an electron. NA62 has made detailed studies of this process for their liquid krypton calorimeter [25]. From this work, one can conclude that the probability of a 45 GeV muon to deposit more than 95% of its energy in the 27 radiation length (X_0) deep calorimeter is 4×10^{-6}, with good agreement (within 10%) between real and simulated data. If this number would represent the true FCC-ee propability, $P_{\mu\mathrm{e}}$, of a muon to be mis-identified as an electron, a fraction 3×10^{-7} of all Z decays would go through $\mathrm{Z} \to \mu\mu$ and appear mistakenly as eμ final states, eventually limiting the sensitivity of the $\mathrm{Z} \to \mu\mathrm{e}$ search to about an order of magnitude lower than that, if one assumes that $P_{\mu\mathrm{e}}$ can be controlled to about 10% of its value. In practice, however, $P_{\mu\mathrm{e}}$ will likely be somewhat lower than the NA62 number. Firstly, the energy resolution of the FCC-ee ECAL is about 2.5% at 45 GeV, and a more stringent requirement on the energy deposit than 95% can be placed. Secondly, longitudinal segmentation of the ECAL allows setting requirements on the energy deposit in the first few radiation lengths where the bremsstrahlung propability will be lower than for the full depth. Very important for this measurement is the ability to precisely determine $P_{\mu\mathrm{e}}$ from the data themselves. Longitudinal segmentation of the calorimeter certainly helps to this end, but a truely independent method of separating electrons and muons would make a more powerful tool. Even if the e/μ separation of a dE/dx measurement at $p = 45.6$ GeV would not be ideal, it could be still employed to manipulate the electron-to-muon ratio in test samples in order to study the calorimeter response. In this respect, it is encouraging to note that the drift chamber of the IDEA detector, through the use of cluser counting, promises a e/μ separation at $p = 45.6$ GeV of 3–4 standard deviations [26].

In conclusion, a sensitivity for the $\mathrm{Z} \to \mu\mathrm{e}$ mode at the 10^{-8} level should be within reach at FCC-ee. An independent method for e/μ separation, as that provieded by a powerful dE/dx measurement, could potentially improve this sensitivity by one to two orders of magnitude potentially all the way down to the 10^{-10} level.

4.2 $\mathrm{Z} \to \tau\mu$ and $\mathrm{Z} \to \tau\mathrm{e}$

The searches for $\mathrm{Z} \to \tau\mu$ and $\mathrm{Z} \to \tau\mathrm{e}$ have many similarities and they will be here treated under one. All previous searches are characterised by having background events occuring in the "signal box", so that sensitivities scale as $1/\sqrt{\mathcal{L}}$, where $\mathcal{L}$ is the collected luminosity. Ultimately, one can thus expect the LHC sensitivities to approach the 10^{-6} level.

In $\mathrm{e}^+\mathrm{e}^-$ collisions, the pursuit for decays $\mathrm{Z} \to \tau\mu$ (τe) amounts to a search for events with a *clear tau decay* in one hemisphere recoiling against a *beam-momentum muon (electron)* in the other. To illuminate the analysis, we will investigate these two terms.

Firstly, let us consider the term *clear tau decay*. Here, the point is to restrict the analysis to those τ-decay modes, where the probability is minimal of misidentifying a final-state lepton from $\mathrm{Z} \to \mu\mu$ or $\mathrm{Z} \to \mathrm{ee}$ as a τ decay. This immediately excludes the leptonic modes $\tau \to \mu\bar{\nu}\nu$ and $\tau \to \mathrm{e}\bar{\nu}\nu$. For the remaining hadronic modes, there may be a risk of mis-identifying either a muon or an electron as a pion, so that the decay $\tau \to \pi\nu$ may also have to be excluded, at least for large π momenta. To reach very high purities, it may be ultimately necessary to restrict the analysis to reconstructed exclusive modes such as $\tau \to \rho\nu \to \pi\pi^0\nu$ and decays to three (or more) charged particles.

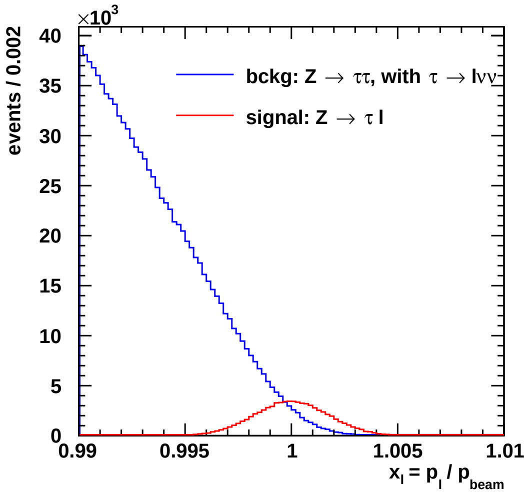

Figure 3: FCC-ee search for the lepton flavour violating decay $Z \to \tau\ell$, $\ell = e, \mu$. Momentum distribution of the final state lepton ℓ for the signal (red) and for the background from $Z \to \tau\tau$, with $\tau \to \ell\nu\nu$ (blue). The shown momentum resolution of 1.8×10^{-3} results from the combination of the spread on the collision energy (0.9×10^{3}) and the detector resolution (1.5×10^{-3}). For illustration, the LVF branching fraction is set here to $\mathcal{B}(Z \to \tau\ell) = 10^{-7}$.

Secondly, we have to consider the term *beam-momentum muon (electron)*. Since neutrinos are (nearly) massless, the muon (electron) momentum distribution in $\tau \to \mu\nu\nu$ ($\tau \to e\nu\nu$) decays will have an end-point at the beam momentum. Ignoring the mass of the final state charged lepton, the muon (electron) momentum distribution is given by the expression [27],

$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \frac{1}{3}\left[\left(5 - 9x^2 + 4x^3\right) + P_\tau\left(1 - 9x^2 + 8x^3\right)\right], \tag{3}$$

where $x = p/p_{\mathrm{beam}}$, and P_τ is the longitudinal polarisation of the τ leptons. Hence, the density of events close to the endpoint depends on P_τ (and thus on $\sin^2\theta_{\mathrm{W}}^{\mathrm{eff}}$), which is here set to the value $P_\tau = -0.15$, consistent with the LEP result [28]. The separation of signal and background now depends on the experimental precision by which a *beam-momentum particle* can be defined. This is illustrated in Figure 3, where the indicated momentum spread of 1.8×10^{-3} arises as a combination of the 0.9×10^{-3} spread of the collision energy and the 1.5×10^{-3} momentum resolution typical for a FCC-ee detector at $p = 45.6$ GeV. By defining the signal box by the simple requirement $x > 1$, it was found that FCC-ee, with the quoted resolution and a signal efficiency of 25%, has a sensitivity to branching fractions down to 10^{-9}. The sensitivity scales linearly in the momentum resolution.

5 Charged Lepton Flavour Violation in τ Decays

Very stringent tests of cLFV have been performed in muon decay experiments where branching fraction limits below 10^{-12} on both of the decay modes $\mu^- \to e^-\gamma$ [29] and $\mu^+ \to e^+e^+e^-$ [30] have been established. All models predicting cLFV in the muon sector imply a violation also in the τ sector, whose strength is often enhanced by several orders of magnitude, usually by some power in the tau-to-muon mass ratio. Studying

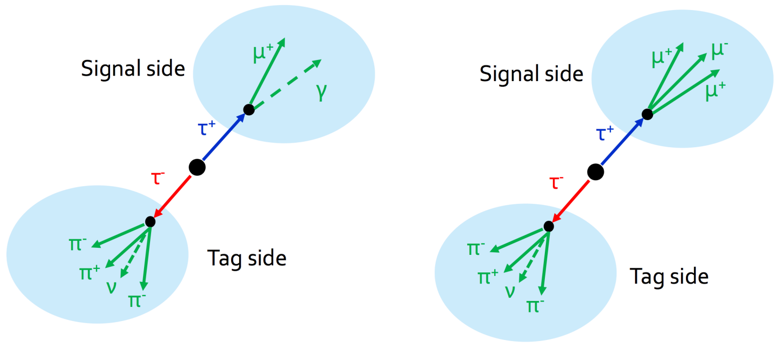

Figure 4: Illustration of the search for lepton flavour violating τ decays

cLFV processes in τ decays offers several advantages compared to muon decays. Since the τ is heavy, more cLFV processes are kinematically allowed. In addition to the modes $\tau \to \mu/\mathrm{e}+\gamma$ and $\tau \to \mu/\mathrm{e}+\ell^+\ell^-$, cLFV can be also studied in several semileptonic modes. The expected 2.6×10^{11} τs produced at FCC-ee exceed the projected Belle II (50 ab^{-1}) statistics by a factor of about three, raising the possibility that FCC-ee may provide competitive sensitivities. The focus here is on $\tau \to 3\mu$ and $\tau \to \mu\gamma$ as benchmark modes for evaluating the sensitivity to cLFV. The analysis strategy is illustrated in Figure 4, with a *tag side* to identify a clear standard-model tau decay and a *signal side* where cLFV decays are searched for. The present $\mathcal{O}(10^{-8})$ bounds on both modes are set at the b factories [31, 32]. As detailed below, about two (one) orders of magnitude improvement can be expected at FCC-ee for the decay $\tau \to 3\mu$ ($\tau \to \mu\gamma$). This turns out to be largely compatible with the recently published estimates for Belle II [33].

5.1 $\tau \to 3\mu$

The present bound of 2.1×10^{-8} on the $\tau \to 3\mu$ mode comes from Belle [31]. With the excellent FCC-ee invariant mass resolution, the search for this mode is expected to be essentially background free, and a sensitivity down to a branching fractions of $\mathcal{O}(10^{-10})$ should be within reach.

5.2 $\tau \to \mu\gamma$

The present bound of 2.7×10^{-8} on the $\tau \to \mu\gamma$ mode comes from BaBar [32]. As shown in their analysis, the search for this mode is limited by backgrounds, namely radiative events $\mathrm{e}^+\mathrm{e}^- \to \tau^+\tau^-\gamma$, with one $\tau \to \mu\bar{\nu}\nu$ decay, and the invariant mass of a $\mu\gamma$ pair accidentially in the signal region. An experimental study of the signal and this dominant background has been performed using the Pythia8 event generator [34, 35], and smearing the output four-vectors with realistic FCC-ee detector resolutions. The decay mode $\tau \to \pi\nu$ was used to fake the signal with the final state particles renamed as $\pi \to \mu$ and $\nu \to \gamma$. For the detector-performance numbers from Table 2, and by further assuming a position resolution for photons of $\sigma_x = \sigma_y = (6\ \mathrm{GeV}/E \oplus 2)$ mm, resolutions on the mass[2] and energy of $\mu\gamma$ pairs of $\sigma(m_{\mu\gamma}) = 26$ MeV and $\sigma(E_{\mu\gamma}) = 850$ MeV were derived. Figure 5 shows the background events in the $E_{\mu\gamma}$ vs. $m_{\mu\gamma}$ plane, with a signal region corresponding to 2σ resolutions indicated in the right-hand plot by the red ellipse. The statistics in the figure corresponds to nearly 7×10^{10} visible Z decays or about 2% of the full FCC-ee statistics.

[2] In order to de-correlate the $E_{\mu\gamma}$ and $m_{\mu\gamma}$ variables, the mass variable used here is defined by $m_{\mu\gamma} = m_{\mu\gamma}^{\mathrm{raw}} \cdot (E_{\mathrm{beam}}/E_{\mu\gamma})$, where $m_{\mu\gamma}^{\mathrm{raw}}$ is the measured "raw" mass.

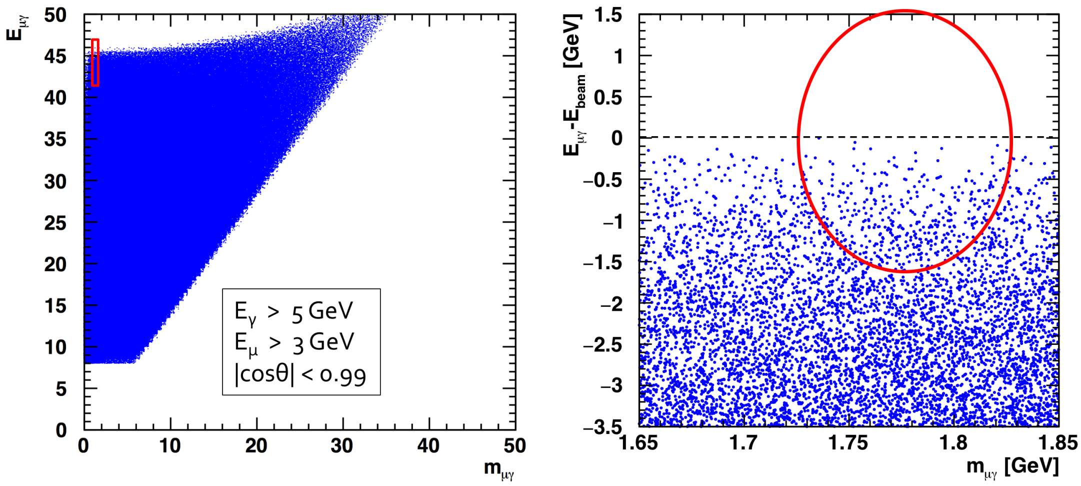

Figure 5: Main background to the $\tau \to \mu\gamma$ search at FCC-ee. Reconstructed energy versus mass of all $\mu\gamma$ combinations in simulated background events $\mathrm{e^+e^-} \to \mathrm{Z} \to \tau^+\tau^-\gamma$ with one $\tau \to \mu\bar{\nu}\nu$ decay. The right-hand diagram, showing an enlargement of the red boxed region from the left-hand diagram, has the signal region shown by a red ellipse.

Scaling to the full statistics, about 20,000 background combinations are expected inside the signal region from which a sensitivity down to branching fractions of 2×10^{-9} can be estimated.

As seen from Figure 5, the density of background events rises away from the (dashed) $E_{\mu\gamma} - E_{\mathrm{beam}} = 0$ line. In fact, the rise is linear, so that the sensitivity scales linearly in $\sigma(E_{\mu\gamma})$ and therefore also in the ECAL energy resolution which is the dominant contribution to $\sigma(E_{\mu\gamma})$.

6 Summary and Conclusions

Among all proposed future lepton colliders, FCC-ee, with 3×10^{12} visible Z decays, offers the largest sample of $\tau\tau$ events. From the LEP experience, we know that a Z factory is particularly well suited for precision τ-physics measurements. Indeed, even today, after $\mathcal{O}(10^3)$ times more τ decays have been collected at the b factories, several LEP measurements still stand unchallenged. With more than five orders of magnitude more events at FCC-ee than at LEP, large steps forward in terms of precision can be therefore foreseen. Whereas almost all LEP measurements were statistics limited, at FCC-ee, systematic effects will be dominant. At this stage, systematic uncertainties are hard to estimate, and all values given here should be taken with this in mind.

A summary of projected measurements of τ-lepton properties is presented in Table 4. More than one order of magnitude improvements are expected in the lifetime and branching fraction measurements. This will enable tests of lepton universality down to a precision at the 0.01% level. Going beyond that, will require also an improved precision on the τ mass. Here, FCC-ee may be able to touch upon the present precision, however, substantial improvements will be more likely obtained via a high statistics scan of the production threshold at a next generation τ factory.

Precise tests of charged lepton flavour violation in Z decays, with branching fraction limits in the range 10^{-6}–10^{-5}, were performed at LEP. LHC measurements have been able

Table 4: Measurement of τ-lepton properties at FCC-ee, compared to the present precisions

Observable	Present value $\pm$ error	FCC-ee stat.	FCC-ee syst.
m_τ (MeV)	1776.86 ± 0.12	0.004	0.1
$\mathcal{B}(\tau \to \mathrm{e}\bar{\nu}\nu)$ (%)	17.82 ± 0.05	0.0001	0.003
$\mathcal{B}(\tau \to \mu\bar{\nu}\nu)$ (%)	17.39 ± 0.05	0.0001	0.003
τ_τ (fs)	290.3 ± 0.5	0.001	0.04

Table 5: FCC-ee sensitivities to cLFV processes in Z decays and in two benchmark τ-decay modes, compared to present bounds. The range of values for the $\mathrm{Z} \to \mu\mathrm{e}$ mode reflects whether or not particle identification via $\mathrm{d}E/\mathrm{d}x$ will be available.

Decay	Present bound	FCC-ee sensitivity
$\mathrm{Z} \to \mu\mathrm{e}$	0.75×10^{-6}	10^{-10}–10^{-8}
$\mathrm{Z} \to \tau\mu$	12×10^{-6}	10^{-9}
$\mathrm{Z} \to \tau\mathrm{e}$	9.8×10^{-6}	10^{-9}
$\tau \to \mu\gamma$	4.4×10^{-8}	2×10^{-9}
$\tau \to 3\mu$	2.1×10^{-8}	10^{-10}

to improve on this, so far, by about a factor of two for the $\mathrm{Z} \to \mu\mathrm{e}$ channel. In general about an order of magnitude improvements can be ultimately expected from the full LHC samples. With its enormous Z statistics, FCC-ee will be able to improve dramatically on this with sensitivities about four orders of magnitude higher than today's. The situation is summerized in Table 5.

Very precise tests of charged lepton flavour violation in τ decays have been carried out at the b factories, and will be further improved at Belle II. With the larger sample of τ decays, and with the higher boost, FCC-ee will be able perform competitive measurements in this area. The situation is presented in Table 5 for two benchmark processes.

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