

From Rindler Fluid to Dark Fluid on the Holographic Cutoff Surface

Rong-Gen Cai^{1,2}, Gansukh Tumurtushaa^{3,4} and Yun-Long Zhang^{5,6*}

1 Institute of Theoretical Physics, Chinese Academy of Sciences(ITP-CAS) and School of Physical Sciences, University of Chinese Academy of Sciences, Beijing, China

2 School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, University of Chinese Academy of Sciences, Hangzhou, China

3 Center for Quantum Spacetime(CQUeST), Sogang University, Seoul, Korea

4 Center for Theoretical Physics of the Universe, Institute for Basic Science, Daejeon, Korea

5 Center for Gravitational Physics, Yukawa Institute for Theoretical Physics(YITP), Kyoto University, Kyoto, Japan

6 National Astronomy Observatories, Chinese Academy of Science, Beijing, China

* [zhangyunlong001@gmail.com]

*4th International Conference on Holography,
String Theory and Discrete Approach
Hanoi, Vietnam, 2020
doi:10.21468/SciPostPhysProc.4*

Abstract

As an approximation to the near horizon regime of black holes, the Rindler fluid was proposed on an accelerating cutoff surface in the flat spacetime. The concept of the Rindler fluid was then generalized into a flat bulk with the cutoff surface of the induced de Sitter and FRW universe, such that an effective description of dark fluid in the accelerating universe can be investigated.



Copyright A. Bee *et al.*

This work is licensed under the Creative Commons
[Attribution 4.0 International License](https://creativecommons.org/licenses/by/4.0/).

Published by the SciPost Foundation.

Received ??-??-20??

Accepted ??-??-20??

Published ??-??-20??

doi:10.21468/SciPostPhysProc.4.??

1

2 Contents

3	1 Introduction	2
4	2 Dark Fluid on Holographic Cutoff	2
5	3 Modified Friedmann equation	3
6	4 Summary	4
7	References	5

8

9

10 **1 Introduction**

11 The origin and properties of the dark fluid, mainly including the dark energy and dark matter,
 12 are still mysterious in the current universe. The model of Lambda Cold Dark Matter (Λ CDM)
 13 treats dark energy as the cosmological constant and dark matter as the collision-less parti-
 14 cles, and explains the cosmic evolution and large-scale structures well. However, the tension
 15 between local measurements of the Hubble constant and the Planck’s observation based on
 16 Λ CDM model becomes more important [1, 2]. Besides, the dark matter particles have not
 17 been detected directly. Thus, alternative models of the dark fluid such as modified gravity
 18 need to be reconsidered. One recent example is the emergent gravity by Verlinde [3], which
 19 is inspired by the volume law correction to the entropy on a holographic screen, whereas the
 20 Einstein gravity is related to the area law [4].

21 So is there a model which can unify these two scenarios of dark fluid and modified gravity?
 22 In this article, we show that a holographic model of the emergent dark universe (hEDU) can
 23 naturally realize the duality between the dark fluid in (3+1)-dimension and a modified gravity
 24 in (4+1)-dimension. We consider that the dark fluid in the universe emerges as the holographic
 25 stress-energy tensor on the hypersurface in one higher dimensional flat bulk [5, 6]. After
 26 adding the localized stress-energy tensor $T_{\mu\nu}$ on the hypersurface with intrinsic metric $g_{\mu\nu}$
 27 and extrinsic curvature $\mathcal{K}_{\mu\nu}$, the induced Einstein field equations on the holographic screen
 28 are modified as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa_4(T_{\mu\nu} + \langle \mathcal{T} \rangle_{\mu\nu}^d), \tag{1}$$

29 where $\langle \mathcal{T} \rangle_{\mu\nu}^d$ denotes the induced Brown-York stress-energy tensor [7],

$$\langle \mathcal{T} \rangle_{\mu\nu}^d \equiv \frac{1}{\kappa_4 L} (\mathcal{K}_{\mu\nu} - \mathcal{K}g_{\mu\nu}). \tag{2}$$

30 Here, $\kappa_4 = 8\pi G_4/c^4$ is the Einstein constant and the length scale $L = \kappa_5/\kappa_4$ is related to the
 31 positive cosmological constant $\Lambda = 3/L^2$. At the cosmological scale, we assume that $T_{\mu\nu}$ only
 32 includes the components of normal matter, and $\langle \mathcal{T} \rangle_{\mu\nu}^d$ represents the total dark components
 33 in our universe, such as dark energy and dark matter. The stress-energy tensor $\langle \mathcal{T} \rangle_{\mu\nu}^d$ as we
 34 formulated is similar to the Verlinde’s elastic response of emergent gravity [3], in the way that
 35 it will back react on the background geometry.

36 The using of the Brown-York stress-energy tensor in (2) is inspired by the Wilsonian renor-
 37 malization group (RG) flow approaches of fluid/gravity duality [8–14]. Where the holographic
 38 stress-energy tensor on the holographic cutoff surface is identified with the stress energy ten-
 39 sor of the dual fluid directly. When taking the near horizon limit, one can reach the so-called
 40 Rindler fluid [15–22], which is a new perspective on the membrane paradigm of black holes,
 41 where the Brown-York stress-energy tensor is used.

42 **2 Dark Fluid on Holographic Cutoff**

43 To see more clearly how the Einstein equation (1) works, it is interesting to consider a de Sitter
 44 hypersurface as the holographic screen in flat spacetime firstly. Then the dual stress tensor
 45 could contribute to the dark energy as $\langle \mathcal{T} \rangle_{\mu\nu}^\Lambda = -(\rho_c \tilde{\Omega}_\Lambda)g_{\mu\nu}$. After adding the baryonic matter
 46 with typical 4-velocity u_μ and stress-energy tensor $T_{\mu\nu} = (\rho_c \tilde{\Omega}_B)u_\mu u_\nu$ on the screen, both of
 47 dark matter and dark energy can be described by the stress-energy tensor of holographic dark
 48 fluid $\langle \mathcal{T} \rangle_{\mu\nu} = \langle \mathcal{T} \rangle_{\mu\nu} + \langle \mathcal{T} \rangle_{\mu\nu}^D$, where $\langle \mathcal{T} \rangle_{\mu\nu}^D = (\rho_c \tilde{\Omega}_D)[(1 + \tilde{w}_D)u_\mu u_\nu + \tilde{w}_D g_{\mu\nu}]$ and \tilde{w}_D is the

49 equation of state of the emergent dark matter. From the Hamiltonian constraint equation
 50 in higher dimensional spacetime, an interesting relation between these components can be
 51 derived [5],

$$\text{hEDU: } \tilde{\Omega}_D^2 = \frac{\tilde{\Omega}_\Lambda}{2(1+3\tilde{w}_D)} [\tilde{\Omega}_D(1-3\tilde{w}_D) - \tilde{\Omega}_B]. \quad (3)$$

52 Once setting $\tilde{w}_D = 0$, we can compare (3) with the Λ CDM parameterization and it is
 53 straightforward to take the values from the observational data by Planck collaboration [23].
 54 The toy constraint relation (3) can be satisfied within the margin of error $\Omega_D^2 - \frac{1}{2}\Omega_L(\Omega_D - \Omega_B) \lesssim 1\%$.
 55 After considering $1 \simeq \Omega_L + \Omega_B + \Omega_D$, we also have $\Omega_B \simeq \Omega_D - 3\Omega_D^2 - \Omega_B^2$. In order to see this
 56 relation more clearly we plot it in Fig. 1, together with Verlinde's relation $\Omega_B = \frac{3}{4}\Omega_D^2$.

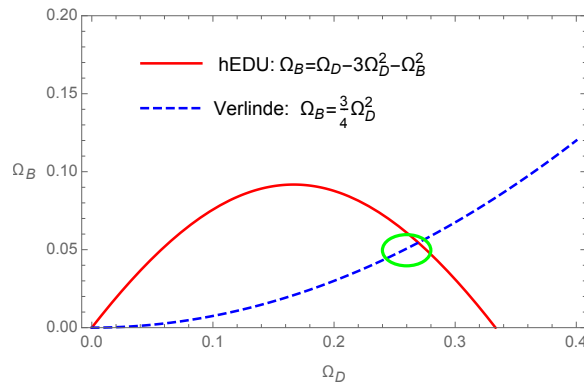


Figure 1: The schematic diagram of the relations between the components of baryonic matter Ω_B and dark matter Ω_D in the present universe. The green circle indicates the rough regime from the observation with $\Omega_B \simeq 0.05 \pm 0.01$, $\Omega_D \simeq 0.26 \pm 0.02$.

57 3 Modified Friedmann equation

58 The consistent embedding of a Friedmann–Lemaître–Robertson–Walker (FLRW) universe in
 59 4 + 1 dimensional flat spacetime has been studied in [24, 25]. In the spirit of the membrane
 60 paradigm [26, 27], we remove half part of the bulk spacetime, which can be effectively replaced
 61 by the holographic stress tensor $\langle T \rangle_{\mu\nu}^d$ in (2). The energy density and pressure in $\langle T \rangle_{\mu\nu}^d$ are
 62 calculated to be $\rho_d(t) = \rho_c \sqrt{\Omega_L} \sqrt{\frac{H(t)^2}{H_0^2} + \frac{\Omega_l}{a(t)^4}}$, where the critical density and other parame-
 63 ters are given by $\rho_c = \frac{3H_0^2 M_p^2}{\hbar c}$, $\Omega_L = \frac{c^2}{L^2 H_0^2}$ and $\Omega_l \equiv \frac{Ic^2}{L^2 H_0^2}$. Considering the relation between the
 64 redshift z and the scale factor via $a(t)/a(t_0) = 1/(1+z)$, we arrive at the normalized Hubble
 65 parameters $H(z)/H_0$ in terms of the redshift z , which is the modified Friedmann equation in
 66 the hEDU model,

$$\frac{H(z)^2}{H_0^2} = \frac{\Omega_L}{2} + \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \frac{\Omega_L}{2} \sqrt{1 + \frac{4}{\Omega_L} [\Omega_m(1+z)^3 + (\Omega_r + \Omega_l)(1+z)^4]}. \quad (4)$$

67 Notice here that at the current universe $z = 0$, we have $1 = \Omega_m + \Omega_r + \sqrt{\Omega_L(1 + \Omega_l)}$, and
 68 we will consider the fact that the radiation components $\Omega_r \ll 1$. By setting $\Omega_l = 0$, we can
 69 recover the usual Friedmann equation of the self-accelerating branch of the DGP braneworld
 70 model (sDGP) [28, 29]. When $\Omega_l \ll 1$, the behavior of $\Omega_l(1+z)^4$ is more like the dark
 71 radiation [30]. However, in this hEDU model, $\Omega_l \gg \Omega_r$ turns out not to be so small, such

72 that the whole dark sector, including dark energy and apparent dark matter, is expected to be
 73 included in the holographic dark fluid [5]. In Fig. 2, we plot the equation of state parameter
 74 of the holographic dark fluid $\tilde{w}_d(z)$ in terms of the redshift z , as well as the $\tilde{w}_D(z)$ of apparent
 75 dark matter where the effective components of cosmological constant Λ has been deducted.

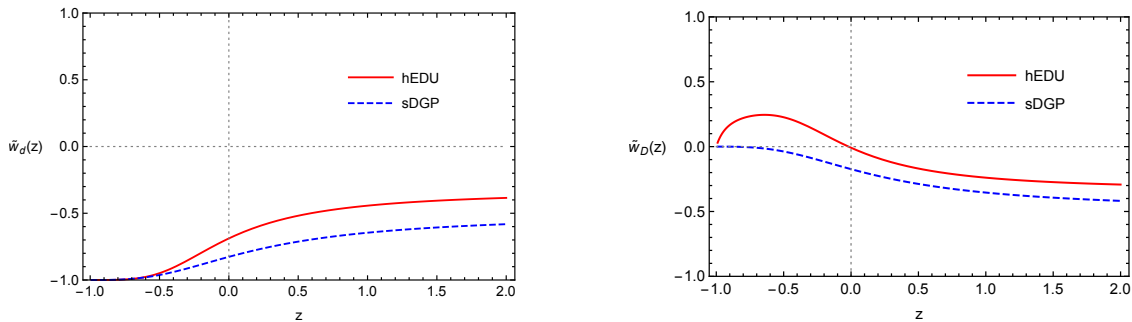


Figure 2: Left: the equation of state of the holographic dark fluid $\tilde{w}_d(z)$ in terms of the redshift z . Right: the equation of state of apparent dark matter $\tilde{w}_D(z)$, after deducting an effective cosmological constant. We adopt the following value for sDGP: $\Omega_I = 0$, $\Omega_m = 0.21$ [31] and hEDU: $\Omega_I = 0.4$, $\Omega_m = 0.04$ [6].

76 In [6], the Markov-chain Monte Carlo (MCMC) sampling analysis together with the obser-
 77 vational data of Type Ia supernovae (SNIa) and the direct measurement of Hubble constant
 78 H_0 [32] are employed. The two-dimensional observational contours are plotted in Fig. 3, with
 79 the 1-3 σ confidence contours for various parameters in the hEDU model [6]. The best-fit
 80 values turn out to be $\Omega_I = 0.43 \pm 0.13$ and $\Omega_m = 0.03 \pm 0.05$. The matter component is
 81 small enough and matches well with our theoretical assumption that only the normal matter
 82 is required.

83 We comment on the possible constraints from gravitational wave observations. It is argued
 84 that in general the modified gravity models are constrained from two aspects [33]. One is
 85 the constraint of the energy loss rate from ultra high energy cosmic rays, which indicates
 86 that gravitational waves should propagate at the speed of light. The other is the observed
 87 gravitational waveforms from LIGO, which are consistent with Einstein’s gravity and suggest
 88 that the gravitational wave should satisfy linear equations of motion in the weak-field limit.
 89 For our model, the Bianchi identity leads to $0 \equiv \nabla^\mu G_{\mu\nu} = \kappa_4 \nabla^\mu T_{\mu\nu} + \kappa_4 \nabla^\mu \langle \mathcal{T} \rangle_{\mu\nu}$. If we do not
 90 put additional sources in the bulk, the Brown-York stress-energy tensor (2) itself is conserved
 91 $\nabla^\mu \langle \mathcal{T} \rangle_{\mu\nu} = 0$. Thus, it is similar to the effects of particle dark matter and it does not conflict
 92 with the observations from LIGO so far [34].

93 4 Summary

94 In summary, we construct a model of the dark fluid in our universe, which originates from the
 95 holographic stress-energy tensor $\langle \mathcal{T} \rangle_{\mu\nu}^d$ of higher dimensional spacetime. The toy hEDU model
 96 on a de-Sitter screen in flat bulk spacetime produces one additional constraint from Λ CDM
 97 parameterization to the components of the late-time universe. We derive the corresponding
 98 Friedmann equation and present a good fitting result with the observational data. Finally, we
 99 would like to mention the literature on modified Newtonian dynamics (MOND) from a brane-
 100 world picture [35, 36], as well as the holographic big bang model in [37, 38] which describes
 101 the early universe with a 3-brane out of a collapsing star in (4+1)- dimensional bulk. These
 102 concepts are all related to our setups in the hEDU model. These models propose a possible
 103 origin of dark matter and dark energy and shed light on the underlying construction of the

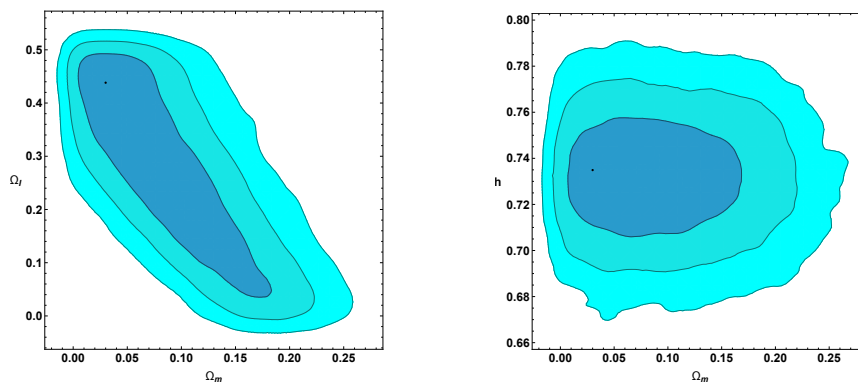


Figure 3: The $1-3\sigma$ confidence contours for various parameters in the hEDU model, Ω_m , Ω_I , $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, with figures taken from [6]. It is based on the MCMC sampling analysis with the observational data of Type Ia supernovae (SNIa) and the direct measurement of Hubble constant H_0 .

104 universe.

105 **Acknowledgement**— We thank Sunly Khimphun, Bum-Hoon Lee and Sichun Sun for the collaboration on
 106 relevant topics. R. -G. Cai was supported by the National Natural Science Foundation of China (No.11690022,
 107 No.11435006, No.11647601, No. 11851302, and No. 11821505), Strategic Priority Research Program of CAS (No.
 108 XDB23030100), Key Research Program of Frontier Sciences of CAS; G. Tumurtushaa was supported by the Institute
 109 for Basic Science (IBS) under the project code(18F18315); Y. -L. Zhang was supported by Grant-in-Aid for JSPS
 110 international research fellow(18F18315).

111 References

- 112 [1] T. M. C. Abbott *et al.* [DES Collaboration], “Dark Energy Survey year 1 results: Cosmological constraints
 113 from galaxy clustering and weak lensing,” *Phys. Rev. D* **98**, no. 4, 043526 (2018) [[arXiv:1708.01530](#) [[astro-ph.CO](#)]].
 114
- 115 [2] A. G. Riess, S. Casertano, W. Yuan, L. M. Macri and D. Scolnic, “Large Magellanic Cloud Cepheid Standards
 116 Provide a 1% Foundation for the Determination of the Hubble Constant and Stronger Evidence for Physics
 117 Beyond Λ CDM,” *Astrophys. J.* **876**, no.1, 85 (2019) [[arXiv:1903.07603](#) [[astro-ph.CO](#)]].
- 118 [3] E. P. Verlinde, “Emergent Gravity and the Dark Universe,” *SciPost Phys.* **2**, 016 (2017) [[arXiv:1611.02269](#)
 119 [[hep-th](#)]].
- 120 [4] E. P. Verlinde, “On the Origin of Gravity and the Laws of Newton,” *JHEP* **1104**, 029 (2011) [[arXiv:1001.0785](#)
 121 [[hep-th](#)]].
- 122 [5] R. G. Cai, S. Sun and Y. L. Zhang, “Emergent Dark Matter in Late Time Universe on Holographic Screen,”
 123 *JHEP* **1810**, 009 (2018) [[arXiv:1712.09326](#) [[hep-th](#)]].
- 124 [6] R. G. Cai, S. Khimphun, B. H. Lee, S. Sun, G. Tumurtushaa and Y. L. Zhang, “Emergent Dark Universe and
 125 the Swampland Criteria,” *Phys. Dark Univ.* **26**, 100387 (2019) [[arXiv:1812.11105](#) [[hep-th](#)]].
- 126 [7] J. D. Brown and J. W. York, Jr., “Quasilocal energy and conserved charges derived from the gravitational
 127 action,” *Phys. Rev. D* **47**, 1407 (1993) [[gr-qc/9209012](#)].
- 128 [8] I. Bredberg, C. Keeler, V. Lysov and A. Strominger, “Wilsonian Approach to Fluid/Gravity Duality,” *JHEP* **1103**,
 129 **141** (2011) [[arXiv:1006.1902](#) [[hep-th](#)]].
- 130 [9] I. Bredberg, C. Keeler, V. Lysov and A. Strominger, “From Navier-Stokes To Einstein,” *JHEP* **1207**, 146 (2012)
 131 [[arXiv:1101.2451](#) [[hep-th](#)]].
- 132 [10] R. G. Cai, L. Li and Y. L. Zhang, “Non-Relativistic Fluid Dual to Asymptotically AdS Gravity at Finite Cutoff
 133 Surface,” *JHEP* **1107**, 027 (2011) [[arXiv:1104.3281](#) [[hep-th](#)]].
- 134 [11] D. Brattán, J. Camps, R. Loganayagam and M. Rangamani, “CFT dual of the AdS Dirichlet problem :
 135 Fluid/Gravity on cut-off surfaces,” *JHEP* **1112**, 090 (2011) [[arXiv:1106.2577](#) [[hep-th](#)]].
- 136 [12] R. G. Cai, L. Li, Z. Y. Nie and Y. L. Zhang, “Holographic Forced Fluid Dynamics in Non-relativistic Limit,”
 137 *Nucl. Phys. B* **864**, 260 (2012) [[arXiv:1202.4091](#) [[hep-th](#)]].

- 138 [13] X. Bai, Y. P. Hu, B. H. Lee and Y. L. Zhang, “Holographic Charged Fluid with Anomalous Current at Finite
139 Cutoff Surface in Einstein-Maxwell Gravity,” *JHEP* **1211**, 054 (2012) [[arXiv:1207.5309 \[hep-th\]](#)].
- 140 [14] R. G. Cai, T. J. Li, Y. H. Qi and Y. L. Zhang, “Incompressible Navier-Stokes Equations from Einstein Gravity
141 with Chern-Simons Term,” *Phys. Rev. D* **86**, 086008 (2012) [[arXiv:1208.0658 \[hep-th\]](#)].
- 142 [15] G. Compere, P. McFadden, K. Skenderis and M. Taylor, “The Holographic fluid dual to vacuum Einstein grav-
143 ity,” *JHEP* **1107**, 050 (2011) [[arXiv:1103.3022 \[hep-th\]](#)].
- 144 [16] G. Compere, P. McFadden, K. Skenderis and M. Taylor, “The relativistic fluid dual to vacuum Einstein gravity,”
145 *JHEP* **1203**, 076 (2012) [[arXiv:1201.2678 \[hep-th\]](#)].
- 146 [17] C. Eling, A. Meyer and Y. Oz, “The Relativistic Rindler Hydrodynamics,” *JHEP* **1205**, 116 (2012)
147 [[arXiv:1201.2705 \[hep-th\]](#)].
- 148 [18] R. G. Cai, L. Li, Q. Yang and Y. L. Zhang, “Petrov type I Condition and Dual Fluid Dynamics,” *JHEP* **1304**,
149 **118** (2013) [[arXiv:1302.2016 \[hep-th\]](#)].
- 150 [19] R. G. Cai, Q. Yang and Y. L. Zhang, “Petrov type I Spacetime and Dual Relativistic Fluids,” *Phys. Rev. D* **90**,
151 **no. 4**, 041901 (2014) [[arXiv:1401.7792 \[hep-th\]](#)].
- 152 [20] N. Pinzani-Fokeeva and M. Taylor, “Towards a general fluid/gravity correspondence,” *Phys. Rev. D* **91**, **no. 4**,
153 **044001** (2015) [[arXiv:1401.5975 \[hep-th\]](#)].
- 154 [21] R. G. Cai, Q. Yang and Y. L. Zhang, “Petrov type I Condition and Rindler Fluid in Vacuum Einstein-Gauss-
155 Bonnet Gravity,” *JHEP* **1412**, 147 (2014) [[arXiv:1408.6488 \[hep-th\]](#)].
- 156 [22] S. Khimphun, B. H. Lee, C. Park and Y. L. Zhang, “Rindler Fluid with Weak Momentum Relaxation,” *JHEP*
157 **1801**, 058 (2018) [[arXiv:1705.05078 \[hep-th\]](#)].
- 158 [23] P. A. R. Ade *et al.* [Planck Collaboration], “Planck 2015 results. XIII. Cosmological parameters,” *Astron. As-*
159 *trophys.* **594**, A13 (2016) [[arXiv:1502.01589 \[astro-ph.CO\]](#)].
- 160 [24] P. Binetruy, C. Deffayet and D. Langlois, “Nonconventional cosmology from a brane universe,” *Nucl. Phys. B*
161 **565**, 269 (2000) [[hep-th/9905012](#)].
- 162 [25] R. Dick, “Brane worlds,” *Class. Quant. Grav.* **18**, **no. 17**, R1 (2001) [[hep-th/0105320](#)].
- 163 [26] R. H. Price and K. S. Thorne, “Membrane Viewpoint on Black Holes: Properties and Evolution of the Stretched
164 Horizon,” *Phys. Rev. D* **33**, 915 (1986).
- 165 [27] M. Parikh and F. Wilczek, “An action for black hole membranes,” *Phys. Rev. D* **58**, 064011 (1998)
- 166 [28] G. R. Dvali, G. Gabadadze and M. Porrati, “4-D gravity on a brane in 5-D Minkowski space,” *Phys. Lett. B*
167 **485**, 208 (2000) [[hep-th/0005016](#)].
- 168 [29] C. Deffayet, “Cosmology on a brane in Minkowski bulk,” *Phys. Lett. B* **502**, 199 (2001) [[hep-th/0010186](#)].
- 169 [30] S. Mukohyama, “Brane world solutions, standard cosmology, and dark radiation,” *Phys. Lett. B* **473**, 241
170 (2000) [[hep-th/9911165](#)].
- 171 [31] A. Lue, “The phenomenology of Dvali-Gabadadze-Porrati cosmologies,” *Phys. Rept.* **423**, 1 (2006) [[astro-](#)
172 [ph/0510068](#)].
- 173 [32] A. G. Riess *et al.*, “A 2.4% Determination of the Local Value of the Hubble Constant,” *Astrophys. J.* **826**, **no.**
174 **1**, 56 (2016) [[arXiv:1604.01424 \[astro-ph.CO\]](#)].
- 175 [33] P. M. Chesler and A. Loeb, “Constraining Relativistic Generalizations of Modified Newtonian Dynamics with
176 Gravitational Waves,” *Phys. Rev. Lett.* **119**, 031102 (2017) [[arXiv:1704.05116 \[astro-ph.HE\]](#)].
- 177 [34] M. A. Green, J. W. Moffat and V. T. Toth, “Modified Gravity (MOG), the speed of gravitational radiation and
178 the event GW170817/GRB170817A,” *Phys. Lett. B* **780**, 300 (2018) [[arXiv:1710.11177 \[gr-qc\]](#)].
- 179 [35] C. M. Ho, D. Minic and Y. J. Ng, “Cold Dark Matter with MOND Scaling,” *Phys. Lett. B* **693**, 567 (2010)
180 [[arXiv:1005.3537 \[hep-th\]](#)].
- 181 [36] M. Milgrom, “MOND from a brane-world picture,” [arXiv:1804.05840 \[gr-qc\]](#).
- 182 [37] R. Pourhasan, N. Afshordi and R. B. Mann, “Out of the White Hole: A Holographic Origin for the Big Bang,”
183 *JCAP* **1404**, 005 (2014) [[arXiv:1309.1487 \[hep-th\]](#)].
- 184 [38] N. Altamirano, E. Gould, N. Afshordi and R. B. Mann, “Cosmological Perturbations in the 5D Holographic
185 Big Bang Model,” [arXiv:1703.00954 \[astro-ph.CO\]](#).