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#### Abstract

The recent ANKE@COSY data on the differential cross section of reaction $p d \rightarrow p d \pi \pi$ demonstrate a peak at invariant mass of the final $d \pi \pi$ system 2.38 GeV , that corresponds to the isocalar $J^{P}=3^{+}$dibaryon $D_{03}$, and also enhancement in the distribution over the invariant mass of two final pions. The two-resonance model involving the $t$-channel $\sigma$ meson exchange between the proton and deuteron in the subprocess $p d \rightarrow p D_{03}$ and the sequential decays $D_{03} \rightarrow D_{12}+\pi$ and $D_{12} \rightarrow d+\pi$ was applied to describe the shapes of these distributions with the lowest orbital angular momenta in the corresponding vertices. A possible role of higher orbital momenta in those vertices is studied here.


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## Introduction

At present as one of the most realistic candidate to dibaryon resonance is considered the resonance $D_{I J}=D_{03}$ observed by the WASA@COSY [1] in the total cross section of the reaction of two-pion production $p n \rightarrow d \pi^{0} \pi^{0}$, here $I$ is the isospin and $J$ is the total angular momentum
of this resonance. The mass of this resonance 2.380 GeV is close to the $\Delta \Delta$-threshold, but its width $\Gamma=70 \mathrm{MeV}$ is twice lower as compared to the width of the free $\Delta$-isobar that is an indication to its unusual structure. Another interesting feathure of this reaction is a resonance behavior of its differential cross section as a function of the invariant mass of the final twopion system $M_{\pi \pi}$ known as the ABC-effect [2]. Possible mechanisms proposed yet in the early 1970s for interpreting the ABC effect in the two-pion production are the $t$-channel excitation of the noninteracting $\Delta \Delta$ or $N N^{*}(1450)$ systems and their subsequent decay into the pion channel ( see Ref. [3] and references therein). These mechanisms made it possible to describe to some extent the old inclusive data, however, fail to explain new exclusive data obtained in $4 \pi$ geometry with very high statistics [1]

After discovery of the dibaryon resonance $D_{03}$ in the reaction $p n \rightarrow d \pi^{0} \pi^{0}$, naturally the question whether dibaryons could be produced in collisions of other particles arose. Recently a very similar resonance structure was observed in the differential cross section of the two-pion production reaction $p d \rightarrow p d \pi \pi$ in experiments performed by the collaboration ANKE@COSY at beam energies $0.8-2.0 \mathrm{GeV}$ with high transferred momentum to the deuteron at small scattering angles of the final proton and deuteron [4]. At proton beam energies at 1.1 GeV and 1.4 GeV the resonance peaks were observed in the distribution over the invariant mass $M_{d \pi \pi}$ of the final $d \pi \pi$ system at $M_{d \pi \pi} \approx 2.38 \mathrm{GeV}$ [4] that is the mass of the isoscalar two-baryon resonance $D_{I J}=D_{03}$, while the kinematic conditions differ considerably from that in [1]. Furthermore, the ABC-like effect was observed in [4] in the distriution over the invariant mass of the two pions $M_{\pi \pi}$. An attempt to explain the observed behavior of this reaction within the modified two-resonance model [5] of the reaction $p n \rightarrow d \pi^{0} \pi^{0}$ was undertaken in Refs. [6] and [7] and the shape of the distribution over the $M_{d \pi \pi}$ was explained qualitatively. However some questions appeared cocerning the origin of the ABC-like structure in the $M_{\pi \pi}$ distribution in the data [4]. Only the lowest orbital momenta in the resonance vertices were considered in $[6,7]$. Here we are focused on the role of possible higher orbital momenta in those vertices.

## The model

Different mechanisms of the ABC effect in the reaction $p n \rightarrow d \pi^{0} \pi^{0}$ were discussed in [8]. One possible mechanism of this reaction suggested in paper [5] involves sequential excitation and decay of two intermediate dibaryon resonances, $D_{03}$ (2380) and $D_{12}$ (2150). We modify this model by including the $t$-channel $\sigma$-meson exchange between the proton and deuteron which leads to excitation of deuteron to the $D_{03}$ dibaryon, $\sigma+d \rightarrow D_{03}$, and after that the dibaryon decays sequentially as $D_{03} \rightarrow D_{12}+\pi \rightarrow d+\pi+\pi$. This mechanim is depicted in Fig. 1 where the second diagram takes into account identity of two $\pi^{0}$ - mesons. Since not all required partial widths are known, we discuss mainly the shapes of the distributions over the invariant masses of the final $d \pi \pi$ and $\pi \pi$ systems. The total amplitude of the reaction $p d \rightarrow p d \pi \pi$ corresponding to the sum of two Feynmann diagrams in Fig. 1 has the following form

$$
\begin{equation*}
T_{\mu_{p} \mu_{d}}^{\mu_{p}^{\prime} \mu_{d}^{\prime}}(p d \rightarrow p d \pi \pi)=T_{\mu_{p}}^{\mu_{p}^{\prime}}\left(p \rightarrow p^{\prime} \sigma\right) \frac{1}{p_{\sigma}^{2}-m_{\sigma}^{2}+i m_{\sigma} \Gamma_{\sigma}} T_{\mu_{d}}^{\mu_{d}^{\prime}}(\sigma d \rightarrow d \pi \pi) \tag{1}
\end{equation*}
$$

where $p_{\sigma}, m_{\sigma}, \Gamma_{\sigma}$ are the 4-momentum, mass, and the total width of the $\sigma$-meson; $\mu_{i}\left(\mu_{i}^{\prime}\right)$ is the spin projection of the initial (final) particle $i$. The amplitude of the virtual process $p \rightarrow p \sigma$ is based on the phenomenological $\sigma N N$ interaction and its spin averaged form $\overline{\left|T_{\mu_{p}}^{\mu_{p}^{\prime}}\left(p \rightarrow p^{\prime} \sigma\right)\right|^{2}}$ is given in [10].


Figure 1: Two resonance mechanism of the reaction $p d \rightarrow p d \pi \pi$

$$
\begin{align*}
& T_{\mu_{d}}^{\mu_{d}^{\prime}}(\sigma d \rightarrow d \pi \pi)= \sum_{\mu_{2}, \mu_{3}, \mu, m_{1}, m_{2}} \frac{F_{D_{03} \rightarrow d \sigma}(q) F_{D_{03} \rightarrow D_{12} \pi_{1}}\left(k_{1}\right)}{P_{D_{03}}^{2}-M_{D_{03}}^{2}+i M_{D_{03}} \Gamma_{D_{03}}} \frac{F_{D_{12} \rightarrow d \pi_{2}}\left(\lambda_{1}\right)}{P_{D_{12}}^{2}-M_{D_{12}}^{2}+i M_{D_{12}} \Gamma_{D_{12}}} \\
& \times C_{1 \mu_{d} L \mu}^{3 \mu_{3}} \mathcal{Y}_{L \mu}(\hat{\mathbf{q}}) C_{2 \mu_{2} l_{1} m_{1}}^{3 \mu_{3}} \mathcal{Y}_{l_{1} m_{1}}\left(\hat{\mathbf{k}}_{1}\right) C_{1 \mu_{d}^{\prime} l_{2} m_{2}}^{2 \mu_{2}} \mathcal{Y}_{l_{2} m_{2}}\left(\hat{\lambda}_{1}\right)+ \\
&+\frac{F_{D_{03} \rightarrow d \sigma}(q) F_{D_{03} \rightarrow D_{12} \pi_{2}}\left(k_{2}\right)}{P_{D_{03}}^{2}-M_{D_{03}}^{2}+i M_{D_{03}} \Gamma_{D_{03}}} \frac{F_{D_{12} \rightarrow d \pi_{1}}\left(\lambda_{2}\right)}{P_{D_{12}}^{2}-M_{D_{12}}^{2}+i M_{D_{12}} \Gamma_{D_{12}}} \\
& \times C_{1 \mu_{d} L \mu}^{3 \mu_{3}} \mathcal{Y}_{L \mu}(\hat{\mathbf{q}}) C_{2 \mu_{2} l_{1} m_{1}}^{3 \mu_{3}} \mathcal{Y}_{l_{1} m_{1}}\left(\hat{\mathbf{k}}_{2}\right) C_{1 \mu_{d}^{\prime} l_{2} m_{2}}^{2 \mu_{2}} \mathcal{Y}_{l_{2} m_{2}}\left(\hat{\boldsymbol{\lambda}}_{2}\right) . \tag{2}
\end{align*}
$$

The matrix element for the amplitude of the decay $D_{03} \rightarrow d \pi^{0} \pi^{0}$ via two decays $D_{03} \rightarrow D_{12}+\pi^{0}$ and $D_{12} \rightarrow d+\pi^{0}$ has the following form

$$
\begin{array}{r}
T_{\left(D_{03} \rightarrow d \pi \pi\right)}=\sum_{\mu_{2} m_{1} m_{2}} F_{D_{03} \rightarrow D_{12} \pi}\left(k_{1}\right) \cdot C_{2 \mu_{2} l_{1} m_{1}}^{3 \mu_{3}} k_{1}^{l_{1}} \sqrt{4 \pi} Y_{l_{1} m_{1}}\left(\hat{\mathbf{k}}_{1}\right) \\
\times F_{D_{12} \rightarrow d \pi}\left(\lambda_{1}\right) C_{1 \mu_{d} l_{2} m_{2}}^{2 \mu_{2}} \lambda_{1}^{l_{2}} \sqrt{4 \pi} Y_{l_{2} m_{2}}\left(\hat{\lambda}_{1}\right) \frac{1}{P_{D_{12}}^{2}-M_{D_{12}}^{2}+i M_{D_{12}} \Gamma_{D_{12}}^{2}}+ \\
\times F_{D_{03} \rightarrow D_{12} \pi}\left(k_{2}\right) \cdot C_{2 \mu_{3} l_{1} m_{1}}^{3 k_{2}} \sqrt{4 \pi} Y_{l_{1} m_{1}}\left(\hat{\mathbf{k}}_{2}\right) \\
\times F_{D_{12} \rightarrow d \pi}\left(\lambda_{2}\right) C_{1 \mu_{d} l_{2} m_{2}}^{2 \mu_{2}} \lambda_{2}^{l_{2}} \sqrt{4 \pi} Y_{l_{2} m_{2}}\left(\hat{\lambda}_{2}\right) \frac{1}{P_{D_{12}}^{\prime 2}-M_{D_{12}}^{2}+i M_{D_{12}} \Gamma_{D_{12}}^{2}} . \tag{3}
\end{array}
$$

Here $P_{D_{03}}, M_{D_{03}}$ and $\Gamma_{D_{03}}\left(P_{D_{12}}, M_{D_{12}}\right.$ and $\left.\Gamma_{D_{12}}\right)$ are the 4-momentum, mass and total width of the dibaryons $D_{03}\left(D_{12}\right)$, respectively, $P_{D_{12}}$ is 4-momentum of dibaryon $D_{12}$, which appeared as a result of the emission of a pion with an impulse $k_{1}$ (first pion): $P_{D_{12}}=P_{D_{03}}-k_{1}$, and $P_{D_{12}}^{\prime}$ appeared as a result of the emission of a pion with an impulse $k_{2}$ (second pion): $P_{D_{12}}^{\prime}=P_{D_{03}}-k_{2}$; $\mathbf{q}$ is the 3 -momentum of the initial deuteron in the cms of the $D_{03}, \mathbf{k}_{\mathbf{1}}\left(\mathbf{k}_{2}\right)$ is the 3 -momentum of the pion $\pi_{1}\left(\pi_{2}\right)$ in the cms of $D_{03}$, and $\lambda_{1}\left(\lambda_{2}\right)$ is the 3 -momentum of the pion $\pi_{2}\left(\pi_{1}\right)$ in the cms of the $D_{12}$. We use in Eq. (3) the standard notations for the Clebsch-Gordan coefficients $C_{j_{1} m_{1} j_{2} m_{2}}^{J M}$ and spherical functions $\mathcal{Y}_{l m}(\hat{\mathbf{k}})=k^{l} Y_{l m}(\hat{\mathbf{k}}) ; l_{1}$ is the orbital momenta of relative motion of dibaryon $D_{12}$ and the first pion in the vertex $D_{03} \rightarrow \pi D_{12}, l_{2}$ is the orbital momenta of relative motion of final deuteron and the second pion in the vertex $D_{12} \rightarrow d \pi$ and $L$ is the orbital momentum of the relative motion of the initial deuteron and the $\sigma$-meson in the vertex $d+\sigma \rightarrow D_{03}$. From the parity and angular momentum conservation one can find the following allowed values for the orbital momenta: $L=2,4 ; l_{1}=1,3,5 ; l_{2}=1,3$. Since we do not know decays parameters for higher orbital momenta, we consider below the differential cross section and the partial width of the decay $\Gamma_{D_{03} \rightarrow d \pi \pi}$ for each allowed set of orbital momenta separately.

The vertex functions $F_{R \rightarrow a b}(q)$ in Eqs. (2)- (3) are related to the corresponding partial


Figure 2: The mechanism of the decay $D_{03} \rightarrow \pi^{0} N N$
widths of the decay of the resonance $R$ to the system $a+b, \Gamma_{R \rightarrow a b}$, as [9]:

$$
\begin{equation*}
F_{R \rightarrow a b}\left(q_{a b}\right)=M_{a b} \sqrt{\frac{8 \pi \Gamma_{R \rightarrow a b}^{(l)}\left(q_{a b}\right)}{q_{a b}^{2 l+1}}} \tag{4}
\end{equation*}
$$

where $M_{a b}$ is the mass of the $a+b$ system, $l$ is the orbital momentum of the relative motion of the particles $a$ and $b$, and $q_{a b}$ is their relative 3-momentum determined as
$q_{a b}=\left[\left(s-\left(m_{a}-m_{b}\right)^{2}\right)\left(s-\left(m_{a}+m_{b}\right)^{2}\right) / 4 s\right]^{1 / 2}$, where $s \equiv M_{a b}^{2}$. We use the following parametrization of the energy dependence of the partial widths [9]:

$$
\begin{equation*}
\Gamma_{R \rightarrow a b}^{(l)}\left(q_{a b}\right)=\Gamma_{R \rightarrow a b}^{(l)}\left(\frac{q_{a b}}{q_{0}}\right)^{2 l+1}\left(\frac{q_{0}^{2}+\lambda_{a b}^{2}}{q_{a b}^{2}+\lambda_{a b}}\right)^{l+1} \tag{5}
\end{equation*}
$$

here $\Gamma_{R \rightarrow a b}^{(l)} \equiv \Gamma_{R \rightarrow a b}^{(l)}\left(q_{0}\right), q_{0}$ is the relative momentum $q_{a b}$ of the particles $a$ and $b$ at the resonance point defined as $M_{a b}\left(q_{0}\right)=M_{R}$, where $M_{R}$ is the nominal mass of the resonance.

According to Eqs. (4), (5), the vertex function
$F_{D_{03} \rightarrow d \sigma}(q)$ is written as

$$
\begin{equation*}
F_{D_{03} \rightarrow d \sigma}(q)=M_{d \sigma}(q) \sqrt{\frac{8 \pi \Gamma_{D_{03} \rightarrow d \sigma}^{(L)}(q)}{q^{2 L+1}}} \tag{6}
\end{equation*}
$$

where the factor $\Gamma_{D_{03} \rightarrow d \sigma}^{(L)}(q)$ is defined as

$$
\begin{equation*}
\Gamma_{D_{03} \rightarrow d \sigma}^{(L)}(q)=\Gamma_{D_{03} \rightarrow d \sigma}^{(L)}\left(\frac{q}{q_{0}}\right)^{2 L+1}\left(\frac{q_{0}^{2}+\lambda_{d \sigma}^{2}}{q^{2}+\lambda_{d \sigma}}\right)^{L+1} \tag{7}
\end{equation*}
$$

Here $q$ is relative 3-momentum of deuteron and $\sigma$-meson in cms of $D_{03}, q_{0}$ is 3-momentum at the resonance point $M_{D_{03}}=2.38 \mathrm{GeV}$ and $q_{0}=0.362 \mathrm{Gev} / \mathrm{c}, \lambda_{d \sigma}=0.18 \mathrm{GeV}$ is the parameter which given in [5].
The vertex function and the partial width of the decay $D_{03} \rightarrow D_{12} \pi$ at $l_{1}(=1,3,5)$ are the following

$$
\begin{array}{r}
F_{D_{03} \rightarrow D_{12} \pi}\left(k_{1}\right)=M_{D_{12} \pi}\left(k_{1}\right) \sqrt{\frac{8 \pi \Gamma_{D_{03} \rightarrow D_{12} \pi}^{\left(l_{1}\right)}\left(k_{1}\right)}{k_{1}^{2 l_{1}+1}}}, \\
\Gamma_{D_{03} \rightarrow D_{12} \pi}^{\left(l_{1}\right)}\left(k_{1}\right)=\Gamma_{D_{03} \rightarrow D_{12} \pi}^{\left(l_{1}\right)}\left(\frac{k_{1}}{k_{10}}\right)^{2 l_{1}+1}\left(\frac{k_{10}^{2}+\lambda_{D_{12} \pi}^{2}}{k_{1}^{2}+\lambda_{D_{12} \pi}}\right)^{l_{1}+1}, \tag{8}
\end{array}
$$

where $k_{1}$ is the relative 3 -momentum of the dibaryon $D_{12}$ and the pion in the cms frame of the $D_{03}, k_{10}$ is the 3 -momentum at the resonance point $M_{D_{03}}=2.38 \mathrm{GeV}$, the parameters $k_{10}=0.177 \mathrm{GeV} / \mathrm{c}$,
and $\lambda_{\pi D_{12}}=0.12 \mathrm{GeV}$ are taken from [5].
The vertex function and the partial width of the decay $D_{12} \rightarrow d \pi$ at $l_{2}=1,3$ are the following

$$
\begin{array}{r}
F_{D_{12} \rightarrow d \pi_{2}}\left(\lambda_{1}\right)=M_{d \pi}\left(\lambda_{1}\right) \sqrt{\frac{8 \pi \Gamma_{D_{12} \rightarrow d \pi}^{\left(l_{2}\right)}\left(k_{2}\right)}{k_{2}^{2 l_{2}+1}}}, \\
\Gamma_{D_{12} \rightarrow d \pi}^{\left(l_{2}\right)}\left(\lambda_{1}\right)=\Gamma_{D_{12} \rightarrow d \pi}^{\left(l_{2}\right)}\left(\frac{\lambda_{1}}{\lambda_{10}}\right)^{2 l_{2}+1}\left(\frac{\lambda_{10}^{2}+\lambda_{d \pi}^{2}}{\lambda_{1}^{2}+\lambda_{d \pi}}\right)^{l_{2}+1}, \tag{9}
\end{array}
$$

where $\lambda_{1}$ is relative 3 -momentum of deuteron and pion in cms of $D_{12}, \lambda_{10}$ is 3 -momentum at the resonance point $M_{D_{12}}=2.15 \mathrm{GeV}$ and $\lambda_{10}=0.224 \mathrm{GeV} / \mathrm{c}, \lambda_{d \pi}=0.25 \mathrm{GeV}$ [5].

The differential cross section of the reaction $p d \rightarrow p d \pi \pi$ within the considered model can be writen as following

$$
\begin{equation*}
d \sigma=\left((2 \pi)^{7} \cdot 64 \cdot q_{p d} \cdot s\right)^{-1} \cdot \overline{\left|T_{f i}(p d \rightarrow p d \pi \pi)\right|^{2}} k \cdot q \cdot p^{\prime} \cdot d \Omega_{\mathbf{k}} d \Omega_{\mathbf{q}} d \cos \theta_{p}^{\prime} d M_{D_{03}} d M_{\pi \pi} \tag{10}
\end{equation*}
$$

and the partial width of the $D_{03} \rightarrow d \pi^{0} \pi^{0}$ decay takes the form

$$
\begin{equation*}
d \Gamma\left(D_{03} \rightarrow d \pi \pi\right)=\frac{1}{(2 \pi)^{5}} \frac{1}{4 M_{D_{03}}^{2}} k \cdot q \cdot \overline{\left|T_{f i}\left(D_{03} \rightarrow d \pi \pi\right)\right|^{2}} d \Omega_{\mathbf{k}} d \Omega_{\mathbf{q}} d M_{\pi \pi} \tag{11}
\end{equation*}
$$

In Eq. (10) $q_{p d}$ is the relative 3-momentum between the initial proton and the deuteron and $p^{\prime}$ is the 3 -momentum of the final proton in the center of mass of the reaction. In Eqs. (10) and (11) $\mathbf{k}$ is the relative 3 -momentum between two pions, and $\mathbf{q}$ is the relative 3 -momentum between the final deuteron and the center mass of the $\pi \pi$ system in the center mass of the of the dibaryon $D_{03}$.

## Numerical results and discussion

The values $M_{D_{03}}=2.380 \mathrm{GeV}$ and $\Gamma_{D_{03}}=70 \mathrm{MeV}\left(M_{D_{12}}=2.15 \mathrm{GeV}, \Gamma_{D_{12}}=0.11 \mathrm{GeV}\right)$ are used in our calculations for the masses and widths of the resonance $D_{03}\left(D_{12}\right)$, respectively, and $m_{\sigma}=0.5 \mathrm{GeV}, \Gamma_{\sigma}=0.55 \mathrm{GeV}$ for the $\sigma$-meson. The values for the vertex parameters appeared in Eqs. (7), (8) and (9) are given in the section 2. The partial widths $\Gamma_{D_{12} \rightarrow d \pi}^{(l=1)}=10 \mathrm{MeV}$ and $\Gamma_{D_{12} \rightarrow p n}^{(l=1)}=10 \mathrm{MeV}$ were found in Ref. [9] from the analysis if the reaction $p p \rightarrow d \pi^{+}$. The partial width $\Gamma_{D_{03} \rightarrow d \sigma}^{(L=2)}=8.5 \mathrm{MeV}$ was found by us [7] from normalization of the calculated differential cross section within this two-resonance model to the data [4]. To get this value we assumed in Ref. [7] that the experimental value of the width $\Gamma_{D_{03} \rightarrow d \pi \pi}^{\text {exp }}=10 \mathrm{MeV}$ [11] is completely determined by the decays chain $D_{03} \rightarrow D_{12} \pi \rightarrow d \pi \pi$ and on this way we found that the value $\Gamma_{D_{03} \rightarrow D_{12} \pi}$ has to be equal to $\approx 140 \mathrm{MeV}$. Here we used for the orbital angular momentum $l_{1}$ in the decay vertex $D_{03} \rightarrow D_{12} \pi$ the value $l_{1}=1$.

The following partial widths are unknown at present: $\Gamma_{D_{03} \rightarrow D_{12} \pi}^{\left(l_{1}\right)}$ at $l_{1}=1,3,5$ and $\Gamma_{D_{12} \rightarrow d \pi}^{\left(l_{2}=3\right)}$. In order to estimate them we use Eqs. (5),(7), (8), (9) with the same parameters $q_{0}$ and $\lambda$ as for the minimal orbital angular momenta $l_{1}=l_{2}=1, L=2$, but substitute the higher orbital momenta $l_{1}=3,5, l_{2}=3$ and $L=4$.

Using the mechanism of the decay $D_{03} \rightarrow \pi N N$ depicted in Fig. 2 and experimental constraints on the width $\Gamma_{D_{03} \rightarrow \pi^{0} N N}<6.3 \mathrm{MeV}$ from [12] we found the corresponding constraints


Figure 3: The distribution over the invariant mass of two final pions system $M_{\pi \pi}$ at different orbital angular momenta. $a: L=2, l_{1}=1, l_{2}=1, b: L=4, l_{1}=1, l_{2}=1$, $c: L=2, l_{1}=1, l_{2}=3, d: L=4, l_{1}=1, l_{2}=3$. Here $L, l_{1}$ and $l_{2}$ are the orbital momenta in the vertices $d+\sigma \rightarrow D_{03}, D_{03} \rightarrow \pi D_{12}$ and $D_{12} \rightarrow d \pi$, respectively. The results of the model calculations (full lines) are normalized to the data [4] (open circles). In the calculations the scattering angle $\theta_{d}^{\text {c.m. }}$ of the final deuteron belongs to the full interval $0^{\circ}<\theta_{d}^{\text {c.m. }}<180^{\circ}$.

Table 1: The partial width $\Gamma_{D_{03} \rightarrow d \pi \pi}$ at different values of the orbital angular momenta $l_{1}$ and $l_{2}$ (see text for details)

| $l_{1}$ | $l_{2}$ | $\Gamma_{D_{03} \rightarrow d \pi \pi}(\mathrm{MeV})$ | $\Gamma_{D_{03} \rightarrow d \pi \pi}(\mathrm{MeV})$ |
| :---: | :---: | :---: | :---: |
| 1 | 1 | $<6.5$ | 10 |
| 1 | 3 | $<2.02$ | 3.1 |
| 3 | 1 | $<1.81$ | 2.78 |
| 3 | 3 | $<0.6$ | 0.92 |
| 5 | 1 | $<4.42$ | 6.8 |
| 5 | 3 | $<3.6$ | 5.5 |

on the partial widths: $\Gamma_{D_{03} \rightarrow D_{12} \pi}^{l=1}<95 \mathrm{MeV}, \Gamma_{D_{03} \rightarrow D_{12} \pi}^{l=3}<82 \mathrm{MeV}, \Gamma_{D_{03} \rightarrow D_{12} \pi}^{l=5}<65 \mathrm{MeV}$, here is $l$ the orbital angular momentum of the relative motion of the $D_{12}$ and the $\pi^{0}$.

Let us consider now the partial width $\Gamma_{D_{03} \rightarrow d \pi \pi}$. We have two possibilities to calculate this partial width. (i) One can take into account the upper limit $\Gamma_{D_{03} \rightarrow \pi^{0} N N}<6.3 \mathrm{MeV}$ [12]. (ii) One may assume that the experimental value $\Gamma_{D_{03} \rightarrow d \pi \pi}^{\exp }=10 \mathrm{MeV}[11]$ is completely determined by the decays chain $D_{03} \rightarrow D_{12} \pi \rightarrow d \pi \pi$. The results of calculations of the $\Gamma_{D_{03} \rightarrow d \pi \pi}$ for the first and the second approximation are given in the third and the fourth columns in the Tab. 1, respectively.

In Ref. [7] we didn't consider the channel $\sigma+d \rightarrow D_{03}$ with the orbital angular momentum $L=4$. Here when calculating the corresponding partial width $\Gamma_{D_{03} \rightarrow d \sigma}^{(L=4)}$ in a similar way as for the channel $L=2$ we find the value $\Gamma_{D_{03} \rightarrow d \sigma}^{(L=4)}=6.4 \mathrm{MeV}$.


Figure 4: The same as in Fig. 3 but with the model calculations at the scatering angle $\theta_{d}^{\text {c.m. }}$ of final deuteron in the interval $0^{\circ}<\theta_{d}^{\text {c.m. }}<11^{\circ}$ as in the experiment [4].

Considering the cross section of reaction $p d \rightarrow p d \pi \pi$ we should note that in our previous work [7] we found that "ABC-type" shape is caused mainly by the collinear kinematics of the experiment [4] and does not occur within the considered model for the case when scattering angle of the final deuteron is in the full interval $0^{\circ}<\theta_{d}^{\text {c.m. }}<180^{\circ}$. In Ref. [7] we performed calculation only for the lowest orbital angular momenta. Here we study the role of other possible orbital angular momenta $L, l_{1}, l_{2}$. We use in Eq. (5) the same partial width $\Gamma_{R \rightarrow a b}^{l}\left(q_{0}\right)$ for the higher orbital angular momenta $l$ as for their lowest values. The calculated differential cross section as a function of the mass of two final pions is shown in Fig. 3. One can see from this figure that at some orbital angular momenta the ABC effect is reproduced qualitatively (Fig. $3 c$ and $d$ ). Other higher orbital angular momenta do not reproduce "ABC-type" shape and we do not show the corresponding results.

From the results of calculations shown in Fig. 4, one can conclude that the contribution of the decay width $\Gamma_{D_{12} \rightarrow d \pi}^{l_{2}=3}$, the reaction $p d \rightarrow d \pi \pi$ is approximately equal to the contribution of tyhe $\Gamma_{D_{12} \rightarrow d \pi}^{l_{2}=1}$, therefore, interference effects, not taken into account here, will be important.

The differential cross section of the reaction $p d \rightarrow p d \pi \pi$ calculated at orbital angular momenta $L=2, l_{1}=5, l_{2}=3$ for the interval of the scattering angle of the deuteron $0^{\circ}<\theta_{d}^{\text {c.m. }}<11^{\circ}$ is depicted in Fig. $5 a$ as a function of the mass $M_{\pi \pi}$. The results of calculations are normalized to the experimental data. One can see that the model fails to reproduce the $A B C$ effect for these (and higher) orbital angular momenta. We should note, that the absolute value of differential cross section decreases with increasing orbital angular momenta.


Figure 5: The distribution over the invariant mass of two final pions $M_{\pi \pi}$ at the orbital angular momenta $L=2, l_{1}=5$ and $l_{2}=3$. The scattering angle $\theta_{d}^{\text {c.m. }}$ of the final deuteron belongs to the experimental interval $0^{\circ}<\theta_{d}^{\text {c.m. }}<11^{\circ}$ (a) and to the full interval $0^{\circ}<\theta_{d}^{\text {c.m. }}<180^{\circ}(b)$. The results of the model calculations are normalized to the data [4].

We do not show distribution of the differential cross section on the $M_{d \pi \pi}$ invariant mass because when changing the orbital angular momenta the shape of the differential cross section practically is not changed, while its absolute value is slowly diminished.

## Conclusion

The two-resonance model of the reaction $p d \rightarrow p d \pi \pi$ considered in Refs. [7] allows one to describe shapes of the measured in [4] distributions over the invariant masses $M_{d \pi \pi}$ and $M_{\pi \pi}$ if the lowest orbital angular momenta are used in the vertices $\sigma+d \rightarrow D_{03}, D_{03} \rightarrow D_{12}+\pi$ and $D_{12} \rightarrow d+\pi$. The ABC-like effect observed in the experiment [4], which was performed at small scattering angles of the deuteron $\theta_{d}^{\text {c.m. }}=0-11^{\circ}$, dissappears in this reaction within the considered model if the full angular interval for the scattered deuteron is allowed, $0 \leq \theta_{d}^{\text {c.m. }} \leq 180^{\circ}$. In the present work the higher orbital angular momenta were included into consideration in the vertices $\sigma+d \rightarrow D_{03}, D_{03} \rightarrow D_{12}+\pi$ and $D_{12} \rightarrow d+\pi$ and it was found that for some of them the ABC-like effect takes the place for the full interval of the deuteron scattering angle $\theta_{d}^{c . m .}=0-180^{\circ}$. The interference effects were not taken into account since the absolute values and relative phases of transition amplitudes with different orbital angular momenta are unknown.

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