Exploring the Unknown Λn Interaction

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Abstract

No published Λn scattering data exist. A relativistic heavy-ion experiment has suggested that a Λnn bound state was seen. However, several theoretical analyses have cast serious doubt on the bound-state assertion. Nevertheless, there could exist a three-body Λnn resonance. Such a resonance could be used to constrain the Λn interaction. We discuss Λnn calculations using nn and Λn pairwise interactions of rank-one, separable form that fit effective range parameters of the nn system and those hypothesized for the as yet unobserved Λn system based upon four different ΛN potentials. The use of rank-one separable potentials allows one to analytically continue the Λnn Faddeev equations onto the second complex energy plane in search of resonance poles, by examining the eigenvalue spectrum of the kernel of the Faddeev equations. Although each of the potential models predicts a Λnn sub-threshold resonance pole, scaling of the Λn interaction by as little as ∼5% does produce a physical resonance. This suggests that one may use photo-(electro-)production of the Λnn system from tritium as a tool to examine the strength of the Λn interaction.

1 Introduction

Unfortunately for hypernuclear physics there exist no published Λn scattering data. This reflects the absence of neutron and Λ targets or beams. Attempts have been made to infer estimates from Λp scattering data combined with the binding energies of few-body Λ hypernuclei. In particular, one utilizes the binding energy difference between the ground states of the mirror hypernuclei ${}^4_\Lambda$H and ${}^4_\Lambda$He to infer charge symmetry breaking in the ΛN interaction. However, recent publications have questioned the existing value of the Λ-separation energy of ${}^4_\Lambda$H [1] as well as the γ transition in ${}^4_\Lambda$He [2]. Moreover, we know not whether the observed $A = 4$ charge symmetry breaking arises from the fundamental two-body ΛN interaction or from a possible ΛNN three-body force.

In a relativistic heavy-ion experiment performed by the HypHi collaboration [3], observation of a ${}^3_\Lambda$n bound state was reported. Specifically, a measurement was reported of the invariant mass of $\pi^- + d$ and $\pi^- + t$ in the scattering of ^{6}Li on graphite, *i.e.*,

$$ {}^6\mathrm{Li} + \mathrm{C} \rightarrow \begin{cases} \cdots + \pi^- + d \\ \cdots + \pi^- + t \end{cases} \tag{1} $$

in which a ${}^{3}_{\Lambda}$n bound state was possibly observed. We note that the peak in the $\pi^- + d$ and $\pi^- + t$ invariant mass is approximately at $m_\Lambda + m_n$ and $m_\Lambda + 2m_n$, respectively. This suggests a Λnn structure that could be associated with a bound state, because the measured lifetime of this state is comparable to the lifetime of a free Λ:

$$\text{Lifetime} = \begin{cases} 181^{+30}_{-24}\ \text{ps} & \pi^- + d \\ 190^{+47}_{-35}\ \text{ps} & \pi^- + t \\ 263.2 \pm 2.0\ \text{ps} & \text{free } \Lambda \end{cases} . \tag{2}$$

Such a ${}^{3}_{\Lambda}$n would be the lightest neutron-rich hypernucleus known. If such a bound state were to exist, our knowledge of the neutron-neutron (nn) interaction would allow us to place strong constraints upon the Λn interaction. Moreover, JLab would be an ideal facility to explore such a bound system, using the ^{3}H(e,e′K$^+$)${}^{3}_{\Lambda}$n electro-production reaction, although a weakly bound system would imply the need to measure a small cross section. [Alternative reactions at J-PARC would be ^{3}H(K$^-$, π^0)${}^{3}_{\Lambda}$n and ^{3}He(K$^-$, π^+)${}^{3}_{\Lambda}$n; the latter being a double-charge-exchange reaction suggests that the cross section would be very small.]

The possible existence of such a bound state was investigated theoretically by a number of groups [4–8] using a variety of few-body methods. The consistent result of these investigations was that there is no ${}^{3}_{\Lambda}$n bound state. To understand this, one need only recall that the hypertriton, a $T = 0$ state, is only barely bound having a Λ separation energy of

$$B_\Lambda({}^{3}_{\Lambda}\text{H}) = 0.13 \pm 0.05\ \text{MeV} . \tag{3}$$

This amounts to a system in which the Λ is very loosely bound to a deuteron. In comparing ${}^{3}_{\Lambda}$H with ${}^{3}_{\Lambda}$n, one is replacing the np interaction that supports a bound state (the deuteron) by an nn interaction that produces a di-neutron that is unbound. In fact, a simple estimate is that a Λnn state should be about 2.224 MeV (the binding energy of the deuteron) above the ground state of the ΛNN system, and therefore would be unbound. In this analysis, it is assumed that: (i) charge symmetry holds; *i.e.*, the Λn interaction is the same as the Λp interaction; (ii) there are no three-body forces acting, other than an effective three-body force that results from the coupling between the ΛN and ΣN channels in the hyperon-nucleon interaction.

Even if there is no ${}^{3}_{\Lambda}$n bound state, there might exist a Λnn resonance. Moreover, such a resonance could be used to constrain the Λn interaction. For that reason, we explore the possible existence of a Λnn resonance even though the underlying nn and Λn interactions are predominantly s-wave and support no two-body bound state. To accomplish this we consider a model in which the pairwise interactions are represented by rank-one separable potentials that reproduce the effective range parameters (scattering length and effective range) of 1) the nn system and 2) those predicted for the yet to be observed Λn system by five different Nijmegen one-boson-exchange potentials [9–13] and the Juelich one-boson-exchange potential [14], and a chiral ΛN potential [15]. All of the potential models are based upon the existing Λp scattering data. The use of rank-one separable potentials makes it possible for us to easily analytically continue the relevant Faddeev equations for the Λnn system onto the second complex energy plane in order to search for resonance poles. We perform the search by examining the eigenvalue spectrum of the kernel of the Faddeev equations. We previously employed this method in our investigation of possible resonances in Λd scattering [16].

In the $A = 4$ Λ-hypernuclei a larger charge symmetry breaking has been observed in the ground states than in the bound excited states:

$$\Delta_{\text{CSB}} \equiv B({}^{4}_{\Lambda}\text{He}) - B({}^{4}_{\Lambda}\text{H}) = \begin{cases} 0.233 \pm 0.092\ \text{MeV} & \text{for the } 0^+ \text{ g.s.} \\ -0.083 \pm 0.094\ \text{MeV} & \text{for the } 1^+ \text{ excited state} \end{cases} . \tag{4}$$

This is significantly larger than the charge symmetry breaking that is observed in the $A = 3$ nuclear system of ∼0.07 MeV, where $a_{\text{pp}} - a_{\text{nn}} \approx 1.5 \pm 0.5$ fm. [17]. This would suggest that there could be a substantial difference between the Λp and Λn scattering lengths. To examine this possible difference we consider the Nijmegen potentials: Model D [9], model NSC89 [10], model NSC97f [11], model NSC08c [12], and model NSC16 [13]. The values shown in Table 1 imply that the difference between the Λp and the Λn scattering lengths in the singlet and triplet channels quite likely requires further investigation, if we are to resolve any charge symmetry breaking at the two-body level.

Table 1: The Λn and Λp singlet and triplet scattering lengths (in fm) for five Nijmegen potentials. The charge symmetry breaking difference is $\Delta a_{\rm CSB} = a_{\Lambda p} - a_{\Lambda n}$.

	Singlet			Triplet		
ΛN Potential	$a_{\Lambda p}$	$a_{\Lambda n}$	$\Delta a_{\rm CSB}$	$a_{\Lambda p}$	$a_{\Lambda n}$	$\Delta a_{\rm CSB}$
Model D	-1.77	-2.03	0.26	-2.06	-1.84	-0.22
NSC89	-2.73	-2.86	0.13	-1.48	-1.24	-0.24
NSC97f	-2.51	-2.68	0.17	-1.75	-1.66	-0.09
NSC08c	-2.46	-2.62	0.16	-1.73	-1.72	-0.01
NSC16	-1.88	-1.96	0.08	-1.86	-1.84	-0.02

We would like to suggest the thesis that if one observes a Λnn resonance, then the energy and width of such a resonance might be used to place some constraint on the Λn scattering lengths, to complement the experimental Λp scattering data.

2 The Λnn Model

The ${}^{3}_{\Lambda}$H hypernucleus is just bound. The hypertriton can be considered to be a Λ loosely bound to the deuteron core by about 0.13 MeV. The small binding suggests that the Λ resides at a considerable distance from the core deuteron. In fact, model calculations show that the rms radius of the Λ is some 6 times the rms radius of the deuteron. Thus, the hypertriton is a true "halo" Λ hypernucleus. Therefore, we can infer that the most important feature of the ΛN interaction for the study of the ΛNN system is the long range component of the interaction (in r-space); this corresponds to the low energy ΛN amplitude. Such a situation can be described in an S-wave effective range approximation; *i.e.*, we can write the amplitude in the well known form

$$f_0(k) = e^{-i\delta_0}\,\sin\delta_0 = \frac{1}{\cot\delta_0 - i} = -\pi\mu k t_0(k)\ , \tag{5}$$

in which μ is the reduced mass of the ΛN , and

$$k\cot\delta_0 \approx -\frac{1}{a} + \frac{1}{2}rk^2\ , \tag{6}$$

is the familiar effective range expansion written in terms of the scattering length a and the effective range r.

2.1 The two-body interaction

The effective range parameters (a, r), which parameterize the ΛN amplitude at low energies, can be used to define a rank one Yamaguchi separable potential [18]. To ensure that the reader can reproduce our numerical results, we repeat here the well known Yamaguchi potential formalism:

$$V(k,k') = g(k)\,C\,g(k') \quad \text{with} \quad g(k) = \frac{1}{k^2+\beta^2}\ . \tag{7}$$

The parameters of the potential, C and β, can be expressed in terms of the effective range parameters [19] as:

$$\beta = \frac{1}{2r}\left[3+\sqrt{9-16\frac{r}{a}}\right] \quad \text{and} \quad C = \frac{4\beta^3}{\pi\mu(1-\beta r)} \ . \tag{8}$$

The corresponding off-shell t-matrix, $t_0(k,k';E)$, has the familiar separable form

$$t_0(k,k';E^+) = g(k)\,\tau(E^+)\,g(k') \ . \tag{9}$$

Here $\tau(E^+)$ is the quasiparticle propagator that can be expressed in terms of the potential form factor $g(k)$ and the strength C as follows:

$$\tau(E^+) = \left\{ C^{-1} - \int_0^\infty dk\,k^2\, \frac{[g(k)]^2}{E^+ - \frac{k^2}{2\mu}} \right\}^{-1} \ . \tag{10}$$

Thus, we are able to construct a ΛN amplitude that satisfies i) two-body unitarity and ii) is uniquely determined by the effective range parameters a and r. This amplitude can be utilized in the three-body Faddeev equations for the ΛNN system in order to determine the low energy spectrum (both bound states and resonances) .

One can show that an alternative way of determining the parameters of the rank one separable potential in the effective range approximation (Eq. (6)) is to note that, because $k\cot\delta_0$ is a quadratic in k, the S-wave amplitude $t_0(k)$ has two poles residing at k_1 and k_2 . The effective range parameters can be expressed in terms of k_1 and k_2 as

$$a = \frac{i\,(k_1+k_2)}{k_1\,k_2} \quad \text{and} \quad r = \frac{2\,i}{k_1+k_2} \ . \tag{11}$$

Thus, the ΛN amplitude can be written: i) in terms of the effective range parameters, or ii) in terms of the poles of the amplitude. The question we address below is whether the Λnn resonance parameters might be more usefully represented in terms of i) the effective range parameters or ii) the poles of the two-body amplitude.

2.2 The separable potential ΛNN equations

It is necessary to utilize a three-body formalism that encompasses both bound states and scattering states including three-body resonances in order to investigate the low energy spectrum of the ΛNN system. The Faddeev formalism [20–22] is optimum for this purpose, because it treats bound states and resonances on equal footing. This allows one to follow the poles of the S-matrix, as the parameters of the input two-body amplitude are varied in a continuous manner, from bound state to resonant state.

It is convenient to consider the AGS [23] form of the Faddeev equations for, *e.g.*, Λd scattering, employing only pairwise interactions [16]:

$$X_{\alpha,\beta}(E) = \bar{\delta}_{\alpha\beta}\,G_0(E) + \sum_\gamma \bar{\delta}_{\alpha\gamma}\,G_0(E)\,t_\gamma(E)X_{\gamma\beta}(E) \ . \tag{12}$$

Here E is the total energy, $G_0(E)$ is the free three-body Green's function, α, β, and γ label the three-body channels, $\bar{\delta}_{\alpha\beta} = 1-\delta_{\alpha\beta}$, and $t_\gamma(E)$ is the two-body amplitude for the interacting pair $(\beta\alpha)$ in the three-body Hilbert space. The solution to this integral equation can be constructed in terms of the eigenfunctions and eigenvalues of the kernel as [16, 24],

$$X_{\alpha\beta}(E) = \sum_n |\phi_{n,\alpha}(E)\rangle\, \frac{\left[\tilde{\lambda}_n(E^*)\right]^*}{1-\lambda_n(E)}\, \langle\tilde{\phi}_{n,\beta}(E^*)| \ , \tag{13}$$

Here $|\phi_{n,\alpha}(E)\rangle$ and $\lambda_n(E)$ are the familiar eigenfunctions and eigenvalues of the homogenous AGS integral equation

$$\lambda_n(E)\,|\phi_{n,\alpha}(E)\rangle = \sum_{\beta} \bar{\delta}_{\alpha\beta}\, G_0(E)\, t_\beta(E)\,|\phi_{n,\beta}(E)\rangle\ , \tag{14}$$

whereas $|\tilde{\phi}_{n,\beta}\rangle$ and $\tilde{\lambda}_n$ are the eigenfunctions and eigenvalues of the corresponding adjoint kernel of the integral equation. For all energies at which $\lambda_n(E)=1$ in Eq. (13) the scattering amplitude has a pole. When E is real and negative, this pole corresponds to a bound state. When E is complex with $\Im[E] < 0$ and $\Re[E] > 0$ and it lies on the second Riemann energy sheet, then the pole corresponds to a physical resonance . In this way one has the ability to explore the trajectory of the pole as one modifies the parameters of the two-body interaction. This approach has been used to explore the trajectory of three- [25,26] and four-neutron [26] resonances on the basis of realistic nucleon-nucleon interactions, which illustrates how one can study bound states and resonances within such a unified scheme.

When investigating a two-body system with a Hermitian Hamiltonian, the Lippmann-Schwinger equation admits poles for real negative energies, which correspond to bound states. The same holds true for the Faddeev equations. This is because these two- and three-body equations are defined on the first Riemann energy sheet, and they correspond to a Hermitian Hamiltonian. To treat bound states and resonances on equal footing, we rotate the contour of integration to expose the region of the second energy plane where resonances reside. This has effectively replaced a Hermitian Hamiltonian which admits S-matrix poles only on the real negative energy axis, with a non-Hermitian Hamiltonian that can have S-matrix poles at both real (bound state) and complex (resonance) energies.

There are no experimental data for Λn scattering. Therefore, we resort to modeling the interaction as rank one Yamaguchi separable potentials defined by the effective range parameters predicted by meson exchange potentials. After partial wave expansion, one can write Eq. (14) as a homogenous one dimensional integral equation of the form [19]

$$\lambda_n(E)\,\phi^{JT}_{n,k_\alpha}(q;E) = \sum_{k_\beta}\int_0^\infty dq'\; K^{JT}_{k_\alpha,k_\beta}(q,q';E)\;\phi^{JT}_{n,k_\beta}(q';E)\ . \tag{15}$$

The sum over k_β runs over all three-body channels for a given total angular momentum J and isospin T. The kernel of the integral equation is

$$K^{JT}_{k_\alpha,k_\beta}(q,q';E) = Z^{JT}_{k_\alpha,k_\beta}(q,q';E)\,\tau_{k_\beta}[E-\epsilon_\beta(q')]\,q'^2\ . \tag{16}$$

Here $Z^{JT}_{k_\alpha,k_\beta}$ is the Born amplitude for the exchange of particle γ, and τ_{k_β} is the quasiparticle propagator for the pair $\gamma\alpha$ defined in Eq. (10). The $\epsilon_\beta(q')$ is the energy of the spectator particle β. In Eq. (15) $\lambda_n(E)$ is the n^{th} eigenvalue of the kernel at energy E. The largest eigenvalue achieving a value of one at the bound state energy corresponds to the ground state of the Λnn system.

Thus, Eq. (15) allows one to determine the bound state of the system. To search for resonances one needs to analytically continue this equation onto the second Riemann energy sheet. To achieve this one deforms the contour of integration making certain not to cross any singularities of the kernel in the integral equation. This can be achieved here by the transformation

$$q \to q\,e^{-i\theta} \quad q' \to q'\,e^{-i\theta} \quad \text{with} \quad \theta > 0\ . \tag{17}$$

This exposes the region of the second Riemann energy sheet for which $|\arg E| < 2\theta$. To locate resonances we require $|\arg E| < \frac{\pi}{2}$. Because both q and q' are rotated by the same angle θ, the Born amplitude $Z^{JT}_{k_\alpha,k_\beta}(q,q';E)$ has no singularity for $\theta < \frac{\pi}{2}$. [27] Additional singularities of the kernel arise from $\tau_{k_\beta}[E-\epsilon(q')]$, the nn and Λn subsystem quasiparticle propagators. Because no nn or Λn bound states exist, the only singularity of τ_{k_β} is a branch point at the two-body subsystem thresholds. (This leads to the three-body threshold branch point at $E=0$.) Therefore, we can analytically continue Eq. (15) to $\theta < \frac{\pi}{2}$. This insures that $\Im[E] < 0$. In this way we are able to investigate bound states, physical resonances, and sub-threshold resonances for the Λnn system.

3 Numerical results

The nn and Λn rank one separable potentials provide the input for Eq. (15). For the nn interaction we use the experimental spin-singlet scattering length $a_s = -18.9 \pm 0.4$ fm and effective range $r_s = 2.75 \pm 0.11$ fm [28] to fix the parameters of the Yamaguchi potential to be

$$\beta_{nn} = 1.1574 \text{ fm.} \quad \text{and} \quad C_{nn} = -0.37986 \text{ fm}^{-2} \,. \tag{18}$$

There are no experimental data for the Λn interaction. As a theoretical model we choose the effective range parameters for the one boson exchange model of the Nijmegen potential Model D [9]. The theoretical Λn effective range parameters and the corresponding Yamaguchi parameters in the singlet and triplet channels are shown in Table 2.

Table 2: The Λn effective range parameters of the Nijmegen Model D [9] for the singlet and triplet channels, plus the parameters of the corresponding Yamaguchi potentials.

channel	a fm	r fm	β fm	C fm^{-2}
Singlet	-2.03 ± 0.32	3.66 ± 0.323	1.2503	-0.2692
Triplet.	-1.84 ± 0.10.	3.32 ± 0.11.	1.3786	-0.3608

In order to explore the Λnn pole trajectory we scale the singlet and triplet Λn potential strengths by the factor s; *i.e*, we use the transformation

$$C_s \to s\,C_s \quad \text{and} \quad C_t \to s\,C_t \,. \tag{19}$$

For $s=1$, the Λnn pole corresponds to a sub-threshold resonance at $E_R = -0.154 - 0.753\,i$ MeV. The largest eigenvalue in Eq. (15) is $\lambda(E_R) = 1.0000 - 0.0001\,i$. We plot the pole trajectory in Fig. 1 as the scaling factor s is increased from 1.0 to 1.4. By the time we reach $s = 1.075$ a physical Λnn resonance has formed. At $s = 1.350$ a Λnn bound state has developed. The pole has transitioned from a sub-threshold resonance, to a physical (observable) resonance, to a bound state by simply scaling the strength of the Λn interaction. This is achieved by using the same homogenous Faddeev integral equations, Eq. (15), after contour rotation by $\theta = 60^o$. For Nijmegen Model D one can see that a Λn potential whose parameters lie within the uncertainty of the experimental low energy Λp scattering parameters could generate a physical resonance in the Λnn system.

We consider four sets of effective range parameters in constructing separable Yamaguchi potentials to demonstrate that the trajectory for the Λnn pole is basically the same for differing models of the Λn interaction. The potentials are: Nijmegen Model *D* [9] used in Fig. 1, Nijmegen NSC97f [11], the Jülich [14] and the ΛN potential based upon the Chiral Lagrangian (Chiral (Λ = 600)) [15] reported by the Jülich group. [Note: The Jülich group reports only the Λp effective range parameters. Therefore, we use those Λp effective range parameters for the Jülich equivalent separable potentials. In contrast, for the Nijmegen equivalent separable

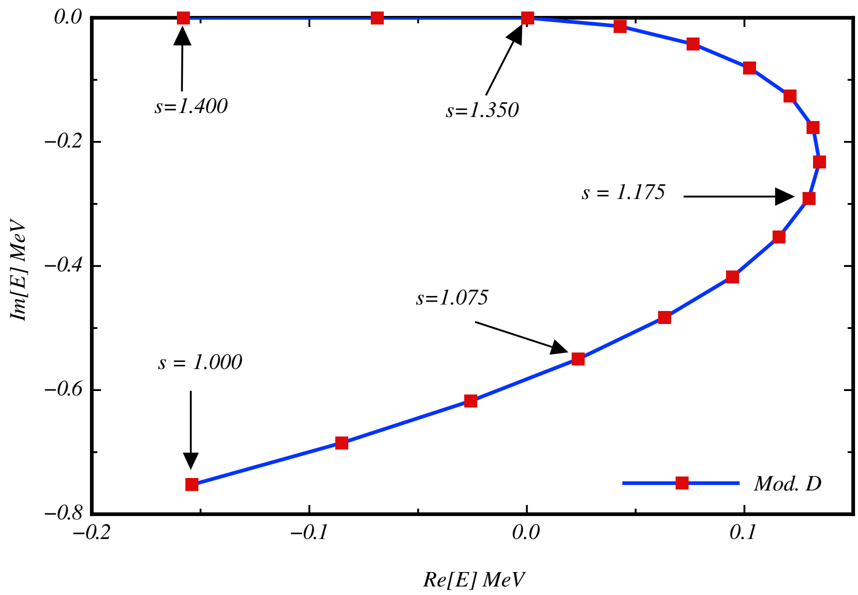

Figure 1: The trajectory of the Λnn pole as one varies the scaling factor s between 1.0 and 1.4 for a Yamaguchi separable potential based upon the effective range parameters of the Nijmegen Model D.

potentials we use the Λn effective range parameters.] In Fig. 2 we show the Λnn pole trajectories for these four separable potentials. Again, with an initial scaling factor $s = 1.000$ each of the four potentials yields a sub-threshold resonance, whereas with the scaling factor of $s = 1.400$ we obtain a Λnn bound state. The scaling factors for which the poles turn into physical resonances and then into bound states differ slightly for each of the potentials. Each of the two Nijmegen potentials have very similar trajectories. The two Jülich potential trajectories lie almost on top of one another. However, the starting pole positions (at $s = 1.000$) differ for each of the four potentials. We emphasize that in these calculations no tensor forces nor any coupling between the ΛN and ΣN channels have been included, as this would add more parameters to the models. One should include such additional sophistications in making a detailed comparison with experimental data.

4 The Λn effective range parameters and Λnn resonance energy

Can one extract the Λn effective range parameters from the position of the Λnn pole? Because we have two parameters defining the position of the pole and we have four effective range parameters (corresponding to the singlet and triplet Λn channels), the answer is not obvious . One might hope that the position of the pole would be less sensitive to one of the two effective range parameters, *e.g.* the effective range. If so, then one might extract the singlet and triplet scattering lengths from the position of the complex Λnn pole. We illustrate the trajectory of the Λnn pole in Fig. 3, as one varies first the singlet scattering length a_s (blue symbols) and then the singlet effective range r_s (red symbols). For these results a Yamaguchi separable potential was fitted to Nijmegen Model D [9], where we scaled the strength of the potential by a factor $s = 1.1$ in order to generate a physical resonance. The position of the Λnn pole is more sensitive to variation in the scattering length than to variation in the effective range. This suggests that one might be able to use the position of the Λnn pole to place some constraint on the singlet scattering length, a_s.

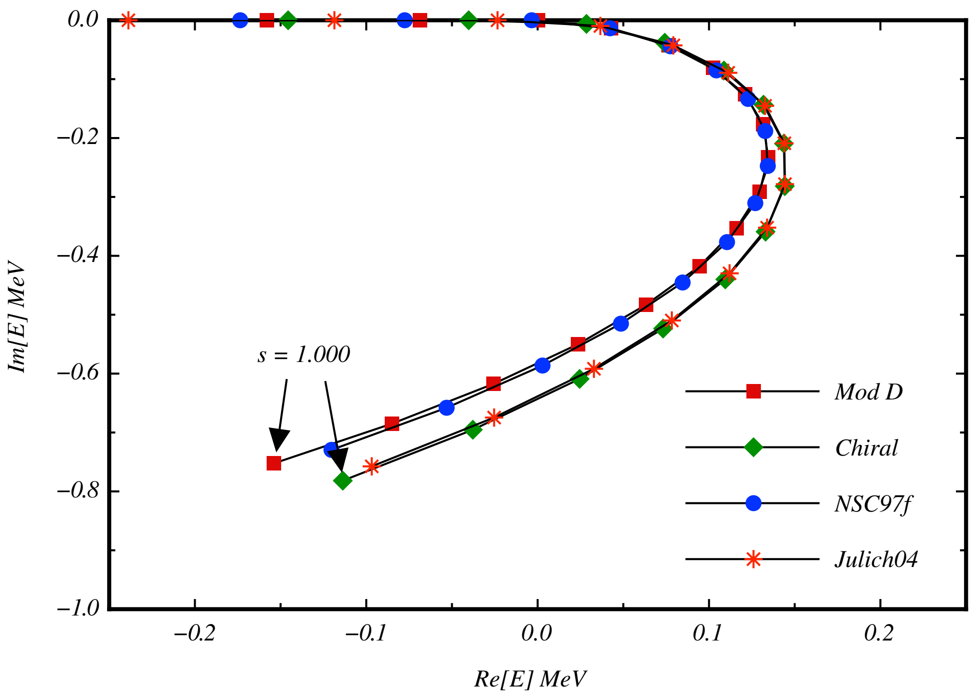

Figure 2: Trajectories of the Λnn pole as one scales the Λn Yamaguchi potentials with effective range parameters equivalent to two Nijmegen (Mod D, $NSC97f$) and two Jülich (Jülich04, Chiral) potentials.

One obtains a similar set of trajectories for the triplet channel. The variation in the position of the Λnn pole is more sensitive to variation in the scattering length a_t than variation in the effective range r_t. In Fig. 4 we compare the trajectory of the Λnn pole as one varies the singlet scattering length a_s and the triplet scattering length a_t, to illustrate the relative sensitivity of the Λnn pole to the triplet and singlet effective range parameters. Here we observe that the position of the pole is more sensitive to variations in the triplet scattering length a_t than to variations in the singlet scattering length a_s. This results from the stronger coupling between the triplet Λn three-body channel and the nn three-body channel.

Based upon the analysis of the movement in the Λnn pole position with variations in the Λn scattering lengths and effective ranges, we surmise that the pole position can place some constraints on the Λn effective range parameters. However, it is not possible to extract a definite set of, *e.g.*, scattering lengths from the pole position. To accomplish that would require an experiment that distinguishes between the singlet and triplet channels. Alternatively one might introduce theoretically meaningful constraints. From the Λnn pole trajectories for the potentials considered in Fig. 2, we infer that a Λnn resonance pole will lie close to the Λnn threshold. Therefore, it should be primarily sensitive to the parameters of the low energy Λn amplitudes. Scattering lengths and effective ranges (the intercept and slope of $k\cot\delta$ at the Λn threshold) are both low energy parameters. Nevertheless, the alternative parametrization in terms of the poles of the amplitude, k_1 and k_2, may provide insight into what theoretical constraints to the parameterization of the Λn amplitude would be helpful.

In order to gain insight into the role of the on-shell Λn amplitude poles in the effective range approximation, we examine the analytic structure of the on-shell amplitude for a given potential. There are two kinds of singularities in this amplitude: i) singularities that arise from the analytic form of the potential and ii) singularities that depend on the strength of the potential. For the Yukawa potential ($\frac{e^{-\mu r}}{r}$) the on-shell amplitude in the k-plane exhibits a branch point at $k = \frac{\mu}{2}i$ with the cut running to $+i\infty$ [30]. If we consider a meson exchange

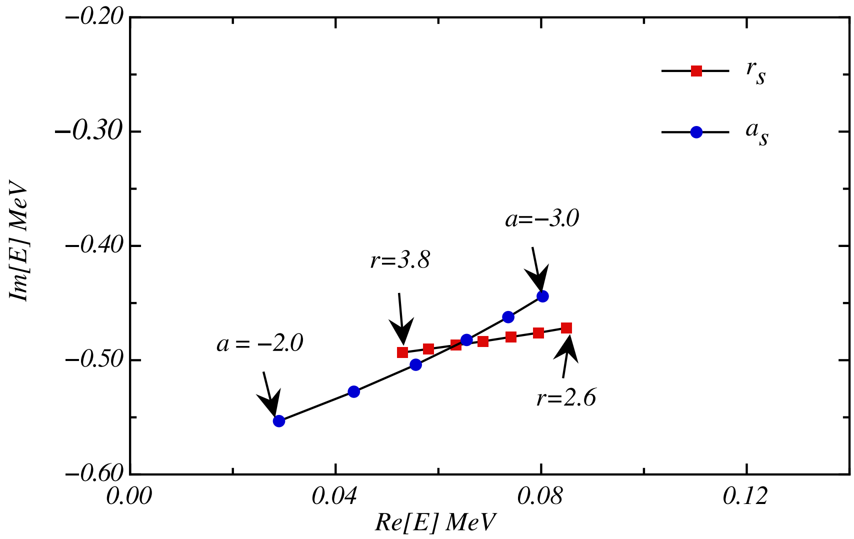

Figure 3: Variation in the position of the Λnn pole as one varies either the singlet scattering length a_s (fm) or the effective range r_s (fm). The starting interaction is a Yamaguchi potential based upon the Nijmegen Model D [9], with a scaling strength factor $s = 1.1$.

potential for the ΛN interaction, then the lightest meson exchanged is the K meson; this would generate a branch point at $k \approx +1.2\, i\ \mathrm{fm}^{-1}$. In addition to this branch point, there can exist a pole whose position is determined by the potential strength. If the potential ha sufficient strength to support a bound state, then a simple pole lies on the positive imaginary k-axis. However, if the strength of the potential is not sufficient to support a bound state, which is the case for the ΛN interaction, then the pole lies on the negative imaginary k-axis. When we replace the Yukawa potential with a separable potential, then the branch point is replaced by poles in the upper half of the k-plane, whereas poles depending on the strength of the interaction reside on the negative k-axis.

Consider the rank one separable Yamaguchi potential based upon the singlet Nijmegen NSC97f. The poles lie at:

$$k_1 = -0.266\,\mathrm{fm}^{-1}\ , \quad k_2 = k_3 = +1.340\,\mathrm{fm}^{-1}\ , \quad k_4 = -2.414\,\mathrm{fm}^{-1}\ .$$

The double pole ($k_2 = k_3$) lies on the positive imaginary axis and is the 'range parameters' β of the separable potential (see Eq. 7). The position of the pole k_1 is almost identical to that in the effective range amplitude, while the value of k_2 is of the same order as that which we see in the effective range approximation. The pole k_4, which depends on the strength of the potential, resides far from the threshold energy; k_4 does not appear in the effective range approximation. When we replace the on-shell amplitude of the potential by the on-shell amplitude generated in the effective range approximation (Eq. 6), then there exist only two poles. One pole lies on the negative imaginary axis near threshold, which we label as k_1, and the second lies on the positive imaginary axis, labeled as k_2.

As explicit examples, we consider the set of Nijmegen potentials NSC97 [11,29], in which the strength parameter α_V^m was varied to generate potentials with different Λn singlet effective range parameters. In Table 3 we list for each NSC97 potential the value of α_V^m, the singlet scattering length and effective range, and the values of k_1 and k_2, the positions of the singlet Λn poles. For comparison, we have also included in Table 3 the scattering length and effective range as well as the position of the poles k_1 and k_2 for the meson exchange potential Jülich04 [14] and the Jülich chiral potential with a cut-off $\Lambda = 600$ [15]. We point out that k_1

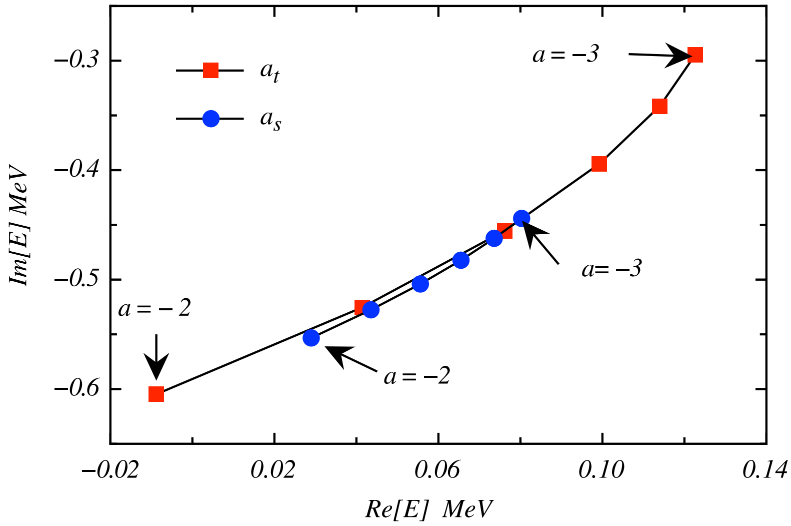

Figure 4: Variation in the position of the Λnn pole as one varies either the singlet scattering length a_s (fm) or the triplet scattering length a_t (fm). The starting interaction is a Yamaguchi potential based upon the Nijmegen Model D [9], with a scaling strength factor $s = 1.1$.

lies on the negative imaginary k-axis and is close to threshold, whereas k_2 lies on the positive imaginary k-axis and farther from threshold (*i.e.*, a higher energy parameter). Moreover, as the strength of the Nijmegen potential is varied due to changes in α_V^m, the pole at k_1 varies by as much as a factor of two, while k_2 varies only by ∼10%. This strongly implies that k_1 is more dependent on the strength of the interaction, whereas k_2 plays a role similar to the 'range' of the interaction. Comparing results for the Nijmegen potentials with those of the Jülich potentials, we find that the values of k_1 and k_2 are qualitatively similar. It would appear that one can make use of the 'range' of the theoretical Λn potential in the singlet and triplet cases to reduce the number of parameters from four to two. That is, one could fix singlet and triplet values of k_2 from a theoretical model and then adjust the values of k_1 to reproduce the position of the Λnn resonance. This would then provide a reasonable estimate of the singlet and triplet effective range parameters.

In Fig. 5 we plot the trajectory of the Λnn pole as one varies the position of pole of the Λn

Table 3: The variation in the position of the poles k_1 and k_2 of the effective range approximation for the Nijmegen NSC97 potentials [11,29] with changes in α_V^m. Also included for comparison are the values of k_1 and k_2 for the Jülich04 [14] and Jülich chiral potential with $\Lambda = 600$, [15].

Model	α_V^m	a_s (fm)	r_s (fm)	k_1 (fm^{-1})	k_2 (fm^{-1})
NSC97a	0.4447	-0.77	6.09	-0.509 i	0.838 i
NSC97b	0.4247	-0.97	5.09	-0.470 i	0.863 i
NSC97c	0.4047	-1.28	4.22	-0.4126 i	0.890 i
NSC97d	0.3847	-1.82	3.52	-0.343 i	0.911 i
NSC97e	0.3747	-2.24	3.24	-0.300 i	0.918 i
NSC97f	0.3647	-2.68	3.07	-0.265 i	0.917 i
Jülich04	–	-2.56	2.74	-0.282 i	1.012 i
chiral ($\Lambda = 600$)	–	-2.91	2.78	-0.254 i	0.973 i

singlet amplitude in the effective range approximation with changes in k_1 (blue symbols) or k_2 (red symbols). As expected the Λnn pole is more sensitive to variation in k_1 than k_2. This also suggests that one will need additional constraints to fix the effective range parameters. Based on the observation from Table 3, it would appear that we may estimate the position of the k_2 pole based on the theoretical range of the Λn potential, and then determine k_1 from the position of the Λnn pole.

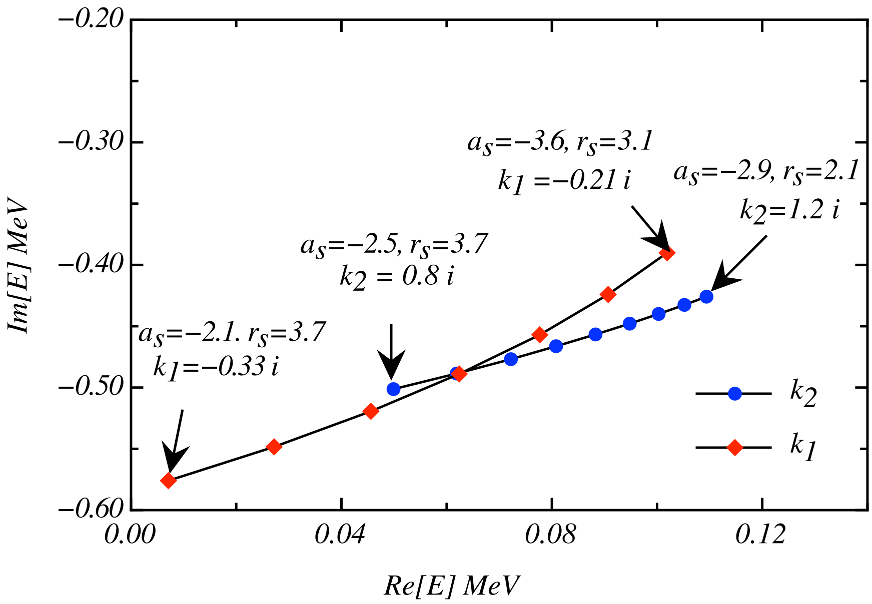

Figure 5: Variation in the position of the Λnn pole as one varies separately the singlet Λn pole positions in the effective range amplitude k_1 (fm^{-1}) or k_2 (fm^{-1}). Also included in the figure are the singlet effective range parameters a_s (fm) and r_s (fm) corresponding the the values of k_1 and k_2. The starting interaction is a Yamaguchi potential based upon the Nijmegen Model D [9], with scaling strength factor $s = 1.1$.

5 Conclusions

We have investigated extracting an experimental constraint upon the Λn interaction from data that may be obtained in an experimental measurement of a Λnn resonance. Our analysis is based on the assumption that states of the ΛNN system, which are close to the three-body threshold, are dominated by the effective range parameters of the pairwise interactions (nn, Λn) governing the Λnn system. This hypothesis enables us to generate rank one Yamaguchi potentials that represent the pairwise interactions. The separable potentials reduce the ΛNN Faddeev equations to a set of coupled one dimensional integral equations that we can analytically continue onto the second complex energy plane where resonances reside. This procedure places bound states and resonance poles on equal footing. After applying a coordinate rotation to expose the second complex energy plane, we are able to follow the trajectory of the S-matrix poles as the strength of the potentials is varied in a continuous manner. If the Λnn system supports a resonance, then the invariant mass of the Λnn system resulting from the reaction $^3\mathrm{H}(e, e'\mathrm{K}^+)X$ can be written as the sum of a Breit-Wigner form plus a smooth background. This suggests that we can extract two parameters from the invariant mass spectrum. These parameters may be used to place constraints on the Λn effective range parameters. We find that such a procedure might be best implemented based upon the poles of the Λn amplitude in the effective range approximation, with some assistance from theory (*i.e.*, existing meson exchange potential models) in terms of providing an estimate of the 'range' of the interaction.

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