Generalized Parton Distribution Functions of ρ Meson

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October 13, 2019

Abstract

We report our recent calculations for the generalized parton distribution functions of the ρ meson with the help of a light-front constituent quark model. The electromagnetic form factors and structure functions of the system are given. Moreover, we also show our results for its gravitational form factors (or energy-momentum tensor form factors) and for other mechanical properties, like its mass distributions, pressures, share-forces, and $D-$term.

Contents

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1 Introduction

The study of the generalized parton distribution functions (GPDs) is a key issue to understand the internal properties since they can give a three-dimensional description of a complicate system [1–4]. The sum rules of GPDs are directly related to the form factors (FFs), and in the forward limit, GPDs directly connect to the parton distribution functions (PDFs) as well. The detailed information of the GPDs can be measured by the deeply virtual Compton scattering or by the vector meson electro-production.

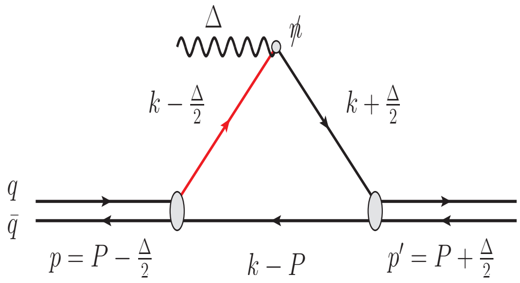
There are many theoretical studies for the GPDs of nucleon (spin-1/2) [5–8], of pion meson (spin-0) [9, 10], and of some nuclei, like ^3He [11], or ^4He [12], and deuteron [13] in the literature. It should be mentioned that the experimental measurements of the Compton form factors of the nuclei (like ^3He or ^4He [14–16]) have already been carried out in Jefferson Lab.

For the spin-1 particles, like the ρ meson and deuteron, there are also some discussions for their FFs, structure functions, transverse momentum distributions, and GPDs in the literature [17–20]. It is addressed that the spin-1 particle, different from spin-1/2 particles, like nucleon and ^3He, and from the spin-0 system, like π-meson, it has three polarizations and therefore has tensor structure function b_1, which is related the parton distribution function of the longitudinally polarized target. There was an experimental measurement for the b_1 of deuteron target at HERMES, however, the available data cannot be simply understood by the deuteron structure functions constructed from the convolution approach by considering the deuteron being a weakly bound state of a proton and neutron [21]. It is expected that future Jefferson Lab. would provide a more precise measurement of the deuteron tensor structure function.

We know that the vector meson ρ is a spin-1 particle as well. It is believed as a two-body system with a quark and antiquark pair and its wave function is expected to be the $S-$wave dominant. Since the electromagnetic (EM) interaction to the quark is much simpler than that to the nucleon (proton or neutron), we focus our attention on a GPDs study of a ρ meson firstly.

2 Generalized parton distribution functions of a spin-1 particle

According to the general analyses of Ref. [13], for each quark flavour and the gluons, there are nine parton helicity conserving GPDs for a spin-1 particle. In the quark sector, there are five unpolarized GPDs $H_i^q(x, \xi, t)$ with $i = 1, 2, \cdots 5$ and the superscript q standing for the contribution of the quark with flavor q, and there are four polarized GPDs



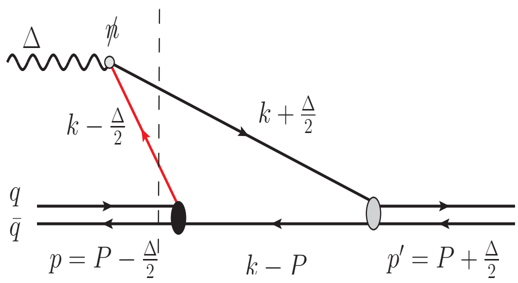


Figure 1: GPDs of the ρ meson. ((a), left) Direct Feynman diagram contributed to the GPDs by the stuck quark (q) in the valence region, and ((b), right) the stuck u quark in the non-valence region.

$\tilde{H}_i^q(x,\xi,t)$ with $i=1,2,\cdots 4$. Those GPDs are defined by the matrix element of

$$\begin{aligned} V_{\lambda'\lambda} &= \frac{1}{2}\int\frac{d\kappa}{2\pi}e^{ix\kappa(\mathcal{P}\cdot n)} < p',\lambda' \mid \bar{\psi}\Big(-\frac{\kappa n}{2}\Big)\not n\psi\Big(\frac{\kappa n}{2}\Big) > \\ &= -(\epsilon'^*\cdot\epsilon)H_1^q + \frac{(\epsilon\cdot n)(\epsilon'^*\cdot\mathcal{P})+(\epsilon\cdot n)(\epsilon'^*\cdot\mathcal{P})}{\mathcal{P}\cdot n}H_2^q - 2\frac{(\epsilon\cdot\mathcal{P})(\epsilon'^*\cdot\mathcal{P})}{M^2}H_3^q \\ &\quad+\frac{(\epsilon\cdot n)(\epsilon'^*\cdot\mathcal{P})-(\epsilon\cdot n)(\epsilon'^*\cdot\mathcal{P})}{\mathcal{P}\cdot n}H_4^q + \Big[M^2\frac{(\epsilon\cdot n)(\epsilon'^*\cdot n)}{(\mathcal{P}\cdot n)^2}+\frac{1}{3}(\epsilon'^*\cdot\epsilon)\Big]H_5^q \\ &= \sum_{i=1,5}\epsilon'^{*\nu}(p',\lambda')V^{(i)}_{\nu\mu}\epsilon^\mu H_i^q(x,\xi,t), \end{aligned} \tag{1}$$

for unpolarized case and

$$\begin{aligned} \tilde{V}_{\lambda'\lambda} &= \frac{1}{2}\int\frac{d\kappa}{2\pi}e^{ix\kappa(\mathcal{P}\cdot n)} < p',\lambda' \mid \bar{\psi}\Big(-\frac{\kappa n}{2}\Big)\not n\gamma_5\psi\Big(\frac{\kappa n}{2}\Big) > \\ &= -i\frac{\epsilon_{\mu\alpha\beta\gamma}n^\mu\epsilon'^{*\alpha}\epsilon^\beta\mathcal{P}^\gamma}{\mathcal{P}\cdot n}\tilde{H}_1^q + 2i\frac{\epsilon_{\mu\alpha\beta\gamma}n^\nu\Delta^\alpha\mathcal{P}^\beta}{\mathcal{P}\cdot n}\frac{\epsilon^\gamma(\epsilon'^*\cdot\mathcal{P})+\epsilon'^*(\epsilon\cdot\mathcal{P})}{M^2}\mathcal{H}_2^q \\ &\quad+2i\frac{\epsilon_{\mu\alpha\beta\gamma}n^\nu\Delta^\alpha\mathcal{P}^\beta}{\mathcal{P}\cdot n}\frac{\epsilon^\gamma(\epsilon'^*\cdot\mathcal{P})-\epsilon'^*(\epsilon\cdot\mathcal{P})}{M^2}\mathcal{H}_3^q \\ &\quad+\frac{i}{2}\frac{\epsilon_{\mu\alpha\beta\gamma}n^\nu\Delta^\alpha\mathcal{P}^\beta}{\mathcal{P}\cdot n}\frac{\epsilon^\gamma(\epsilon'^*\cdot n)+\epsilon'^*(\epsilon\cdot n)}{\mathcal{P}\cdot n}\mathcal{H}_4^q \\ &= \sum_{i=1}^4\epsilon'^{*\beta}\tilde{V}^{(i)}_{\beta\alpha}\epsilon^\alpha\tilde{H}_i(x,\xi,t), \end{aligned} \tag{2}$$

for polarized case. In the above two equations, ψ stands for the quark field, M is the mass of the system, $\epsilon'(p',\lambda')$ ($\epsilon(p,\lambda)$) is the polarization vector of the final (initial) spin-1 particle with the momentum and polarization of (p',λ') ((p,λ)), respectively. In eqs. (1-2), n is a light-like 4-vector with $n^2=0$, $\mathcal{P}=(p'+p)/2$, $t=\Delta^2=(p'-p)^2$, and $\xi=-\frac{\Delta\cdot n}{2\mathcal{P}\cdot n}=-\frac{\Delta^+}{2\mathcal{P}^+}$. It should be mentioned that ξ is called skewness parameter describing the longitudinal momentum asymmetry. Figure 1 shows the GPDs of the ρ meson with the valence and non-valence contributions, respectively.

2.1 Form factors

The sum rules of GPDs give the form factors of the system as

$$\int_{-1}^{1} dx H_i^q(x,\xi,t) = G_i^q(t) \quad (i=1,2,3), \qquad \int_{-1}^{1} dx H_i^q(x,\xi,t) = 0 \quad (i=4,5);$$
$$\int_{-1}^{1} dx \tilde{H}_i^q(x,\xi,t) = \tilde{G}_i^q(t) \quad (i=1,2), \qquad \int_{-1}^{1} dx \tilde{H}_i^q(x,\xi,t) = 0 \quad (i=3,4) \tag{3}$$

where $G_{1,2,3}(t)$ are the known three form factors of the spin-1 particle which relate to the EM vector current

$$\begin{aligned} I^{\mu}_{\lambda'\lambda} &= < p'\lambda'|\bar{\psi}(0)\gamma^\mu\psi(0)|p,\lambda > \\ &= \epsilon'^{*\beta}\epsilon^{\alpha}\Big[-2\Big(G_1(t)g_{\beta\alpha} - G_3(t)\frac{\Delta_\beta\Delta_\alpha}{2M^2}\Big)\mathcal{P}^\mu - G_2(t)\Big(g^\mu_\alpha\Delta_\beta - g^\mu_\beta\Delta_\alpha\Big)\Big]. \end{aligned} \tag{4}$$

The three form factors $G_{1,2,3}(t)$ give the electromagnetic charge $G_C(t)$, magnetic $G_M(t)$ and quadrupole form factors $G_Q(t)$. $\tilde{G}_{1,2}(t)$ are the two axial vector form factors defined by the electro-weak (EW) matrix element of

$$\begin{aligned} \tilde{I}^{\mu}_{\lambda'\lambda} &= < p'\lambda'|\bar{\psi}(0)\gamma^\mu\gamma_5\psi(0)|p,\lambda > \\ &= -2i\epsilon^{\mu}{}_{\alpha\beta\gamma}\epsilon'^{*\alpha}\epsilon^{\beta}\mathcal{P}^\gamma\tilde{G}_1(t) + 4i\epsilon^{\mu}{}_{\alpha\beta\gamma}\Delta^\alpha\mathcal{P}^\beta\frac{\epsilon^\gamma(\epsilon'^*\cdot\mathcal{P}) + \epsilon'^\gamma(\epsilon'\cdot\mathcal{P})}{M^2}\tilde{G}_2(t). \end{aligned} \tag{5}$$

It should be stressed that the form factors are only t-dependent and the explicit ξ-dependence in GPDs of H and $\tilde{H}$, showed in eq. (3), vanishes after the integral with respect to x due to the analytic properties of GPDs.

In addition, the energy-momentum tensor $T^{\mu\nu}$ can also relate to the moment of GPDs as [22, 23]

$$\begin{aligned} < p',\lambda'|\hat{T}^{\mu\nu}(0)|p,\lambda > &= (\mathcal{P}\cdot n)\mathcal{P}^\nu\int xdx\int\frac{d\kappa}{2\pi}e^{ix\kappa(\mathcal{P}\cdot n)}\bar{\psi}\Big(-\frac{\kappa\cdot n}{2}\Big)\gamma^\mu\psi\Big(\frac{\kappa\cdot n}{2}\Big) \\ &= \Big\{2\mathcal{P}^\mu\mathcal{P}^\nu\Big(-\epsilon'^*\cdot\epsilon A_0(t)\frac{\epsilon'^*\cdot\mathcal{P}\epsilon\cdot\mathcal{P}}{M^2}A_1(t)\Big) \\ &\quad +\frac{1}{2}(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon D_0(t) + \frac{\epsilon'^*\cdot\mathcal{P}\epsilon\cdot\mathcal{P}}{M^2}D_1(t)\Big) \\ &\quad +2\Big[\mathcal{P}^\mu\big(\epsilon^\nu\epsilon'^*\cdot\mathcal{P} + \epsilon'^{*\nu}\epsilon\cdot\mathcal{P}\big) + \mathcal{P}^\nu\big(\epsilon^\mu\epsilon'^*\cdot\mathcal{P} + \epsilon'^{*\mu}\epsilon\cdot\mathcal{P}\big)\Big]J(t) \\ &\quad +\Big[\frac{1}{2}\big(\epsilon^\mu\epsilon'^{*\nu} + \epsilon^\nu\epsilon'^{*\mu}\big)\Delta^2 - \big(\epsilon'^{*\mu}\Delta^\nu + \epsilon'^{*\nu}\Delta^\mu\big)\epsilon\cdot\mathcal{P} \\ &\quad +\big(\epsilon^\mu\Delta^\nu + \epsilon^\nu\Delta^\mu\big)\epsilon'^*\cdot\mathcal{P} - 4\epsilon\cdot\mathcal{P}\epsilon'^*\cdot\mathcal{P}g^{\mu\nu}\Big]E(t)\Big\} + ... \end{aligned} \tag{6}$$

where $A_{0,1}(t)$, $D_{0,1}(t)$, $J(t)$, and $E(t)$ are the six energy-momentum conserved gravitational form factors of the spin-1 system, and $\cdots$ denotes the other energy-momentum non-conserved form factors. All those gravitational form factors (GFFs) can be extracted from the moments of GPDs, and they give the mechanical properties, like the mass distributions, share forces, pressures, and the $D-$term, of the considered system. For example, the mass radius is defined as

$$< |r^2| >_{Grav.} = \frac{1}{M^2}\int d^3r r^2 T^{00}(\vec{r}) = -6\frac{dA_0(t)}{dt}\Big|_{t\to 0}, \tag{7}$$

where the static EMT $T^{\mu\nu}(\vec{r},\sigma',\sigma)$ of the spin-1 particle is defined by the Fourier transformation of the EMT with respect to $\vec{\Delta}$. The pressures and share forces $p_i(r)$ and $s_i(r)$ are

$$\begin{aligned}&\int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} < p',\lambda'|T^{ij}(0)|p,\lambda > \\ &= p_0(r)\delta^{ij}\delta_{\lambda'\lambda} + s_0(r)Y_2^{ij}\delta_{\lambda'\lambda} \\ &\quad + p_2(r)\hat{Q}^{ij}_{\lambda'\lambda} + 2s_2(r)\big[\hat{Q}^{ip}_{\lambda'\lambda}Y_2^{pj} + \hat{Q}^{jp}_{\lambda'\lambda} - \delta^{ij}\hat{Q}^{pq}_{\lambda'\lambda}Y_2^{pq}\big] + ...,\end{aligned} \tag{8}$$

where $Y_2^{ij} = \frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}$ and the quadrupole operator $\hat{Q}^{ij}_{\lambda'\lambda} =< p,\lambda'|\hat{Q}^{ij}|p,\lambda >$ with $\hat{Q}^{ij} = \frac{1}{2}\big(\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\big)$. In the above equation, the appearance of p_2 and s_2 etc. is due to the fact that the system is a spin-1 and it has quadrupole form factor. In unpolarized case, i.e. under averaging over polarizations, $\hat{Q}^{ij}_{\lambda'\lambda} = 0$ and therefore only p_0 and s_0 survive.

Finally, the $D-$term of the system is

$$\begin{aligned} D &= -\frac{2}{5}m\int d^3r(r^2Y_2^{ij})\sum_{\lambda'\lambda}\frac{\delta_{\lambda'\lambda}}{3}T^{ij}(\vec{r},\lambda',\lambda) = -\frac{4}{15}m\int d^3r^2 s_0(r) && (9)\\ &= -D_0(0) + \frac{4}{3}E(0), && (10)\end{aligned}$$

which stands for the fundamental property and is negative characterizing a stable system.

2.2 Parton distribution functions

In the forward limit, namely $\xi \to 0$, the parton distribution functions relate to GPDs as

$$\begin{aligned} H_1(x,0,0) &= \frac{q^1(x) + q^{-1}(x) + q^0(x)}{3} = q(x) \to F_1(x) \\ H_5(x,0,0) &= q^0(x) - \frac{q^1(x) + q^{-1}(x)}{2} \to b_1(x) \\ \tilde{H}_1(x,0,0) &= q^1_\uparrow(x) - q^1_\downarrow(x) = \Delta q(x) \to g_1(x)\end{aligned} \tag{11}$$

where $q^\lambda = q^\lambda_\uparrow + q^\lambda_\downarrow$ stands for the parton distributions with the polarization parallel $\uparrow$ or anti-parallel $\downarrow$ to the motion of the spin-1 particle with polarization of λ. $b_1(x)$ is the tensor structure function, which is unique for the spin-1 particle.

3 Calculations of the ρ meson

It is known the ρ meson and deuteron are the typical spin-1 meson and nuclei, respectively. Since the interaction of a probe to a quark is much simpler than the one to a nucleon, and moreover, it is believed that the wave function of the ρ meson is almost pure S-wave, we consider the ρ meson system firstly.

3.1 Light-front quark model

To describe the ρ meson in a quark degrees of freedom, we follow Ref. [24] to write an effective Lagrangian of meson-quark-quark as

$$\begin{aligned}\mathcal{L}_{qq\rho} &= -i\frac{m}{f_\rho}\bar{q}\Gamma^\mu\vec{\tau}q\cdot\vec{\rho}_\mu \\ &= -i\frac{m}{f_\rho}\Big[\bar{u}\Gamma^\mu u\rho^0_\mu+\sqrt{2}\bar{u}\Gamma^\mu d\rho^+_\mu+\sqrt{2}\bar{d}\Gamma^\mu u\rho^-_\mu+\bar{d}\Gamma^\mu d\rho^0_\mu\Big],\end{aligned} \tag{12}$$

where f_ρ is the decay constant of ρ, m is the constituent quark mass, and the phenomenological vertex Γ^μ equals to γ^μ plus the momentum-dependent term of the parton. To consider the bound state properties, we phenomenologically employe the quark momentum distribution inside the ρ meson as

$$\Lambda\big(k-\frac{1}{2}\mathcal{P},p\big)=\frac{c}{[(k-\frac{1}{2}\mathcal{P})^2-M_R^2+i\epsilon][(k-\frac{1}{2}\Delta)^2-M_R^2+i\epsilon]}, \tag{13}$$

where M_R is the regulator mass and k stands for the momentum of the active quark. This vertex of momentum distribution stands for the wave function of a bound state. Then, we calculate the matrix elements of eqs. (1-2) by performing a loop integral (see Fig. 1), and extract the GPDs of the ρ meson. In our phenomenological approach, we have three model-parameters. The quark mass, regulator mass M_R and the constant c in the momentum distribution. The last one can be determined by the normalization of the ρ^+ meson charge, and the former two parameters are optimally selected as $m = 0.403$ and $M_R = 1.61$ GeV, respectively. After the extraction of GPDs, we can get the EM and EW form factors from the sum rules of GPDs, the structure functions in the forward limit ($\xi \to 0$), as well as other mechanical properties like the pressures and mass distributions of the ρ meson. It should be addressed that in our calculation we simultaneously consider the valence and non-valence contributions (see the two figures in Fig. 1). In the non-forward limit, namely $\xi \neq 0$ and $|\xi| < 1/\sqrt{1-4M^2/t}$, the non-valence contribution is found to be sizeable, and we simply employe the prescription of Ref. [25] for the non-valence contribution. In our calculation, we also reach the continuity from the valence to the non-valence regions, and moreover, we preserve the sum rules of eq. (3) numerically at different ξ.

3.2 Form factors

The "3-dimensional" GPDs have been explicitly plotted in our calculation of Refs. [26, 27] at different skewness ξ, where the EM and EW form factors are also obtained according to eq. (3). Our calculated magnetic moment is $2.06/2M_\rho$, which fairly agrees with other model calculations as analyzed by Refs. [19, 20]. The estimated quadrupole moment is $-0.323/M_\rho^2$. This value also consistent with other model calculations. Our estimated charge radius is about 0.72 fm. Fig. 2 shows our calculated EM form factors and EW form factors $\tilde{G}_1(t)$ contributed by u quark. Since we also calculate the GPDs of the ρ meson in the non-forward limit $\xi \neq 0$, by considering the contribution of the non-valence region, we check the sum rule of eq. (3) and find that the sum of the contributions of the valence and non-valence regions at some value of ξ is almost the same as the contribution from the valence region to the form factors at the forward limit. Namely, our numerical results almost verify the sum rule.

3.3 Structure functions

In the forward limit ($\xi = 0$), we get the calculated structure functions of the spin-1 particle from our estimated GPDs. Fig. 3 show our two model calculated structure func-

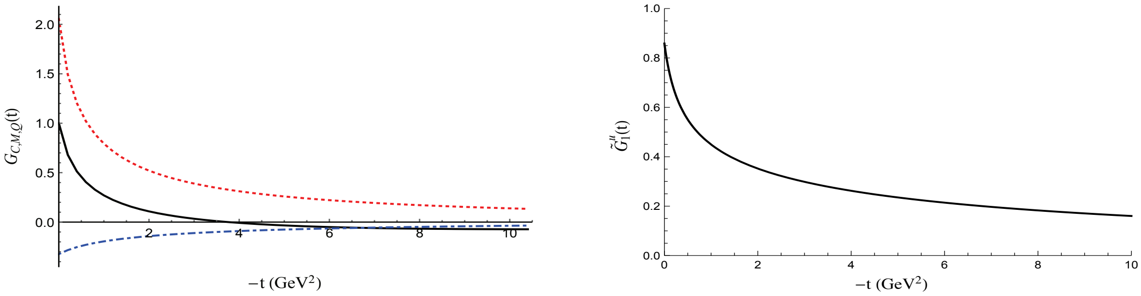


Figure 2: ρ meson form factors. ((a), left) EM form factors, charge G_c (solid black curve), magnetic G_M (dashed red curve), and quadrupole G_Q (dotted-dashed blue curve) form factors, and (right (b)) EW axial form factor $\tilde{G}_1^u(t)$ contributed by the u quark.

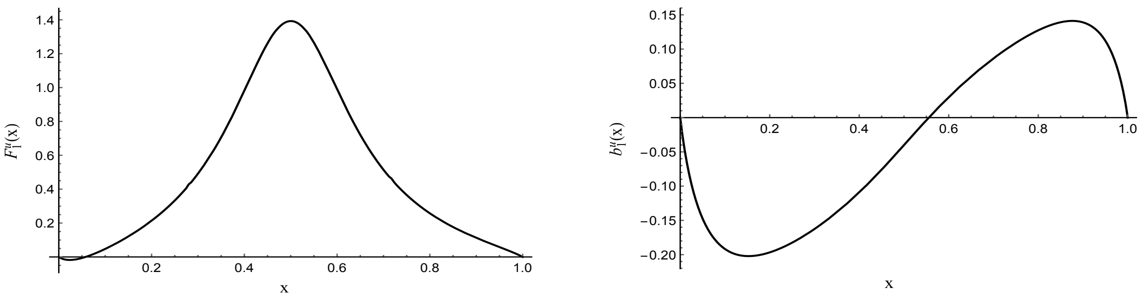


Figure 3: ρ meson Structure functions. ((a), left) $F_1^{(u)}$ contributed by u quark, and ((b), right) $b_1^{(u)}(x)$ contributed by the u quark.

tions of $F_1(x)$ and $b_1(x)$.

It has been mentioned that a system with spin-0 or spin-1/2 does not contain the tensor structure function. Since we only consider the constituent quark in the ρ meson, the calculated tensor structure function b_1 is resulted from the constituent quarks of the system. Our numerical results in Fig. 3(b) show that the known Close-Kumano [28] sum rule for the tensor structure function $\int dx b_1(x) = 0$ in the parton model almost preserves.

3.4 Mechanical properties

Form the calculated GPDs, we can get the gravitational form factors of the system. Fig. 4 show four typical GFFs of $A_0(t)$, $A_1(t)$, $J(t)$, and $E(t)$, respectively. It should be reiterated that in the non-forward limit ($\xi \neq 0$), we also consider the non-valence contribution and the sum of the valence and non-valence contributions to the GFFs are found to be numerically almost the same as the valence contribution in the forward limit ($\xi = 0$). Other mechanical properties, like the mass distributions, pressures, share-forces, and $D-$term of the ρ meson can be calculated as well from the obtained GPDs. Fig. 5 display the results for the unpolarized mass distributions, share forces, and pressures of the ρ meson in our approach.

From the mass distribution and eq. (7), we can get the mass radius. Our phenomenological approach gives $\sqrt{< |r^2| >_{Grav.}} \sim 0.54$ fm, which is smaller than the calculated charge radius. This feature is reasonable and consistent with the nucleon case [29, 30]. The pressure in Fig. 5(b) is similar to the pressure obtained for the nucleon case as well.

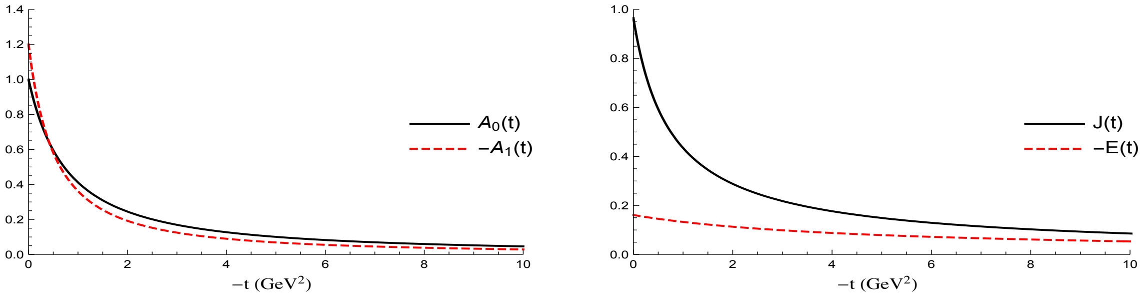


Figure 4: Some gravitational form factors of the ρ meson. ((a), left) $A_0(t)$ and $A_1(t)$, and ((b), right) $J(t)$ and $E(t)$.

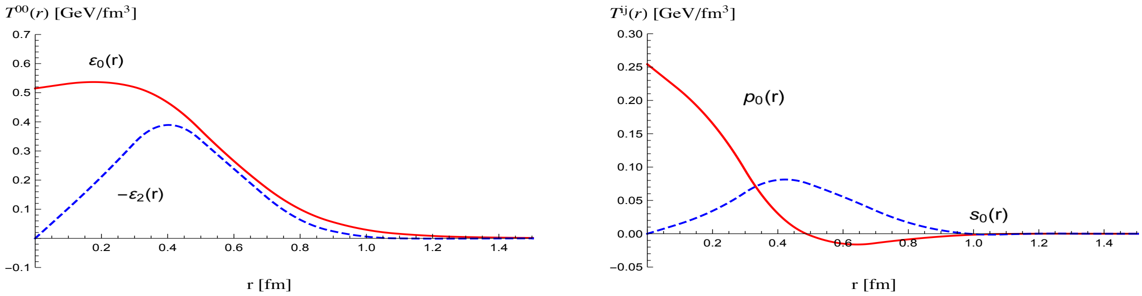


Figure 5: ρ meson mechanical properties. ((a), left) mass distributions $\epsilon_0(r)$ and $\epsilon_2(r)$, and ((b), right) share force $s_0(r)$ and pressure $p_0(r)$.

Moreover, our calculated $D = -0.21$, explicitly show that our spin-1 system is a stable one.

4 Summary

We summarize our recent studies for the properties of the ρ meson (a spin-1 particle). After performing the loop calculation, we, first of all, extract GPDs of the system. Both the contributions from the valence and non-valence regions are explicitly considered in the non-forward limit ($\xi \neq 0$). Our calculated low-energy observables, such as the form factors, are in a good agreement with other model and Lattice calculations. We also check the valence and non-valence contributions for the form factors and the continuity from the valence to the non-valence regions. Our numerical results display that the obtained form factors are almost ξ-independent.

By employing the forward limit, we obtain the structure functions, like $F_1(x)$, $g_1(x)$ and $b_1(x)$. The tensor structure function is unique for a spin-1 system. We find that our calculated b_1 contributed the constituent quark and antiquark inside the system almost satisfies the known Close-Kumano sum rule [28].

We also calculate the moments of our obtained GPDs and then extract the gravitational form factors for quarks. For the spin-1 system, it has six energy-momentum conserved gravitational form factors. The resulted GFFs give the mechanical properties of the system, like mass distributions, share forces, pressures, and the D-term. Our model calculation shows the mass radius is about 0.54 fm which is smaller than its calculated charge radius ~ 0.72 fm. The D term ~ -0.21 tells that the considered ρ meson is stable.

This feature is the consequence of the spin-1 system in terms of the quark-antiquark pair bound state.

Finally, our GPDs give a "3-dimensional" description for the space-like properties of the system. We may further apply our approach to the deuteron target, which can be explained by a weakly bound state of a proton and neutron. Moreover, we can also calculate the time-like properties of those spin-1 systems, like to calculate the generalized distribution amplitudes.

Acknowledgements

This work is supported by the National Natural Sciences Foundations of China under the grant Nos. 11975245 and 11475192, the Sino-German CRC 110 by NSFC under the grant No.11621131001, and the Key Research Program of CAS, under the grant No. Y7292610K1. Y. B. Dong thanks the hospitality of Institute of Theoretical Physics, Tuebingen University.

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