

Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions I

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June 22, 2021

1 Abstract

2 We compute the one-loop renormalisation group running of the bosonic Stan-
3 dard Model effective operators to order v^4/Λ^4 , with $v \sim 246$ GeV being the
4 electroweak scale and Λ the unknown new physics threshold. We concentrate
5 on the effects triggered by pairs of the leading dimension-six interactions,
6 namely those that can arise at tree level in weakly-coupled ultraviolet com-
7 pletions of the Standard Model. We highlight some interesting consequences,
8 including the interplay between positivity bounds and the form of the anoma-
9 lous dimensions; the non renormalisation of the S and U parameters; or the
10 importance of radiative corrections to the Higgs potential for the electroweak
11 phase transition. As a byproduct of this work, we provide a complete Green
12 basis of operators involving only the Higgs and derivatives at dimension-eight,
13 comprising 13 redundant interactions.

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27 1 Introduction

28 The Standard Model (SM) extended with effective interactions, also known as SM effective field theory (SMEFT) [1], is increasingly becoming one of the favourite options for
 29 describing particle physics at currently explored energies. The main reasons are the apparent absence of new resonances below the TeV scale [2] and the fact that in general the
 30 SMEFT explains the experimental data better than the SM alone [3].

31 Relatively to the SM, the impact of effective operators of dimension $d > 4$ on observables computed at energy $\sim E$ is of order $(E/\Lambda)^{d-4}$, with $\Lambda \gg E$ being the (unknown) new
 32 physics threshold. Thus, the most relevant interactions are those of lowest energy dimension, which, ignoring lepton number violation (LNV), are the ones of dimension six. These
 33 operators have been experimentally tested from very different angles at all kind of particle physics facilities. In particular, the knowledge of the corresponding renormalisation group
 34 running [4–10] has allowed the high-energy physics community to probe the SMEFT to order E^2/Λ^2 combining experimental information gathered across very different energies;
 35 see for example Refs. [11–14].

36 However, there is by now convincing evidence that dimension-six operators do not suffice for making predictions within the SMEFT in a number of situations. For example,
 37 dimension-six interactions do not provide the dominant contribution to some observables [15] or even they do not arise at tree level in concrete ultraviolet (UV) completions
 38 of the SM [16, 17]. It can be also that relatively low values of Λ are favoured by data in some interactions, and therefore corrections involving higher powers of E/Λ are not
 39 negligible [16, 18]; or simply that some observables are so well measured (or constrained) that they are sensitive to higher-dimensional operators [18–20].

40 In either case, dimension-eight operators must be retained when using the SMEFT. (Dimension-seven interactions [21, 22] are also LNV.) This has been in fact the approach
 41 adopted in a number of recent theoretical works [16, 18–20, 23–28], but so far mostly at tree level. Our goal is to make a first step forward towards the renormalisation of the
 42 SMEFT to order E^4/Λ^4 . We think that, beyond opening the door to using the SMEFT precisely and consistently across energy scales, there are several motivations to address
 43 this challenge. For example:

44 **1.** Several classes of dimension-eight operators (including purely bosonic) that arise only at one loop in weakly-coupled UV completions of the SM can be renormalised by
 45 dimension-eight terms that can be generated at tree level [29]. (While at dimension six this occurs solely in one case.) This implies that the running of some operators can
 46 provide the leading SMEFT corrections to SM predictions in observables in which only loop-induced interactions contribute at tree level.

47 **2.** Eight is the lowest dimension at which there exist two co-leading contributions to renormalisation within the SMEFT: one involving single insertions of dimension-eight
 48 operators, and another one consisting of pairs of dimension-six interactions. (Pairs of dimension-five operators renormalise dimension-six ones [30], but they are LNV and therefore
 49 sub-leading with respect to single dimension-six terms.) Non-renormalisation theorems have been established only in relation to the first contribution [17, 29]. Thus, whether
 50 tree-level dimension-six operators renormalise loop-induced dimension-eight interactions is, to the best of our knowledge, still unknown.

71 **3.** Dimension-eight operators are subject to positivity bounds [31–37]. Thus, precisely
 72 because dimension-six interactions mix into dimension-eight ones, it is *a priori* conceivable
 73 that theoretical constraints on combinations of dimension-six Wilson coefficients can be
 74 established if the corresponding renormalisation group equations (RGEs) are precisely
 75 known.

76 Inspired by these observations, and in particular by **2**, in this paper we will focus
 77 on renormalisation triggered by dimension-six operators. (We will consider the effects of
 78 higher-dimensional operators in loops in subsequent works.) Also, we will concentrate on
 79 the running of the bosonic sector of the SMEFT.

80 This article is organised as follows. In section 2 we introduce the relevant Lagrangian
 81 and clarify the notation used thorough the rest of the paper. In section 3 we describe the
 82 technical details of the renormalisation programme. In section 4 we unravel the global
 83 structure of the renormalisation group equations (RGEs). We finalise with a discussion of
 84 the results in section 5. We dedicate Appendix A to relations that hold on-shell between
 85 different operators. In Appendix B we write explicitly all RGEs, while in Appendix C we
 86 describe briefly a UV model that accounts for generic tree-level generated dimension-six
 87 bosonic operators.

88 2 Theory and conventions

89 We denote by e , u and d the right-handed (RH) leptons and quarks; while l and q refer to
 90 the left-handed (LH) counterparts. The electroweak (EW) gauge bosons and the gluon are
 91 named by W , B and by G , respectively. We represent the Higgs doublet by $\phi = (\phi^+, \phi^0)^T$,
 92 and $\tilde{\phi} = i\sigma_2\phi^*$ with σ_I ($I = 1, 2, 3$) being the Pauli matrices. Thus, the renormalisable
 93 SM Lagrangian reads:

$$\begin{aligned} \mathcal{L}_{\text{SM}} = & -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \\ & + \bar{q}_L^\alpha i\not{D}q_L^\alpha + \bar{l}_L^\alpha i\not{D}l_L^\alpha + \bar{u}_R^\alpha i\not{D}u_R^\alpha + \bar{d}_R^\alpha i\not{D}d_R^\alpha + \bar{e}_R^\alpha i\not{D}e_R^\alpha \\ & + (D_\mu\phi)^\dagger (D^\mu\phi) - \mu^2|\phi|^2 - \lambda|\phi|^4 - \left(y_{\alpha\beta}^u \bar{q}_L^\alpha \tilde{\phi} u_R^\beta + y_{\alpha\beta}^d \bar{q}_L^\alpha \phi d_R^\beta + y_{\alpha\beta}^e \bar{l}_L^\alpha \phi e_R^\beta + \text{h.c.} \right). \end{aligned} \quad (1)$$

94 We adopt the minus-sign convention for the covariant derivative:

$$D_\mu = \partial_\mu - ig_1 Y B_\mu - ig_2 \frac{\sigma^I}{2} W_\mu^I - ig_3 \frac{\lambda^A}{2} G_\mu^A, \quad (2)$$

95 where g_1 , g_2 and g_3 represent, respectively, the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge couplings,
 96 Y stands for the hypercharge and λ^A are the Gell-Mann matrices.

97 We use the Warsaw basis [38] for the dimension-six SMEFT Lagrangian $\mathcal{L}^{(6)}$, and the
 98 basis of Ref. [17] for the dimension-eight part $\mathcal{L}^{(8)}$. (An equivalent basis can be found
 99 in Ref. [39].) While the renormalisation of $\mathcal{L}^{(6)}$ has been studied at length [7–9], the
 100 running of $\mathcal{L}^{(8)}$ is largely unknown. Assuming lepton-number conservation, the running
 101 of dimension-eight Wilson coefficients receives contributions from loops involving single
 102 insertions of dimension-eight couplings as well as from pairs of dimension-six operators.
 103 Schematically:

$$16\pi^2\mu \frac{dc_i^{(8)}}{d\mu} = \gamma_{ij} c_j^{(8)} + \gamma'_{ijk} c_j^{(6)} c_k^{(6)}. \quad (3)$$

104 Although $c^{(6)}$ (and $c^{(8)}$) are in general unknown, fits of the SMEFT to the data favour
 105 relatively large values of some of these coefficients [3]. This implies that the γ' term, which

	Operator	Notation	Operator	Notation
ϕ^8	$(\phi^\dagger\phi)^4$	\mathcal{O}_{ϕ^8}		
$\phi^6 D^2$	$(\phi^\dagger\phi)^2(D_\mu\phi^\dagger D^\mu\phi)$	$\mathcal{O}_{\phi^6}^{(1)}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)(D_\mu\phi^\dagger\sigma^I D^\mu\phi)$	$\mathcal{O}_{\phi^6}^{(2)}$
$\phi^4 D^4$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\nu\phi^\dagger D^\mu\phi)$	$\mathcal{O}_{\phi^4}^{(1)}$	$(D_\mu\phi^\dagger D_\nu\phi)(D^\mu\phi^\dagger D^\nu\phi)$	$\mathcal{O}_{\phi^4}^{(2)}$
	$(D^\mu\phi^\dagger D_\mu\phi)(D^\nu\phi^\dagger D_\nu\phi)$	$\mathcal{O}_{\phi^4}^{(3)}$		
$X^3\phi^2$	$\epsilon^{IJK}(\phi^\dagger\phi)W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{W^3\phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger\phi)W_\mu^{I\nu}W_\nu^{J\rho}\widetilde{W}_\rho^{K\mu}$	$\mathcal{O}_{W^3\phi^2}^{(2)}$
	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)B_\mu^\nu W_\nu^{J\rho}W_\rho^{K\mu}$	$\mathcal{O}_{W^2 B\phi^2}^{(1)}$	$\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(\widetilde{B}^{\mu\nu}W_\nu^J W_\mu^{K\rho} + B^{\mu\nu}W_\nu^J \widetilde{W}_\mu^{K\rho})$	$\mathcal{O}_{W^2 B\phi^2}^{(2)}$
$X^2\phi^4$	$(\phi^\dagger\phi)^2 G_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{G^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2 \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	$\mathcal{O}_{G^2\phi^4}^{(2)}$
	$(\phi^\dagger\phi)^2 W_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2 \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(2)}$
	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)W_{\mu\nu}^I W^{J\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(3)}$	$(\phi^\dagger\sigma^I\phi)(\phi^\dagger\sigma^J\phi)\widetilde{W}_{\mu\nu}^I W^{J\mu\nu}$	$\mathcal{O}_{W^2\phi^4}^{(4)}$
	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)W_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{WB\phi^4}^{(1)}$	$(\phi^\dagger\phi)(\phi^\dagger\sigma^I\phi)\widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	$\mathcal{O}_{WB\phi^4}^{(2)}$
	$(\phi^\dagger\phi)^2 B_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{B^2\phi^4}^{(1)}$	$(\phi^\dagger\phi)^2 \widetilde{B}_{\mu\nu} B^{\mu\nu}$	$\mathcal{O}_{B^2\phi^4}^{(2)}$
$X^2\phi^2 D^2$	$(D^\mu\phi^\dagger D^\nu\phi)W_{\mu\rho}^I W_\nu^{I\rho}$	$\mathcal{O}_{W^2\phi^2 D^2}^{(1)}$	$(D^\mu\phi^\dagger D_\mu\phi)W_{\nu\rho}^I W^{I\nu\rho}$	$\mathcal{O}_{W^2\phi^2 D^2}^{(2)}$
	$(D^\mu\phi^\dagger D_\mu\phi)W_{\nu\rho}^I \widetilde{W}^{I\nu\rho}$	$\mathcal{O}_{W^2\phi^2 D^2}^{(3)}$	$i\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W_{\mu\rho}^J W_\nu^{K\rho}$	$\mathcal{O}_{W^2\phi^2 D^2}^{(4)}$
	$\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(W_{\mu\rho}^J \widetilde{W}_\nu^{K\rho} - \widetilde{W}_{\mu\rho}^J W_\nu^{K\rho})$	$\mathcal{O}_{W^2\phi^2 D^2}^{(5)}$	$i\epsilon^{IJK}(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(W_{\mu\rho}^J \widetilde{W}_\nu^{K\rho} + \widetilde{W}_{\mu\rho}^J W_\nu^{K\rho})$	$\mathcal{O}_{W^2\phi^2 D^2}^{(6)}$
	$(D^\mu\phi^\dagger\sigma^I D_\mu\phi)B_{\nu\rho} W^{I\nu\rho}$	$\mathcal{O}_{WB\phi^2 D^2}^{(1)}$	$(D^\mu\phi^\dagger\sigma^I D_\mu\phi)B_{\nu\rho} \widetilde{W}^{I\nu\rho}$	$\mathcal{O}_{WB\phi^2 D^2}^{(2)}$
	$i(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho} W_\nu^{I\rho} - B_{\nu\rho} W_\mu^{I\rho})$	$\mathcal{O}_{WB\phi^2 D^2}^{(3)}$	$(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho} W_\nu^{I\rho} + B_{\nu\rho} W_\mu^{I\rho})$	$\mathcal{O}_{WB\phi^2 D^2}^{(4)}$
	$i(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho} \widetilde{W}_\nu^{I\rho} - B_{\nu\rho} \widetilde{W}_\mu^{I\rho})$	$\mathcal{O}_{WB\phi^2 D^2}^{(5)}$	$(D^\mu\phi^\dagger\sigma^I D^\nu\phi)(B_{\mu\rho} \widetilde{W}_\nu^{I\rho} + B_{\nu\rho} \widetilde{W}_\mu^{I\rho})$	$\mathcal{O}_{WB\phi^2 D^2}^{(6)}$
	$(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\rho} B_\nu^\rho$	$\mathcal{O}_{B^2\phi^2 D^2}^{(1)}$	$(D^\mu\phi^\dagger D_\mu\phi)B_{\nu\rho} B^{\nu\rho}$	$\mathcal{O}_{B^2\phi^2 D^2}^{(2)}$
$(D^\mu\phi^\dagger D_\mu\phi)B_{\nu\rho} \widetilde{B}^{\nu\rho}$	$\mathcal{O}_{B^2\phi^2 D^2}^{(3)}$			
$X\phi^4 D^2$	$i(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)W_{\mu\nu}^I$	$\mathcal{O}_{W\phi^4 D^2}^{(1)}$	$i(\phi^\dagger\phi)(D^\mu\phi^\dagger\sigma^I D^\nu\phi)\widetilde{W}_{\mu\nu}^I$	$\mathcal{O}_{W\phi^4 D^2}^{(2)}$
	$i\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)W_{\mu\nu}^K$	$\mathcal{O}_{W\phi^4 D^2}^{(3)}$	$i\epsilon^{IJK}(\phi^\dagger\sigma^I\phi)(D^\mu\phi^\dagger\sigma^J D^\nu\phi)\widetilde{W}_{\mu\nu}^K$	$\mathcal{O}_{W\phi^4 D^2}^{(4)}$
	$i(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)B_{\mu\nu}$	$\mathcal{O}_{B\phi^4 D^2}^{(1)}$	$i(\phi^\dagger\phi)(D^\mu\phi^\dagger D^\nu\phi)\widetilde{B}_{\mu\nu}$	$\mathcal{O}_{B\phi^4 D^2}^{(2)}$

Table 1: *Basis of bosonic dimension-eight operators involving the Higgs. We follow the notation from Ref. [17]. All interactions are hermitian. The operators in grey arise only at one loop in weakly-coupled renormalisable UV completions of the SM [29].*

106 is quadratic in the dimension-six couplings, can dominate the running of dimension-eight
107 Wilson coefficients even if the latter are equally large. As such, the computation of this
108 piece of the running is especially appealing.

109 Moreover, non-renormalisation theorems [29, 40–42] have not been yet established for
110 the mixing triggered by pairs of dimension-six operators. Consequently, for now the zeros
111 in γ' can be only obtained upon explicit calculation.

112 We therefore focus on this part of the dimension-eight running in what follows. Like-
113 wise, and as a first attack to the problem, we will concentrate on the bosonic sector
114 of the theory. The advantage of this is that bosonic operators are not renormalised by
115 field-redefining away redundant operators involving fermions (the opposite is not true).
116 Besides, we consider loops involving only dimension-six operators that can arise at tree
117 level in weakly-coupled UV completions of the SM. These can be found in Refs. [43–47].

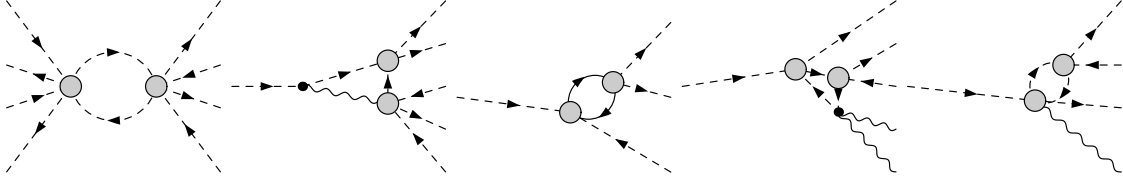


Figure 1: *Example diagrams for the renormalisation of operators in classes ϕ^8 (first), $\phi^6 D^2$ (second), $\phi^4 D^2$ (third), $X^2 \phi^4$ (fourth) and $X \phi^4 D^2$ (fifth). The gray blobs represent dimension-six interactions.*

118 Thus, our starting Lagrangian is:

$$\begin{aligned} \mathcal{L}_{\text{UV}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda^2} & \left\{ c_\phi (\phi^\dagger \phi)^3 + c_{\phi \square} (\phi^\dagger \phi) \square (\phi^\dagger \phi) + c_{\phi D} (\phi^\dagger D^\mu \phi)^* (\phi^\dagger D_\mu \phi) \right. \\ & + c_{\phi \psi_L}^{(1)} (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\overline{\psi}_L \gamma^\mu \psi_L) + c_{\phi \psi_L}^{(3)} (\phi^\dagger i \overleftrightarrow{D}_\mu^I \phi) (\overline{\psi}_L \gamma^\mu \sigma^I \psi_L) + c_{\phi \psi_R} (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) (\overline{\psi}_R \gamma^\mu \psi_R) \\ & \left. + [c_{\phi ud} (\tilde{\phi} i D_\mu \phi) (\overline{u}_R \gamma^\mu d_R) + c_{\psi_R \phi} (\phi^\dagger \phi) \overline{\psi}_L \tilde{\phi} \psi_R + \text{h.c.}] \right\}, \end{aligned} \quad (4)$$

119 with $\psi_R = u_R, d_R, e_R$ and $\psi_L = q_L, l_L$. Restricting to the bosonic sector of the SMEFT,
120 only dimension-eight operators involving Higgses can be renormalised at one loop from
121 the Lagrangian above. For clarity, we reproduce them in Table 1 following the notation
122 of Ref. [17].

123 3 Computation

124 We use the background field method and work in the Feynman gauge in dimensional
125 regularisation with space-time dimension $d = 4 - 2\epsilon$. We compute the one-loop divergences
126 generated by \mathcal{L}_{UV} using dedicated routines that rely on `FeynRules` [48], `FeynArts` [49] and
127 `FormCalc` [50]. Most of the calculations have been cross-checked using `Match-Maker` [51].
128 All amplitudes of the kind $X^3 \phi^2$ and $X^2 \phi^2 D^2$ are finite, hence operators in these classes
129 do not renormalise within our theory.

130 The bosonic one-loop divergent Lagrangian, involving Higgses, can be written as:

$$16\pi^2 \epsilon \mathcal{L}_{\text{DIV}} = \tilde{K}_\phi (D_\mu \phi)^\dagger (D^\mu \phi) - \tilde{\mu}^2 |\phi|^2 - \tilde{\lambda} |\phi|^4 + \tilde{c}_i^{(6)} \frac{\mathcal{O}_i^{(6)}}{\Lambda^2} + \tilde{c}_j^{(8)} \frac{\mathcal{O}_j^{(8)}}{\Lambda^4}, \quad (5)$$

131 where i and j run over elements in the Green bases of operators of dimension-six and
132 dimension-eight, respectively. The former extends the Warsaw basis with the interactions
133 given in Table 2. The bosonic Higgs operators expanding the dimension-eight Green basis
134 and which are redundant in the basis of Table 1 are shown in Table 3. To the best of our
135 knowledge, this last result is completely new.

136 We are interested in the unknown $\mathcal{O}(E^4/\Lambda^4)$ piece of the renormalisation of bosonic
137 operators. As such, we only provide this new contribution to the aforementioned diver-
138 gences. The only exception are the Higgs kinetic term and the dimension-six redundant
139 operators, for which we also compute E^2/Λ^2 corrections, as these generate E^4/Λ^4 terms
140 when moving to the physical basis by means of field redefinitions. Note also that, since we
141 are dealing with only bosonic operators, we omit flavour indices. Flavourful couplings are
142 written in matrix form (keeping the correct order in the matrix multiplication) and a trace
143 over indices is implicit. We also use the shorthand notation for matrices $A^2 \equiv \text{Tr } A^\dagger A$,
144 where A is an arbitrary flavour matrix.

	Operator	Notation	Operator	Notation
$\phi^2 D^4$	$(D_\mu D^\mu \phi^\dagger)(D_\nu D^\nu \phi)$	$\mathcal{O}_{D\phi}$		
$\phi^4 D^2$	$(\phi^\dagger \phi)(D_\mu \phi)^\dagger (D^\mu \phi)$	$\mathcal{O}'_{\phi D}$	$(\phi^\dagger \phi) D^\mu (\phi^\dagger \overleftrightarrow{D}_\mu \phi)$	$\mathcal{O}''_{\phi D}$
$X \phi^2 D^2$	$D_\nu W^{I\mu\nu} (\phi^\dagger \overleftrightarrow{D}_\mu^I \phi)$	$\mathcal{O}_{WD\phi}$	$\partial_\nu B^{\mu\nu} (\phi^\dagger \overleftrightarrow{D}_\mu \phi)$	$\mathcal{O}_{BD\phi}$

Table 2: Independent dimension-six bosonic operators involving the Higgs which are redundant with respect to the Warsaw basis. We adopt the notation of Ref. [52].

	Operator	Notation	Operator	Notation
$\phi^2 D^6$	$D^2 \phi^\dagger D_\mu D_\nu D^\mu D^\nu \phi$	\mathcal{O}_{ϕ^2}		
$\phi^4 D^4$	$D_\mu \phi^\dagger D^\mu \phi (\phi^\dagger D^2 \phi + \text{h.c.})$	$\mathcal{O}_{\phi^4}^{(4)}$	$D_\mu \phi^\dagger D^\mu \phi (\phi^\dagger i D^2 \phi + \text{h.c.})$	$\mathcal{O}_{\phi^4}^{(5)}$
	$(D_\mu \phi^\dagger \phi)(D^2 \phi^\dagger D_\mu \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(6)}$	$(D_\mu \phi^\dagger \phi)(D^2 \phi^\dagger i D_\mu \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(7)}$
	$(D^2 \phi^\dagger \phi)(D^2 \phi^\dagger \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(8)}$	$(D^2 \phi^\dagger \phi)(i D^2 \phi^\dagger \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(9)}$
	$(D^2 \phi^\dagger D^2 \phi)(\phi^\dagger \phi)$	$\mathcal{O}_{\phi^4}^{(10)}$	$(\phi^\dagger D^2 \phi)(D^2 \phi^\dagger \phi)$	$\mathcal{O}_{\phi^4}^{(11)}$
	$(D_\mu \phi^\dagger \phi)(D^\mu \phi^\dagger D^2 \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(12)}$	$(D_\mu \phi^\dagger \phi)(D^\mu \phi^\dagger i D^2 \phi) + \text{h.c.}$	$\mathcal{O}_{\phi^4}^{(13)}$
$\phi^6 D^2$	$(\phi^\dagger \phi)^2 (\phi^\dagger D^2 \phi + \text{h.c.})$	$\mathcal{O}_{\phi^6}^{(3)}$	$(\phi^\dagger \phi)^2 D_\mu (\phi^\dagger \overleftrightarrow{D}^\mu \phi)$	$\mathcal{O}_{\phi^6}^{(4)}$
$X \phi^4 D^2$	$(\phi^\dagger \phi) D_\nu W^{I\mu\nu} (D_\mu \phi^\dagger \sigma^I \phi + \text{h.c.})$	$\mathcal{O}_{W\phi^4 D^2}^{(5)}$	$(\phi^\dagger \phi) D_\nu W^{I\mu\nu} (D_\mu \phi^\dagger i \sigma^I \phi + \text{h.c.})$	$\mathcal{O}_{W\phi^4 D^2}^{(6)}$
	$\epsilon^{IJK} (D_\mu \phi^\dagger \sigma^I \phi) (\phi^\dagger \sigma^J D_\nu \phi) W^{K\mu\nu}$	$\mathcal{O}_{W\phi^4 D^2}^{(7)}$	$(\phi^\dagger \phi) D_\nu B^{\mu\nu} (D_\mu \phi^\dagger i \phi + \text{h.c.})$	$\mathcal{O}_{B\phi^4 D^2}^{(3)}$

Table 3: Independent dimension-eight bosonic operators involving the Higgs which are redundant with respect to the basis of Ref. [17]. Redundant operators in the class $X^2 \phi^2 D^2$ are not shown. The addition of h.c. when needed implies that all operators are hermitian.

145 Thus, the divergences of the couplings of dimension $d \leq 4$ read:

$$\tilde{K}_\phi \supset -\frac{1}{2}(c_{\phi D} + 2c_{\phi\Box})\frac{\mu^2}{\Lambda^2}, \quad (6)$$

$$\tilde{\lambda} \supset -\frac{3}{2}(c_{\phi D}^2 - 4c_{\phi D}c_{\phi\Box} + 8c_{\phi\Box}^2)\frac{\mu^4}{\Lambda^4}. \quad (7)$$

146 We use the symbol \supset to make explicit that corrections irrelevant for the computation of
147 the E^4/Λ^4 terms are disregarded.

148 The $\tilde{c}^{(6)}$ couplings in Eq. (5) read:

$$\tilde{c}_{\phi D} \supset c_{\phi D}(5c_{\phi D} - 8c_{\phi\Box})\frac{\mu^2}{\Lambda^2}, \quad (8)$$

$$\tilde{c}_{\phi\Box} \supset \frac{1}{4}(c_{\phi D}^2 + 24c_{\phi D}c_{\phi\Box} - 48c_{\phi\Box}^2)\frac{\mu^2}{\Lambda^2}, \quad (9)$$

$$\tilde{c}_\phi \supset 6c_\phi(3c_{\phi D} - 10c_{\phi\Box})\frac{\mu^2}{\Lambda^2} + 12\lambda(c_{\phi D}^2 - 6c_{\phi D}c_{\phi\Box} + 12c_{\phi\Box}^2)\frac{\mu^2}{\Lambda^2}, \quad (10)$$

$$\begin{aligned} \tilde{c}'_{\phi D} &= \frac{1}{2} \left[(3g_2^2 - 3g_1^2 - 4\lambda)c_{\phi D} + (8\lambda - 6g_2^2)c_{\phi\Box} \right] \\ &\quad - \left[3(c_{u\phi}y^{u\dagger} + y^u c_{u\phi}^\dagger + y^d c_{d\phi}^\dagger + c_{d\phi}y^{d\dagger}) + y^e c_{e\phi}^\dagger + c_{e\phi}y^{e\dagger} \right] \\ &\quad + 12c_{\phi q}^{(3)}(y^u y^{u\dagger} + y^d y^{d\dagger}) - 6(c_{\phi ud}y^{d\dagger}y^u + c_{\phi ud}^\dagger y^{u\dagger}y^d) + 4c_{\phi l}^{(3)}y^e y^{e\dagger} \end{aligned} \quad (11)$$

$$- (c_{\phi D}^2 + 4c_{\phi D}c_{\phi\Box} - 8c_{\phi\Box}^2) \frac{\mu^2}{\Lambda^2}, \quad (12)$$

$$\tilde{c}''_{\phi D} = -\frac{i}{2} \left[3(c_{u\phi}y^{u\dagger} - y^u c_{u\phi}^\dagger + y^d c_{d\phi}^\dagger - c_{d\phi}y^{d\dagger}) + (y^e c_{e\phi}^\dagger - c_{e\phi}y^{e\dagger}) \right], \quad (13)$$

$$\tilde{c}_{BD\phi} = -\frac{g_1}{6} \left[c_{\phi D} + c_{\phi\Box} - 4c_{\phi e} - 4c_{\phi l}^{(1)} + 8c_{\phi u} - 4c_{\phi d} + 4c_{\phi q}^{(1)} \right], \quad (14)$$

$$\tilde{c}_{WD\phi} = -\frac{g_2}{6} \left[c_{\phi\Box} + 4c_{\phi l}^{(3)} + 12c_{\phi q}^{(3)} \right]; \quad (15)$$

149 while for the $\tilde{c}^{(8)}$ we get:

$$\begin{aligned} \tilde{c}_{\phi^8} &= \frac{3}{16} \left[336c_\phi^2 + (g_1^4 + g_2^4 + 2g_1^2g_2^2 + 160\lambda^2)c_{\phi D}^2 + 2304\lambda^2c_{\phi\Box}^2 \right. \\ &\quad \left. + 512\lambda c_{\phi\Box}c_\phi - 1920\lambda c_{\phi D}c_\phi - 1152\lambda^2c_{\phi\Box}c_\phi \right] \\ &\quad - \left[c_{e\phi}y^{e\dagger}c_{e\phi}y^{e\dagger} + c_{e\phi}^\dagger y^e c_{e\phi}^\dagger y^e + 2c_{e\phi}c_{e\phi}^\dagger y^e y^{e\dagger} + 2c_{e\phi}^\dagger c_{e\phi} y^{e\dagger} y^e \right] \\ &\quad - 3 \left[c_{u\phi}y^{u\dagger}c_{u\phi}y^{u\dagger} + c_{u\phi}^\dagger y^u c_{u\phi}^\dagger y^u + 2c_{u\phi}c_{u\phi}^\dagger y^u y^{u\dagger} + 2c_{u\phi}^\dagger c_{u\phi} y^{u\dagger} y^u \right] \\ &\quad - 3 \left[c_{d\phi}y^{d\dagger}c_{d\phi}y^{d\dagger} + c_{d\phi}^\dagger y^d c_{d\phi}^\dagger y^d + 2c_{d\phi}c_{d\phi}^\dagger y^d y^{d\dagger} + 2c_{d\phi}^\dagger c_{d\phi} y^{d\dagger} y^d \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \tilde{c}_{\phi^4}^{(1)} &= \frac{1}{6} \left[11c_{\phi D}^2 - 32c_{\phi D}c_{\phi\Box} + 16c_{\phi\Box}^2 + 24c_{\phi ud}^2 - 24c_{\phi d}^2 - 8c_{\phi e}^2 - 16(c_{\phi l}^{(1)})^2 \right. \\ &\quad \left. + 16(c_{\phi l}^{(3)})^2 - 48(c_{\phi q}^{(1)})^2 + 48(c_{\phi q}^{(3)})^2 - 24c_{\phi u}^2 \right], \end{aligned} \quad (17)$$

$$\begin{aligned} \tilde{c}_{\phi^4}^{(2)} &= \frac{1}{6} \left[5c_{\phi D}^2 + 16c_{\phi D}c_{\phi\Box} + 16c_{\phi\Box}^2 + 24c_{\phi d}^2 + 8c_{\phi e}^2 + 16(c_{\phi l}^{(1)})^2 + 16(c_{\phi l}^{(3)})^2 + 48(c_{\phi q}^{(1)})^2 \right. \\ &\quad \left. + 48(c_{\phi q}^{(3)})^2 + 24c_{\phi u}^2 \right], \end{aligned} \quad (18)$$

$$\tilde{c}_{\phi^4}^{(3)} = \frac{1}{6} \left[-7c_{\phi D}^2 + 16c_{\phi D}c_{\phi\Box} + 40c_{\phi\Box}^2 - 32(c_{\phi l}^{(3)})^2 - 96(c_{\phi q}^{(3)})^2 - 24c_{\phi ud}^2 \right], \quad (19)$$

$$\tilde{c}_{\phi^4}^{(4)} = \frac{1}{2} \left[-c_{\phi D}^2 - 2c_{\phi D}c_{\phi\Box} + 24c_{\phi\Box}^2 \right], \quad (20)$$

$$\tilde{c}_{\phi^4}^{(6)} = c_{\phi D} (c_{\phi D} - c_{\phi\Box}), \quad (21)$$

$$\tilde{c}_{\phi^4}^{(8)} = \frac{1}{8} \left[c_{\phi D}^2 - 8c_{\phi D}c_{\phi\Box} + 32c_{\phi\Box}^2 \right], \quad (22)$$

$$\tilde{c}_{\phi^4}^{(10)} = \frac{1}{2} c_{\phi D}^2, \quad (23)$$

$$\tilde{c}_{\phi^4}^{(11)} = \frac{1}{4} \left[c_{\phi D}^2 - 8c_{\phi D}c_{\phi\Box} + 32c_{\phi\Box}^2 \right], \quad (24)$$

$$\tilde{c}_{\phi^4}^{(12)} = \frac{1}{2} c_{\phi D} (c_{\phi D} + 2c_{\phi\Box}), \quad (25)$$

$$\begin{aligned}
\tilde{c}_{\phi^6}^{(1)} = & \frac{1}{4} \left[24c_\phi(c_{\phi D} + 8c_{\phi\Box}) - 2c_{\phi D}c_{\phi\Box}(9g_1^2 + 3g_2^2 - 32\lambda) \right. \\
& \left. + c_{\phi D}^2(-3g_1^2 - 9g_2^2 + 34\lambda) - 4c_{\phi\Box}^2(9g_1^2 + 15g_2^2 + 112\lambda) \right] \\
& - 3(c_{d\phi}^2 + c_{u\phi}^2) - c_{e\phi}^2 - (3c_{\phi l}^{(1)} + 5c_{\phi l}^{(3)}) \left[y^e c_{e\phi}^\dagger + c_{e\phi} y^{e\dagger} \right] \\
& + 9c_{\phi q}^{(1)} \left[y^u c_{u\phi}^\dagger + c_{u\phi} y^{u\dagger} - y^d c_{d\phi}^\dagger - c_{d\phi} y^{d\dagger} \right] - 15c_{\phi q}^{(3)} \left[y^u c_{u\phi}^\dagger + c_{u\phi} y^{u\dagger} + y^d c_{d\phi}^\dagger + c_{d\phi} y^{d\dagger} \right] \\
& - 3 \left[3(c_{\phi u} c_{u\phi}^\dagger y^u + c_{u\phi} c_{\phi u} y^{u\dagger} - c_{d\phi} c_{\phi d} y^{d\dagger} - c_{\phi d} c_{d\phi}^\dagger y^d) - c_{e\phi} c_{\phi e} y^{e\dagger} - c_{\phi e} c_{e\phi}^\dagger y^e \right] \\
& - 9 \left[2(c_{\phi q}^{(1)} - c_{\phi q}^{(3)}) y^u c_{\phi u} y^{u\dagger} + 2(c_{\phi q}^{(1)} + c_{\phi q}^{(3)}) y^d c_{\phi d} y^{d\dagger} \right] + 9(c_{\phi q}^{(1)} c_{\phi q}^{(3)} + c_{\phi q}^{(3)} c_{\phi q}^{(1)}) \left[-y^u y^{u\dagger} + y^d y^{d\dagger} \right] \\
& + 15c_{\phi q}^{(3)} \left[y^u y^{u\dagger} + y^d y^{d\dagger} \right] c_{\phi q}^{(3)} + 9c_{\phi q}^{(1)} \left[y^u y^{u\dagger} + y^d y^{d\dagger} \right] c_{\phi q}^{(1)} \\
& - 3 \left[2(c_{\phi l}^{(1)} + c_{\phi l}^{(3)}) y^e c_{\phi e} y^{e\dagger} - (c_{\phi l}^{(1)} c_{\phi l}^{(3)} + c_{\phi l}^{(3)} c_{\phi l}^{(1)}) y^e y^{e\dagger} \right] + 5c_{\phi l}^{(3)} y^e y^{e\dagger} c_{\phi l}^{(3)} + 3c_{\phi l}^{(1)} y^e y^{e\dagger} c_{\phi l}^{(1)} \\
& + \frac{3}{2} \left[c_{\phi u d} c_{\phi u d}^\dagger y^{u\dagger} y^u + c_{\phi u d}^\dagger c_{\phi u d} y^{d\dagger} y^d \right] + 3 \left[3(c_{\phi u} y^{u\dagger} y^u c_{\phi u} + c_{\phi d} y^{d\dagger} y^d c_{\phi d}) + c_{\phi e} y^{e\dagger} y^e c_{\phi e} \right] \\
& - 6c_{\phi q}^{(3)} \left[y^u c_{\phi u d} y^{d\dagger} + y^d c_{\phi u d}^\dagger y^{u\dagger} \right] + 3 \left[c_{\phi u d} c_{d\phi}^\dagger y^u + c_{d\phi} c_{\phi u d}^\dagger y^{u\dagger} + c_{u\phi} c_{\phi u d} y^{d\dagger} + y^d c_{\phi u d}^\dagger c_{u\phi}^\dagger \right],
\end{aligned} \tag{26}$$

$$\begin{aligned}
\tilde{c}_{\phi^6}^{(2)} = & \frac{1}{8} \left[72c_\phi c_{\phi D} - 24c_{\phi\Box}(c_{\phi D} + 2c_{\phi\Box})g_1^2 - 3c_{\phi D}(c_{\phi D} + 16c_{\phi\Box})g_2^2 \right. \\
& \left. + 32c_{\phi D}(3c_{\phi D} - 8c_{\phi\Box})\lambda \right] - 3 \left[c_{\phi u d} c_{d\phi}^\dagger y^u + c_{d\phi} c_{\phi u d}^\dagger y^{u\dagger} + y^d c_{\phi u d}^\dagger c_{u\phi}^\dagger + c_{u\phi} c_{\phi u d} y^{d\dagger} \right] \\
& - 2 \left[c_{\phi l}^{(1)} y^e c_{e\phi}^\dagger + c_{e\phi} y^{e\dagger} c_{\phi l}^{(1)} - 3(c_{\phi q}^{(1)} y^u c_{u\phi}^\dagger + c_{u\phi} y^{u\dagger} c_{\phi q}^{(1)} - c_{\phi q}^{(1)} y^d c_{d\phi}^\dagger - c_{d\phi} y^{d\dagger} c_{\phi q}^{(1)}) \right] \\
& + 2 \left[c_{\phi e} c_{e\phi}^\dagger y^e + c_{e\phi} c_{\phi e} y^{e\dagger} - 3(c_{\phi u} c_{u\phi}^\dagger y^u + c_{u\phi} c_{\phi u} y^{u\dagger} - c_{\phi d} c_{d\phi}^\dagger y^d - c_{d\phi} c_{\phi d} y^{d\dagger}) \right] \\
& - 4 \left[c_{\phi l}^{(1)} + c_{\phi l}^{(3)} \right] y^e c_{\phi e} y^{e\dagger} - 12 \left[(c_{\phi q}^{(1)} - c_{\phi q}^{(3)}) y^u c_{\phi u} y^{u\dagger} + (c_{\phi q}^{(1)} + c_{\phi q}^{(3)}) y^d c_{\phi d} y^{d\dagger} \right] \\
& + 2 \left[c_{\phi l}^{(1)} c_{\phi l}^{(3)} + c_{\phi l}^{(3)} c_{\phi l}^{(1)} + c_{\phi l}^{(1)} c_{\phi l}^{(1)} \right] y^e y^{e\dagger} + 2c_{\phi e} c_{\phi e} y^{e\dagger} y^e - 6 \left[c_{\phi q}^{(1)} c_{\phi q}^{(3)} + c_{\phi q}^{(3)} c_{\phi q}^{(1)} - c_{\phi q}^{(1)} c_{\phi q}^{(1)} \right] y^u y^{u\dagger} \\
& + 6c_{\phi u} c_{\phi u} y^{u\dagger} y^u + 6 \left[c_{\phi q}^{(1)} c_{\phi q}^{(3)} + c_{\phi q}^{(3)} c_{\phi q}^{(1)} + c_{\phi q}^{(1)} c_{\phi q}^{(1)} \right] y^d y^{d\dagger} + 6c_{\phi d} c_{\phi d} y^{d\dagger} y^d \\
& - \frac{3}{2} \left[c_{\phi u d} c_{\phi u d}^\dagger y^{u\dagger} y^u + c_{\phi u d}^\dagger c_{\phi u d} y^{d\dagger} y^d \right] + 6c_{\phi q}^{(3)} \left[y^d c_{\phi u d}^\dagger y^{u\dagger} + y^u c_{\phi u d} y^{d\dagger} \right],
\end{aligned} \tag{27}$$

$$\begin{aligned}
\tilde{c}_{\phi^6}^{(3)} = & \frac{1}{16} \left[432c_\phi c_{\phi\Box} + (448\lambda - 12g_1^2 - 12g_2^2)c_{\phi D}c_{\phi\Box} - 48c_\phi c_{\phi D} \right. \\
& \left. - 8(3g_1^2 + 3g_2^2 + 128\lambda)c_{\phi\Box}^2 - (3g_2^2 + 12\lambda)c_{\phi D}^2 \right] \\
& - c_{e\phi}^2 - 3(c_{d\phi}^2 + c_{u\phi}^2) - \frac{1}{2} \left[(c_{\phi l}^{(1)} + c_{\phi l}^{(3)})(y^e c_{e\phi}^\dagger + c_{e\phi} y^{e\dagger}) - c_{\phi e} c_{e\phi}^\dagger y^e - c_{e\phi} c_{\phi e} y^{e\dagger} \right] \\
& - \left[c_{\phi l}^{(1)} + c_{\phi l}^{(3)} \right] y^e c_{\phi e} y^{e\dagger} + \frac{1}{2} \left[c_{\phi l}^{(1)} c_{\phi l}^{(3)} + c_{\phi l}^{(1)} c_{\phi l}^{(1)} + c_{\phi l}^{(3)} c_{\phi l}^{(1)} + c_{\phi l}^{(3)} c_{\phi l}^{(3)} \right] y^e y^{e\dagger} + \frac{1}{2} c_{\phi e} c_{\phi e} y^{e\dagger} y^e \\
& - \frac{3}{2} \left[c_{\phi q}^{(1)} + c_{\phi q}^{(3)} \right] (y^d c_{d\phi}^\dagger + c_{d\phi} y^{d\dagger}) + \frac{3}{2} \left[c_{\phi d} c_{d\phi}^\dagger y^d + c_{d\phi} c_{\phi d} y^{d\dagger} \right] - 3 \left[c_{\phi q}^{(1)} + c_{\phi q}^{(3)} \right] y^d c_{\phi d} y^{d\dagger}
\end{aligned} \tag{28}$$

$$\begin{aligned}
& + \frac{3}{2} \left[c_{\phi q}^{(1)} c_{\phi q}^{(3)} + c_{\phi q}^{(1)} c_{\phi q}^{(1)} + c_{\phi q}^{(3)} c_{\phi q}^{(1)} + c_{\phi q}^{(3)} c_{\phi q}^{(3)} \right] y^d y^{d\dagger} + \frac{3}{2} c_{\phi d} c_{\phi d} y^{d\dagger} y^d \\
& - \frac{3}{2} \left[(-c_{\phi q}^{(1)} + c_{\phi q}^{(3)}) (y^u c_{u\phi}^\dagger + c_{u\phi} y^{u\dagger}) + c_{\phi u} c_{u\phi}^\dagger y^u + c_{u\phi} c_{\phi u} y^{u\dagger} \right] - 3 \left[c_{\phi q}^{(1)} - c_{\phi q}^{(3)} \right] y^u c_{\phi u} y^{u\dagger} \\
& + \frac{3}{2} \left[-c_{\phi q}^{(1)} c_{\phi q}^{(3)} + c_{\phi q}^{(1)} c_{\phi q}^{(1)} - c_{\phi q}^{(3)} c_{\phi q}^{(1)} + c_{\phi q}^{(3)} c_{\phi q}^{(3)} \right] y^u y^{u\dagger} + \frac{3}{2} c_{\phi u} c_{\phi u} y^{u\dagger} y^u, \\
\tilde{c}_{\phi^6}^{(4)} &= -\frac{3i}{2} \left[c_{d\phi}^\dagger \left(c_{\phi q}^{(1)} + c_{\phi q}^{(3)} \right) y^d - c_{\phi d} c_{d\phi}^\dagger y^d + c_{d\phi} c_{\phi d} y^{d\dagger} - \left(c_{\phi q}^{(1)} + c_{\phi q}^{(3)} \right) c_{d\phi} y^{d\dagger} \right. \\
& \left. + c_{u\phi}^\dagger \left(c_{\phi q}^{(1)} - c_{\phi q}^{(3)} \right) y^u - c_{\phi u} c_{u\phi}^\dagger y^u + c_{u\phi} c_{\phi u} y^{u\dagger} - \left(c_{\phi q}^{(1)} - c_{\phi q}^{(3)} \right) c_{u\phi} y^{u\dagger} \right] \\
& - \frac{i}{2} \left[c_{e\phi}^\dagger \left(c_{\phi l}^{(1)} + c_{\phi l}^{(3)} \right) y^e - c_{\phi e} c_{e\phi}^\dagger y^e + c_{e\phi} c_{\phi e} y^{e\dagger} - \left(c_{\phi l}^{(1)} + c_{\phi l}^{(3)} \right) c_{e\phi} y^{e\dagger} \right], \\
\tilde{c}_{W^2\phi^4}^{(1)} &= \frac{g_2^2}{12} \left[c_{\phi D}^2 - 3c_{\phi D} c_{\phi\Box} - 12(c_{\phi l}^{(3)})^2 - 36(c_{\phi q}^{(3)})^2 + 9c_{\phi ud}^2 \right], \tag{30} \\
\tilde{c}_{W^2\phi^4}^{(3)} &= \frac{g_2^2}{48} \left[c_{\phi D}^2 - 12c_{\phi D} c_{\phi\Box} - 24c_{\phi d}^2 - 8c_{\phi e}^2 - 16(c_{\phi l}^{(1)})^2 - 48(c_{\phi q}^{(1)})^2 - 24c_{\phi u}^2 + 12c_{\phi ud}^2 \right], \tag{31} \\
\tilde{c}_{WB\phi^4}^{(1)} &= \frac{g_1 g_2}{24} \left[c_{\phi D}^2 - 12c_{\phi D} c_{\phi\Box} - 8c_{\phi e}^2 - 16(c_{\phi l}^{(1)})^2 - 48(c_{\phi q}^{(1)})^2 - 24c_{\phi u}^2 + 12c_{\phi ud} - 24c_{\phi d}^2 \right], \tag{32} \\
\tilde{c}_{B^2\phi^4}^{(1)} &= -\frac{g_1^2}{48} \left[3c_{\phi D}^2 + 24c_{\phi d}^2 + 8c_{\phi e}^2 + 16(c_{\phi l}^{(1)})^2 - 48(c_{\phi l}^{(3)})^2 + 48(c_{\phi q}^{(1)})^2 \right. \\
& \left. - 144(c_{\phi q}^{(3)})^2 + 24c_{\phi u}^2 + 24c_{\phi ud}^2 \right], \tag{33} \\
\tilde{c}_{W\phi^4 D^2}^{(1)} &= \frac{g_2}{6} \left[5c_{\phi D}^2 - 24c_{\phi D} c_{\phi\Box} - 8c_{\phi e}^2 - 16(c_{\phi l}^{(1)})^2 - 48(c_{\phi l}^{(3)})^2 - 48(c_{\phi q}^{(1)})^2 \right. \\
& \left. - 144(c_{\phi q}^{(3)})^2 - 24c_{\phi u}^2 + 48c_{\phi ud}^2 - 24c_{\phi d}^2 \right], \tag{34} \\
\tilde{c}_{B\phi^4 D^2}^{(1)} &= -\frac{g_1}{6} \left[24c_{\phi d}^2 + 3c_{\phi D}^2 + 8(c_{\phi e}^2 + 2(c_{\phi l}^{(1)})^2 - 6(c_{\phi l}^{(3)})^2 + 6(c_{\phi q}^{(1)})^2 - 18(c_{\phi q}^{(3)})^2 + 3c_{\phi u}^2) + 24c_{\phi ud}^2 \right], \tag{35} \\
\tilde{c}_{W\phi^4 D^2}^{(6)} &= \frac{g_2}{24} \left[5c_{\phi D}^2 - 20c_{\phi D} c_{\phi\Box} + 16(c_{\phi\Box}^2 - 2(c_{\phi l}^{(3)})^2 - 6(c_{\phi q}^{(3)})^2) + 48c_{\phi ud}^2 \right], \tag{36} \\
\tilde{c}_{W\phi^4 D^2}^{(7)} &= \frac{g_2}{6} \left[24c_{\phi d}^2 - c_{\phi D} (c_{\phi D} - 12c_{\phi\Box}) + 8(c_{\phi e}^2 + 2(c_{\phi l}^{(1)})^2 + 6(c_{\phi q}^{(1)})^2 + 3c_{\phi u}^2) - 12c_{\phi ud}^2 \right], \tag{37} \\
\tilde{c}_{B\phi^4 D^2}^{(3)} &= \frac{g_1}{24} \left[-3c_{\phi D}^2 + 4c_{\phi D} c_{\phi\Box} + 16(c_{\phi\Box}^2 + 4(c_{\phi l}^{(3)})^2 + 12(c_{\phi q}^{(3)})^2) - 24c_{\phi ud}^2 \right]. \tag{38}
\end{aligned}$$

150 All other relevant counterterms ($\tilde{\mu}^2$, $\tilde{c}_{D\phi}$, $\tilde{c}_{\phi^4}^{(5,7,9,13)}$, etc.) vanish at the order of E/Λ so
151 we are interested in.

152 4 The structure of the renormalisation group equations

153 The explicit form of the divergences in the Green basis, shown in the equations above, is
154 of utmost importance, or else the computation of RGEs at higher orders or involving other
155 light degrees of freedom could not be built on our results [53]. Subsequently, though, one

156 can reduce the redundant operators to the physical basis. We do that following the results
157 in Appendix A.

158 In the physical basis, the RGEs of the different dimension-eight couplings read:

$$16\pi^2\mu\frac{dc_i^{(8)}}{d\mu} = -c_i^{(8)}\sum_j x_j n_j \frac{\partial}{\partial x_j} \frac{\tilde{c}_i^{(8)}}{c_i^{(8)}}, \quad (39)$$

159 with x running over all couplings, renormalisable or not, and with n representing the
160 corresponding tree-level anomalous dimension, defined as the value required to keep the
161 couplings dimensionless in $4 - 2\epsilon$ dimensions. The minus sign results from requiring that
162 the counterterms cancel the divergences. The complete set of RGEs can be found in
163 Appendix B, including those of renormalisable and dimension-six terms. In this section,
164 we limit ourselves to discussing the structure of the γ' matrices defined in Eq. (3).

165 Since the contribution comes from pairs of dimension-six operators, we provide a (sym-
166 metric) matrix for each $c_i^{(8)}$ in which we represent with a \times a non-vanishing entry, with 0
167 a trivial zero, for which all contributions in the Green basis vanish (for example in some
168 cases there are no diagrams contributing to the corresponding amplitude), and with \emptyset a
169 non-trivial zero, for which several non-vanishing contributions cancel in the physical basis.
170 We find:

171

$\gamma'_{\mathbf{c}_\phi^8}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	\times	\times	\times	0	\times	0	\times	\times
$c_{\phi D}$		\times	\times	\times	\times	\times	\times	\times
$c_{\phi\Box}$			\times	0	\times	0	\times	\times
$c_{\phi\psi_L}^{(1)}$				\times	\times	\times	0	\times
$c_{\phi\psi_L}^{(3)}$					\times	\times	0	\times
$c_{\phi\psi_R}$						\times	0	\times
$c_{\phi ud}$							\times	0
$c_{\psi_R\phi}$								\times

$\gamma'_{\mathbf{c}_{\phi^4}^{(1)}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	0	0	0	0	0	0	0
$c_{\phi D}$		\times	\times	0	0	0	0	0
$c_{\phi\Box}$			\times	0	0	0	0	0
$c_{\phi\psi_L}^{(1)}$				\times	0	0	0	0
$c_{\phi\psi_L}^{(3)}$					\times	0	0	0
$c_{\phi\psi_R}$						\times	0	0
$c_{\phi ud}$							\times	0
$c_{\psi_R\phi}$								0

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$\gamma'_{\mathbf{c}_{\phi^4}^{(2)}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	0	0	0	0	0	0	0
$c_{\phi D}$		\times	\times	0	0	0	0	0
$c_{\phi\Box}$			\times	0	0	0	0	0
$c_{\phi\psi_L}^{(1)}$				\times	0	0	0	0
$c_{\phi\psi_L}^{(3)}$					\times	0	0	0
$c_{\phi\psi_R}$						\times	0	0
$c_{\phi ud}$							0	0
$c_{\psi_R\phi}$								0

$\gamma'_{\mathbf{c}_{\phi^4}^{(3)}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	0	0	0	0	0	0	0
$c_{\phi D}$		\times	\times	0	0	0	0	0
$c_{\phi\Box}$			\times	0	0	0	0	0
$c_{\phi\psi_L}^{(1)}$				0	0	0	0	0
$c_{\phi\psi_L}^{(3)}$					\times	0	0	0
$c_{\phi\psi_R}$						0	0	0
$c_{\phi ud}$							\times	0
$c_{\psi_R\phi}$								0

173

$\gamma'_{\mathbf{c}_{\phi^6}^{(1)}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	\times	\times	0	0	0	0	0
$c_{\phi D}$		\times	\times	\times	\times	\times	\times	\times
$c_{\phi\Box}$			\times	0	\times	0	\times	\times
$c_{\phi\psi_L}^{(1)}$				\times	\times	\times	0	\times
$c_{\phi\psi_L}^{(3)}$					\times	\times	\times	\times
$c_{\phi\psi_R}$						\times	0	\times
$c_{\phi ud}$							\times	\times
$c_{\psi_R\phi}$								\times

$\gamma'_{\mathbf{c}_{\phi^6}^{(2)}}$	c_ϕ	$c_{\phi D}$	$c_{\phi\Box}$	$c_{\phi\psi_L}^{(1)}$	$c_{\phi\psi_L}^{(3)}$	$c_{\phi\psi_R}$	$c_{\phi ud}$	$c_{\psi_R\phi}$
c_ϕ	0	\times	0	0	0	0	0	0
$c_{\phi D}$		\times	\times	\times	\times	\times	\times	\times
$c_{\phi\Box}$			\times	0	0	0	0	0
$c_{\phi\psi_L}^{(1)}$				\times	\times	\times	0	\times
$c_{\phi\psi_L}^{(3)}$					\times	\times	\times	0
$c_{\phi\psi_R}$						\times	0	\times
$c_{\phi ud}$							\times	\times
$c_{\psi_R\phi}$								\times

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$$\gamma'_{\mathbf{c}_{\mathbf{W}2\phi^4}^{(1)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \times & \emptyset & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \times & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & \times & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \times & 0 & 0 \\ C_{\phi ud} & & & & & & & \times & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

175

$$\gamma'_{\mathbf{c}_{\mathbf{W}2\phi^4}^{(3)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \emptyset & \emptyset & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \emptyset & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & 0 & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \emptyset & 0 & 0 \\ C_{\phi ud} & & & & & & & \emptyset & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

$$\gamma'_{\mathbf{c}_{\mathbf{W}B\phi^4}^{(1)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \emptyset & \emptyset & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \emptyset & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & 0 & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \emptyset & 0 & 0 \\ C_{\phi ud} & & & & & & & \emptyset & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

176

$$\gamma'_{\mathbf{c}_{\mathbf{B}2\phi^4}^{(1)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \times & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \times & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & \times & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \times & 0 & 0 \\ C_{\phi ud} & & & & & & & \times & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

$$\gamma'_{\mathbf{c}_{\mathbf{W}\phi^4\mathbf{D}2}^{(1)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \times & \emptyset & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \times & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & \times & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \times & 0 & 0 \\ C_{\phi ud} & & & & & & & \times & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

$$\gamma'_{\mathbf{c}_{\mathbf{B}\phi^4\mathbf{D}2}^{(1)}} \begin{array}{c|cccccccc} & C_\phi & C_{\phi D} & C_{\phi\Box} & C_{\phi\psi_L}^{(1)} & C_{\phi\psi_L}^{(3)} & C_{\phi\psi_R} & C_{\phi ud} & C_{\psi_R\phi} \\ \hline C_\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi D} & & \times & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\Box} & & & 0 & 0 & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(1)} & & & & \times & 0 & 0 & 0 & 0 \\ C_{\phi\psi_L}^{(3)} & & & & & \times & 0 & 0 & 0 \\ C_{\phi\psi_R} & & & & & & \times & 0 & 0 \\ C_{\phi ud} & & & & & & & \times & 0 \\ C_{\psi_R\phi} & & & & & & & & 0 \end{array}$$

 177 All other γ' matrices vanish identically, with all their zeros being trivial.

178 Finally, in Table 4 we provide a different view on the global structure of the anomalous
 179 dimensions, by showing, for each pair of dimension-six interactions, the dimension-eight
 180 operators that get renormalised by them. Despite being not explicitly shown, contributions
 181 proportional to two fermionic operators involve only leptons or quarks, but not both.

\mathbf{C}_ϕ	$\mathbf{C}_{\phi D}$	$\mathbf{C}_{\phi\Box}$	$\mathbf{C}_{\phi\psi_L}^{(1)}$	$\mathbf{C}_{\phi\psi_L}^{(3)}$	$\mathbf{C}_{\phi\psi_R}$	$\mathbf{C}_{\phi ud}$	$\mathbf{C}_{\psi_R\phi}$
\mathbf{C}_ϕ	$\phi^8, \phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1)$		ϕ^8		ϕ^8	ϕ^8
$\mathbf{C}_{\phi D}$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2), \phi^4(3)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2), \phi^4(3)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$
$\mathbf{C}_{\phi\Box}$		$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2), \phi^4(3)$		$\phi^8, \phi^6(1)$		$\phi^8, \phi^6(1)$	$\phi^8, \phi^6(1)$
$\mathbf{C}_{\phi\psi_L}^{(1)}$		$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$
$\mathbf{C}_{\phi\psi_L}^{(3)}$			$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2), \phi^4(3)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2), \phi^4(3)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1)$
$\mathbf{C}_{\phi\psi_R}$				$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(2)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$
$\mathbf{C}_{\phi ud}$				$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(3)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$ $\phi^4(1), \phi^4(3)$ $V^2\phi^4(1), V\phi^4D^2(1)$	$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^6(1), \phi^6(2)$
$\mathbf{C}_{\psi_R\phi}$						$\phi^8, \phi^6(1), \phi^6(2)$	$\phi^8, \phi^6(1), \phi^6(2)$

Table 4: Dimension-eight operators that are renormalised by the pairs of dimension-six interactions in the corresponding column and row. V stands for either W or B .

5 Discussion and outlook

We conclude this article highlighting several observations that can be made on the basis of the RGE matrices above:

1. All the dimension-eight operators that are renormalised can arise at tree level in UV completions of the SM [29]. The reason is simply that those operators that arise only at loop level involve two Higgs fields (see Table 1), unlike any one-loop diagram containing two insertions of the dimension-six terms. The same holds for dimension-six operators. Thus, we conclude that within the bosonic sector of the SMEFT, dimension-six tree-level operators do not mix into loop-level operators to order E^4/Λ^4 . This extends previous findings at order E^2/Λ^2 [41].

2. Several of the γ' matrices above exhibit a number of zeros (denoted by 0) for which all contributions in the Green basis are vanishing (they can result simply from the absence of Feynman diagrams or from CP conservation reasons). For example, the first row in $\gamma'_{c_{\phi^4}^{(1)}}$ reflects that there are no one-particle-irreducible diagrams with four Higgses involving the insertion of one six-Higgs operator and one four-Higgs interaction. Instead, those denoted by \emptyset ensue from non-trivial cancellations between different counterterms in the Green basis which, on-shell, add to zero. For example, the (23) entry of $\gamma'_{c_{W^2\phi^4}^{(1)}}$ vanishes because the terms proportional to $c_{\phi D}c_{\phi\Box}$ in $\tilde{c}_{W^2\phi^4}^{(1)}$ and $\tilde{c}_{W\phi^4 D^2}^{(7)}$ cancel in Eq. (73). Zeros as this one might be understood on the basis of the helicity-amplitude formalism [54, 55].

3. Related to the previous point, we find the very surprising result that the Peskin-Takeuchi parameters [56] S and U are not renormalised by tree-level dimension-six operators to order v^4/Λ^4 . Indeed, these observables read [17, 27]:

$$\frac{1}{16\pi}S = \frac{v^2}{\Lambda^2} \left[c_{\phi WB} + c_{WB\phi^4}^{(1)} \frac{v^4}{\Lambda^4} \right], \quad \frac{1}{16\pi}U = \frac{v^4}{\Lambda^4} c_{W^2\phi^4}^{(3)}, \quad (40)$$

with $\mathcal{O}_{\phi WB} = (\phi^\dagger \sigma^I \phi) W_{\mu\nu}^I B^{\mu\nu}$. (Note that U arises only at dimension eight.) What we find is that $c_{WB\phi^4}^{(1)}$ and $c_{W^2\phi^4}^{(3)}$ do not renormalise because the direct contribution cancels that from the redundant operator $\mathcal{O}_{W\phi^4 D^2}^{(7)}$. This fact, together with the non-renormalisation of $c_{\phi WB}$ found in Refs. [9], shows that both S and U are not triggered by dimension-six tree-level interactions at one loop.

4. The Wilson coefficients $c_{\phi^4}^{(1)}$, $c_{\phi^4}^{(2)}$ and $c_{\phi^4}^{(3)}$ are subject to positivity constraints [33]. In particular, $c_{\phi^4}^{(2)} \geq 0$, $c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0$ and $c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$. These inequalities should reflect in the corresponding matrices.

To see how, let us first note that there exist well-behaved UV completions of the SM that induce, at tree level, the operators c_ϕ , $c_{\phi D}$ and $c_{\phi\Box}$ with *arbitrary values*; see Appendix C for a particular example. The values of the dimension-eight Wilson coefficients $c_{\phi^4}^{(1)}$, $c_{\phi^4}^{(2)}$ and $c_{\phi^4}^{(3)}$ at any energy $\mu < M$ triggered by double insertions of the dimension-six operators scale differently with the model couplings than the tree-level contribution (which in general can not be avoided). In particular, within the model of Appendix C, we have $\sim \kappa^4/M^4$ versus $\sim \kappa^2/M^2$. This suggests that both contributions must satisfy the positivity bounds separately. Indeed, in the limit of scale-invariant dimension-six Wilson coefficients, we can check that:

$$16\pi^2 c_{\phi^4}^{(2)} = \frac{1}{3}(5c_{\phi D}^2 + 16c_{\phi D}c_{\phi\Box} + 16c_{\phi\Box}^2) \log \frac{M}{\mu} > 0, \quad (41)$$

$$16\pi^2 \left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \right] = \frac{16}{3} (c_{\phi D}^2 - c_{\phi D} c_{\phi \square} + 2c_{\phi \square}^2) \log \frac{M}{\mu} > 0, \quad (42)$$

$$16\pi^2 \left[c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \right] = 3(c_{\phi D}^2 + 8c_{\phi \square}^2) \log \frac{M}{\mu} > 0; \quad (43)$$

224 for arbitrary values of $c_{\phi D}$ and $c_{\phi \square}$. (Fermionic Wilson coefficients do not modify these
 225 relations because they contribute as sums of modulus squared and therefore positively, as
 226 a result also of very fine cancellations between positive and negative terms in Eqs. 17–19.)
 227 Note that these inequalities hold non-trivially; for example $c_{\phi^4}^{(1)}$ is negative in a neigh-
 228 bourhood of its minimum. It should be possible to extend this kind of analysis to other
 229 operators (which do not renormalise within the assumptions we make in this work), thus
 230 providing interesting cross-checks of the anomalous dimensions (or new bounds on Wilson
 231 coefficients).

232

233 **5.** Among the non-vanishing entries in the different γ 's, we find values that depart
 234 significantly from the naive estimate of $\mathcal{O}(1)$. The most notable of these, not suppressed
 235 by gauge or λ couplings, are the 126 in γ'_{ϕ^8} and the 96 in $\gamma'_{c_{\phi^6}^{(1)}}$; see Appendix B. As we
 236 discuss below, these large numbers can have important low-energy implications.

237

238 **6.** Although we do not aim to exhaust all possible phenomenological implications of
 239 the running of the dimension-eight operators, we would like to stress that the T parameter,
 240 defined by [17]

$$\alpha T = -\frac{1}{2} \frac{v^2}{\Lambda^2} \left[c_{\phi D} + c_{\phi^6}^{(2)} \frac{v^2}{\Lambda^2} \right], \quad (44)$$

241 with $\alpha \sim 1/137$ being the fine-structure constant, receives contributions from the operator
 242 $\mathcal{O}_{\phi ud}$ only at order v^4/Λ^4 (because $c_{\phi D}$ is not renormalised by one insertion of $\mathcal{O}_{\phi ud}$; see
 243 Ref. [8]). Using bounds on T from Ref. [57], and assuming that only $c_{\phi tb}$ is non-vanishing,
 244 we obtain $c_{\phi tb} \leq 5.9$ for $\Lambda = 1$ TeV. This constraint is competitive with the value $c_{\phi tb} \leq 5.3$
 245 reported in Ref. [58].

246

247 **7.** We would also like to emphasize the importance of one-loop v^4/Λ^4 effects for the EW
 248 phase transition (EWPT) ensuing from modifications of the Higgs potential [16, 59–63].
 249 To this aim, let us assume that c_ϕ is the only non-vanishing Wilson coefficient in the UV,
 250 and let us neglect gauge and Yukawa couplings. The Higgs potential in the infrared is then
 251 provided by running \mathcal{L}_{UV} down to the EW scale. In the leading-logarithm approximation,
 252 this reads:

$$V \sim \mu^2 |\phi|^2 + \lambda |\phi|^4 + \frac{c_\phi}{\Lambda^2} \left(1 - \frac{108}{16\pi^2} \lambda \log \frac{\Lambda}{v} \right) |\phi|^6 + \frac{126}{16\pi^2 \Lambda^4} \log \frac{\Lambda}{v} c_\phi^2 |\phi|^8, \quad (45)$$

253 where c_ϕ is evaluated in the UV, and the renormalisable couplings are assumed scale-
 254 invariant for simplicity. (The first logarithm can be read from Ref. [7].) In both $|\phi|^6$ and
 255 $|\phi|^8$ we have included only the dominant corrections.

256 Following Refs. [16, 64], we know that the EWPT is first order and strong as required
 257 by EW baryogenesis [65] provided that $500 \text{ GeV} \lesssim \Lambda/\sqrt{c_{\text{eff}}} \lesssim 750 \text{ GeV}$, where we have
 258 defined $c_{\text{eff}} = c_\phi + 3/2 v^2/\Lambda^2 c_{\phi^8}$. Fixing, as a matter of example, $\Lambda = 1$ TeV, it can be
 259 easily checked that this occurs for:

$$1.7 \text{ TeV}^{-2} \lesssim c_\phi \lesssim 3.7 \text{ TeV}^{-2} \quad (46)$$

260 if the running of c_{ϕ^8} is neglected, whereas if we account for it we obtain:

$$1.5 \text{ TeV}^{-2} \lesssim c_\phi \lesssim 2.6 \text{ TeV}^{-2}. \quad (47)$$

	d_5	d_5^2	d_6	d_5^3	$d_5 \times d_6$	d_7	d_5^4	$d_5^2 \times d_6$	d_6^2	$d_5 \times d_7$	d_8
$d_{<4}$ (bosonic)			✓ [7]						This work		✗
$d_{<4}$ (fermionic)			✓ [7]						✗		✗
d_5	✓ [66–68]				✓ [71]	✓ [71]					
d_6 (bosonic)		✓ [30]	✓ [7–9]					✗	This work	✗	✗
d_6 (fermionic)		✓ [30]	✓ [7–9, 69]					✗	✗	✗	✗
d_7				✓ [71]	✓ [71]	✓ [22, 70]					
d_8 (bosonic)							✗	✗	This work	✗	✗
d_8 (fermionic)							✗	✗	✗	✗	✗

Table 5: *State of the art of SMEFT renormalisation. The rows represent the operators (defined by their dimension d) being renormalised, while the columns show the operators entering the loops. Note that there are no bosonic interactions at odd dimension. Blank entries vanish. A tick ✓ represents that the complete contribution is known. The ✓ indicates that only (but substantial) partial results have been already obtained. The ✗ indicates that nothing, or very little, is known. The contribution made in this paper is marked by ■.*

261 The 30 % difference in the upper limit evidences the potential importance of both dimension-
 262 eight operators and their running.

263

264 To conclude, let us remark that our results comprise one step further towards the
 265 one-loop renormalisation of the SMEFT to order v^4/Λ^4 . This endeavor was initiated
 266 in Refs. [4–9] (see also Refs. [66–68] for the renormalisation of the Weinberg operator)
 267 and continued in Ref. [69] (for baryon-number violating interactions), Ref. [30] (which
 268 includes the renormalisation of dimension-six operators by pairs of Weinberg interactions),
 269 Refs. [22, 70] (which involves the renormalisation of dimension-seven operators triggered
 270 by relevant couplings) and Ref. [71] (in which neutrino masses are renormalised to order
 271 v^3/Λ^3 , including arbitrary combinations of dimension-five, -six and -seven operators).
 272 Some partial results of renormalisation within the dimension-eight sector of the SMEFT
 273 can be found in Refs. [19, 72]. See Table 5 for a summary of the state of the art.

274 In forecoming works, we plan to extend the results of this paper with the inclusion, in
 275 the UV, of the operators of dimension eight that can be generated at tree level. We will
 276 also consider the renormalisation of non-bosonic operators. The latter can be induced, in
 277 particular, by field redefinitions aimed at removing the operators $\mathcal{O}_{BD\phi}$, $\mathcal{O}'_{\phi D}$, $\mathcal{O}_{WD\phi}$ and
 278 $\mathcal{O}_{W\phi^4 D^2}^{(6)}$; see Appendix A. Consequently, our current findings lay the basis for future work
 279 in this direction.

280 Acknowledgments

281 We would like to thank Jorge de Blas for useful discussions. MC and JS are supported
 282 by the Ministry of Science and Innovation under grant number FPA2016-78220-C3-1/3-
 283 P (FEDER), SRA under grant PID2019-106087GB-C21/C22 (10.13039/501100011033),
 284 and by the Junta de Andalucía grants FQM 101, A-FQM-211-UGR18 and P18-FR-4314
 285 (FEDER). MC is also supported by the Spanish MINECO under the Ramón y Cajal pro-
 286 gramme. GG and MR acknowledge support by LIP (FCT, COMPETE2020-Portugal2020,
 287 FEDER, POCI-01-0145-FEDER-007334) as well as by FCT under project CERN/FIS-
 288 PAR/0024/2019. GG is also supported by FCT under the grant SFRH/BD/144244/2019.
 289 MR is also supported by Fundação para a Ciência e Tecnologia (FCT) under the grant
 290 PD/BD/142773/2018.

291 A Removing redundant operators

292 The redundant operators generated in the process of renormalisation can be removed upon
 293 performing suitable perturbative field redefinitions, for example $\phi \rightarrow \phi + \frac{1}{\Lambda^2} \mathcal{O}$, where \mathcal{O}
 294 is called the *perturbation*. We are interested in the effect of these field redefinitions to
 295 linear order in the perturbation (because \mathcal{O} is loop suppressed and therefore quadratic
 296 powers of this term are formally two-loop corrections), which can be implemented through
 297 the equations of motion of the SMEFT to order v^2/Λ^2 [53]. These read [73]:

$$D^2 \phi^i = -\mu^2 \phi^i - 2\lambda(\phi^\dagger \phi) \phi^i + \frac{1}{\Lambda^2} \left\{ 3c_\phi (\phi^\dagger \phi)^2 \phi^i + 2c_{\phi\Box} \phi^i \Box (\phi^\dagger \phi) \right. \\ \left. - c_{\phi D} \left[(D^\mu \phi)^i (\phi^\dagger \overleftrightarrow{D}_\mu \phi) + \phi^i \partial^\mu (\phi^\dagger D_\mu \phi) \right] \right\} + \dots, \quad (48)$$

$$\partial^\nu B_{\mu\nu} = \frac{g_1}{2} \phi^\dagger i \overleftrightarrow{D}_\mu \phi + \frac{c_{\phi D}}{\Lambda^2} \frac{g_1}{2} (\phi^\dagger \phi) (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) + \dots, \quad (49)$$

$$D^\nu W_{\mu\nu}^I = \frac{g_2}{2} \phi^\dagger i \overleftrightarrow{D}_\mu^I \phi + \frac{c_{\phi D}}{\Lambda^2} g_2 (\phi^\dagger \sigma^I \phi) (\phi^\dagger i \overleftrightarrow{D}_\mu \phi) + \dots, \quad (50)$$

298 where the ellipses represent fermionic operators, on which we are not interested. The
 299 following relations hold on-shell:

$$\mathcal{O}_{BD\phi} = \frac{g_1}{2} \left[\mathcal{O}_{\phi\Box} + 4\mathcal{O}_{\phi D} - \frac{\mu^2}{\Lambda^2} c_{\phi D} \mathcal{O}_\phi \right] + \frac{g_1}{2} \frac{c_{\phi D}}{\Lambda^2} \left[-2\lambda \mathcal{O}_{\phi^8} + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)} \right] + \dots, \quad (51)$$

$$\mathcal{O}'_{\phi D} = -\frac{1}{2} \left[-\mathcal{O}_{\phi\Box} - 2\mu^2 |\phi|^4 - 4\lambda \mathcal{O}_\phi + \frac{\mu^2}{\Lambda^2} (c_{\phi D} - 8c_{\phi\Box}) \mathcal{O}_\phi \right] \\ - \frac{1}{2\Lambda^2} \left[(6c_\phi - 16\lambda c_{\phi\Box} + 2\lambda c_{\phi D}) \mathcal{O}_{\phi^8} + (8c_{\phi\Box} + c_{\phi D}) \mathcal{O}_{\phi^6}^{(1)} + 2c_{\phi D} \mathcal{O}_{\phi^6}^{(2)} \right] + \dots, \quad (52)$$

$$\mathcal{O}_{WD\phi} = -g_2 \left[-\frac{3}{2} \mathcal{O}_{\phi\Box} - 2\mu^2 |\phi|^4 - 4\lambda \mathcal{O}_\phi + \frac{\mu^2}{\Lambda^2} (2c_{\phi D} - 8c_{\phi\Box}) \mathcal{O}_\phi \right] \\ - \frac{g_2}{\Lambda^2} \left[(6c_\phi - 16\lambda c_{\phi\Box} + 4\lambda c_{\phi D}) \mathcal{O}_{\phi^8} + (8c_{\phi\Box} - 2c_{\phi D}) \mathcal{O}_{\phi^6}^{(1)} \right] + \dots; \quad (53)$$

300 where the ellipses encode again fermionic interactions. The operator $\mathcal{O}''_{\phi D}$ gives no contri-
 301 butions to the bosonic sector.

302 To arrive at these results, we used the following identities:

$$|\phi|^2 (\phi^\dagger D_\mu \phi) (D^\mu \phi^\dagger \phi) = \frac{1}{2} \left[\mathcal{O}_{\phi^6}^{(1)} + \mathcal{O}_{\phi^6}^{(2)} \right], \quad (54)$$

$$|\phi|^2 (\phi^\dagger i \overleftrightarrow{D}_\mu \phi)^2 = 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)} + \frac{1}{2} \mathcal{O}_{\phi^6}^{(3)}, \quad (55)$$

$$|\phi|^4 \Box |\phi|^2 = 2\mathcal{O}_{\phi^6}^{(1)} + \mathcal{O}_{\phi^6}^{(3)}. \quad (56)$$

303 In turn, the redundant dimension-eight operators become, on-shell:

$$\mathcal{O}_{\phi^6}^{(3)} = -2\mu^2 \mathcal{O}_\phi - 4\lambda \mathcal{O}_{\phi^8}, \quad (57)$$

$$\mathcal{O}_{\phi^4}^{(4)} = \mu^2 \left[-\mathcal{O}_{\phi D} - 2\mu^2 |\phi|^4 - 4\lambda \mathcal{O}_\phi \right] - 4\lambda \mathcal{O}_{\phi^6}^{(1)}, \quad (58)$$

$$\mathcal{O}_{\phi^4}^{(6)} = -2\mu^2 \mathcal{O}_{\phi D} - 2\lambda \left[\mathcal{O}_{\phi^6}^{(1)} + \mathcal{O}_{\phi^6}^{(2)} \right], \quad (59)$$

$$\mathcal{O}_{\phi^4}^{(8)} = 2\mu^2 \left[\mu^2 |\phi|^4 + 4\lambda \mathcal{O}_\phi \right] + 8\lambda^2 \mathcal{O}_{\phi^8}, \quad (60)$$

$$\mathcal{O}_{\phi^4}^{(10)} = \mu^2 \left[\mu^2 |\phi|^4 + 4\lambda \mathcal{O}_\phi \right] + 4\lambda^2 \mathcal{O}_{\phi^8}, \quad (61)$$

$$\mathcal{O}_{\phi^4}^{(11)} = \mu^2 \left[\mu^2 |\phi|^4 + 4\lambda \mathcal{O}_\phi \right] + 4\lambda^2 \mathcal{O}_{\phi^8}, \quad (62)$$

$$\mathcal{O}_{\phi^4}^{(12)} = \mu^2 \left[2\mathcal{O}_{\phi D} + \mathcal{O}_{\phi\Box} - 2\lambda \mathcal{O}_\phi \right] + 2 \left[2\lambda \mathcal{O}_{\phi^6}^{(1)} + \lambda \mathcal{O}_{\phi^6}^{(2)} - 2\lambda^2 \mathcal{O}_{\phi^8} \right], \quad (63)$$

$$\mathcal{O}_{W\phi^4 D^2}^{(6)} = -\frac{g_2}{2} \left[-\mu^2 \mathcal{O}_\phi + 5\mathcal{O}_{\phi^6}^{(1)} - 2\lambda \mathcal{O}_{\phi^8} + \dots \right], \quad (64)$$

$$\begin{aligned} \mathcal{O}_{W\phi^4 D^2}^{(7)} = & \frac{1}{4} \left\{ \frac{g_2}{2} \left[2\mu^2 \mathcal{O}_\phi + \mathcal{O}_{W^2\phi^4}^{(1)} + \mathcal{O}_{W^2\phi^4}^{(3)} \right] + g_1 \mathcal{O}_{WB\phi^4}^{(1)} + 8\mathcal{O}_{W\phi^4 D^2}^{(1)} \right. \\ & \left. - 6g_2 \mathcal{O}_{\phi^6}^{(1)} + g_2 \mathcal{O}_{\phi^6}^{(2)} + 2g_2 \lambda \mathcal{O}_{\phi^8}^{(1)} \right\}, \quad (65) \end{aligned}$$

$$\mathcal{O}_{B\phi^4 D^2}^{(3)} = -\frac{g_1}{2} \left[-\mu^2 \mathcal{O}_\phi + 3\mathcal{O}_{\phi^6}^{(1)} + 2\mathcal{O}_{\phi^6}^{(2)} - 2\lambda \mathcal{O}_{\phi^8} \right]; \quad (66)$$

304 where again the ellipses represent terms on which we are not interested in this work. The
305 operator $\mathcal{O}_{\phi^6}^{(4)}$ contributes only to fermionic interactions.

306 Altogether, these equations lead to:

$$\lambda \rightarrow \lambda - \frac{\mu^2}{\Lambda^2} c'_{\phi D} - \frac{\mu^4}{\Lambda^4} \left[-2 \left(c_{\phi^4}^{(4)} - c_{\phi^4}^{(8)} \right) + c_{\phi^4}^{(10)} + c_{\phi^4}^{(11)} \right], \quad (67)$$

$$\begin{aligned} c_\phi \rightarrow & c_\phi + 2\lambda c'_{\phi D} + \frac{\mu^2}{2\Lambda^4} \left[3(c_{\phi D} + 2c_{\phi\Box})c_\phi - (c_{\phi D} - 8c_{\phi\Box})(c'_{\phi D} + 2g_2 c_{WD\phi}) \right. \\ & \left. - 2g_2 c_{\phi D} c_{WD\phi} - g_1 c_{\phi D} c_{BD\phi} - 4c_{\phi^6}^{(3)} + 4\lambda \left\{ -2c_{\phi^4}^{(4)} + 4c_{\phi^4}^{(8)} + 2c_{\phi^4}^{(10)} + 2c_{\phi^4}^{(11)} - c_{\phi^4}^{(12)} \right\} \right. \\ & \left. + g_2 c_{W\phi^4 D^2}^{(6)} + \frac{g_2}{2} c_{W\phi^4 D^2}^{(7)} + g_1 c_{B\phi^4 D^2}^{(3)} \right], \quad (68) \end{aligned}$$

$$c_{\phi D} \rightarrow c_{\phi D} + 2\frac{\mu^2}{\Lambda^4} \left[\frac{1}{2}(c_{\phi D} + 2c_{\phi\Box})c_{\phi D} - c_{\phi^4}^{(6)} + c_{\phi^4}^{(12)} \right], \quad (69)$$

$$c_{\phi\Box} \rightarrow c_{\phi\Box} + \frac{1}{2}c'_{\phi D} + \frac{\mu^2}{\Lambda^4} \left[(c_{\phi D} + 2c_{\phi\Box})c_{\phi\Box} - c_{\phi^4}^{(4)} + c_{\phi^4}^{(12)} \right], \quad (70)$$

307 where we have already normalised canonically the Higgs kinetic term; as well as

$$\begin{aligned} c_{\phi^8} \rightarrow & c_{\phi^8} - g_1 \lambda c_{BD\phi} c_{\phi D} - (c'_{\phi D} + 2g_2 c_{WD\phi}) (3c_\phi - 8\lambda c_{\phi\Box} + \lambda c_{\phi D}) \\ & - 2g_2 \lambda c_{WD\phi} c_{\phi D} - 4\lambda c_{\phi^6}^{(3)} + 4\lambda^2 \left[2c_{\phi^4}^{(8)} + c_{\phi^4}^{(10)} + c_{\phi^4}^{(11)} - c_{\phi^4}^{(12)} \right] \\ & + g_2 \lambda c_{W\phi^4 D^2}^{(6)} + \frac{g_2}{2} \lambda c_{W\phi^4 D^2}^{(7)} + g_1 \lambda c_{B\phi^4 D^2}^{(3)}, \quad (71) \end{aligned}$$

$$\begin{aligned} c_{\phi^6}^{(1)} \rightarrow & c_{\phi^6}^{(1)} + \frac{3}{2} g_1 c_{BD\phi} c_{\phi D} - \frac{(c'_{\phi D} + 2g_2 c_{WD\phi})}{2} (8c_{\phi\Box} + c_{\phi D}) + 3g_2 c_{WD\phi} c_{\phi D} \\ & - 2\lambda \left[2c_{\phi^4}^{(4)} + c_{\phi^4}^{(6)} - 2c_{\phi^4}^{(12)} \right] - \frac{5}{2} g_2 c_{W\phi^4 D^2}^{(6)} - \frac{3}{2} g_2 c_{W\phi^4 D^2}^{(7)} - \frac{3}{2} g_1 c_{B\phi^4 D^2}^{(3)}, \quad (72) \end{aligned}$$

$$c_{\phi^6}^{(2)} \rightarrow c_{\phi^6}^{(2)} + g_1 c_{BD\phi} c_{\phi D} - c'_{\phi D} c_{\phi D} - 2\lambda \left[c_{\phi^4}^{(6)} - c_{\phi^4}^{(12)} \right] + \frac{g_2}{4} c_{W\phi^4 D^2}^{(7)} - g_1 c_{B\phi^4 D^2}^{(3)},$$

$$c_{W^2\phi^4}^{(1)} \rightarrow c_{W^2\phi^4}^{(1)} + \frac{g_2}{8} c_{W\phi^4 D^2}^{(7)}, \quad (73)$$

$$c_{W^2\phi^4}^{(3)} \rightarrow c_{W^2\phi^4}^{(3)} + \frac{g_2}{8} c_{W\phi^4 D^2}^{(7)}, \quad (74)$$

$$c_{WB\phi^4}^{(1)} \rightarrow c_{WB\phi^4}^{(1)} + \frac{g_1}{4} c_{W\phi^4 D^2}^{(7)} \quad (75)$$

$$c_{W\phi^4 D^2}^{(1)} \rightarrow c_{W\phi^4 D^2}^{(1)} + 2c_{W\phi^4 D^2}^{(7)}. \quad (76)$$

308 B Renormalisation group equations

309 For a given coupling c , we define:

$$\dot{c} \equiv 16\pi^2 \mu \frac{dc}{d\mu}. \quad (77)$$

310 Thus, we have:

$$\dot{\lambda} \supset (5c_{\phi D}^2 - 24c_{\phi D}c_{\phi\Box} + 24c_{\phi\Box}^2) \frac{\mu^4}{\Lambda^4}, \quad (78)$$

311

$$\begin{aligned} \dot{c}_\phi \supset & \left\{ \frac{-37}{24} g_1^2 c_{\phi D}^2 + \frac{5}{8} g_2^2 c_{\phi D}^2 - 35\lambda c_{\phi D}^2 + \frac{26}{3} c_{\phi\Box} g_1^2 c_{\phi D} - \frac{113}{6} c_{\phi\Box} g_2^2 c_{\phi D} - 51c_\phi c_{\phi D} \right. & (79) \\ & + 320\lambda c_{\phi\Box} c_{\phi D} - 3\text{Tr}[c_{d\phi} y^{d\dagger}] c_{\phi D} - \text{Tr}[c_{e\phi} y^{e\dagger}] c_{\phi D} - 3\text{Tr}[c_{u\phi} y^{u\dagger}] c_{\phi D} - 3\text{Tr}[(c_{d\phi})^\dagger y^d] c_{\phi D} \\ & - \text{Tr}[(c_{e\phi})^\dagger y^e] c_{\phi D} - 3\text{Tr}[(c_{u\phi})^\dagger y^u] c_{\phi D} - 6\text{Tr}[y^{u\dagger} y^d (c_{\phi ud})^\dagger] c_{\phi D} + 4\text{Tr}[c_{\phi l}^{(3)} y^e y^{e\dagger}] c_{\phi D} \\ & + 12\text{Tr}[c_{\phi q}^{(3)} y^d y^{d\dagger}] c_{\phi D} + 12\text{Tr}[c_{\phi q}^{(3)} y^u y^{u\dagger}] c_{\phi D} - 6\text{Tr}[c_{\phi ud} y^{d\dagger} y^u] c_{\phi D} + \frac{2}{3} g_1^2 \text{Tr}[c_{\phi d}] c_{\phi D} \\ & + \frac{2}{3} g_1^2 \text{Tr}[c_{\phi e}] c_{\phi D} + \frac{2}{3} g_1^2 \text{Tr}[c_{\phi l}^{(1)}] c_{\phi D} - \frac{8}{3} g_2^2 \text{Tr}[c_{\phi l}^{(3)}] c_{\phi D} - \frac{2}{3} g_1^2 \text{Tr}[c_{\phi q}^{(1)}] c_{\phi D} - 8g_2^2 \text{Tr}[c_{\phi q}^{(3)}] c_{\phi D} \\ & - \frac{4}{3} g_1^2 \text{Tr}[c_{\phi u}] c_{\phi D} - 640\lambda c_{\phi\Box}^2 - \frac{20}{3} c_{\phi\Box}^2 g_1^2 + 20c_{\phi\Box}^2 g_2^2 + 222c_\phi c_{\phi\Box} + 24c_{\phi\Box} \text{Tr}[c_{d\phi} y^{d\dagger}] \\ & + 8c_{\phi\Box} \text{Tr}[c_{e\phi} y^{e\dagger}] + 24c_{\phi\Box} \text{Tr}[c_{u\phi} y^{u\dagger}] + 24c_{\phi\Box} \text{Tr}[(c_{d\phi})^\dagger y^d] - 12\text{Tr}[(c_{d\phi})^\dagger c_{d\phi}] \\ & + 8c_{\phi\Box} \text{Tr}[(c_{e\phi})^\dagger y^e] - 4\text{Tr}[(c_{e\phi})^\dagger c_{e\phi}] + 24c_{\phi\Box} \text{Tr}[(c_{u\phi})^\dagger y^u] - 12\text{Tr}[(c_{u\phi})^\dagger c_{u\phi}] \\ & - 2g_2^2 \text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] - \frac{2}{3} g_2^2 \text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] - \frac{4}{3} g_2^2 \text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] - \frac{8}{3} g_1^2 \text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] \\ & + \frac{4}{3} g_2^2 \text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] - 4g_2^2 \text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] - 8g_2^2 \text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 4g_2^2 \text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] \\ & - 2g_2^2 \text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + g_1^2 \text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] - g_2^2 \text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 48c_{\phi\Box} \text{Tr}[y^{u\dagger} y^d (c_{\phi ud})^\dagger] \\ & - 6\text{Tr}[c_{d\phi} y^{d\dagger} c_{\phi q}^{(1)}] - 6\text{Tr}[c_{d\phi} y^{d\dagger} c_{\phi q}^{(3)}] - 2\text{Tr}[c_{e\phi} y^{e\dagger} c_{\phi l}^{(1)}] - 2\text{Tr}[c_{e\phi} y^{e\dagger} c_{\phi l}^{(3)}] + 6\text{Tr}[c_{u\phi} y^{u\dagger} c_{\phi q}^{(1)}] \\ & - 6\text{Tr}[c_{u\phi} y^{u\dagger} c_{\phi q}^{(3)}] + 6\text{Tr}[c_{\phi d} y^{d\dagger} c_{d\phi}] + 2\text{Tr}[c_{\phi e} y^{e\dagger} c_{e\phi}] - 2\text{Tr}[c_{\phi l}^{(1)} y^e (c_{e\phi})^\dagger] - 32c_{\phi\Box} \text{Tr}[c_{\phi l}^{(3)} y^e y^{e\dagger}] \\ & - 2\text{Tr}[c_{\phi l}^{(3)} y^e (c_{e\phi})^\dagger] - 6\text{Tr}[c_{\phi q}^{(1)} y^d (c_{d\phi})^\dagger] + 6\text{Tr}[c_{\phi q}^{(1)} y^u (c_{u\phi})^\dagger] - 96c_{\phi\Box} \text{Tr}[c_{\phi q}^{(3)} y^d y^{d\dagger}] \\ & - 6\text{Tr}[c_{\phi q}^{(3)} y^d (c_{d\phi})^\dagger] - 96c_{\phi\Box} \text{Tr}[c_{\phi q}^{(3)} y^u y^{u\dagger}] - 6\text{Tr}[c_{\phi q}^{(3)} y^u (c_{u\phi})^\dagger] - 6\text{Tr}[c_{\phi u} y^{u\dagger} c_{u\phi}] \\ & + 48c_{\phi\Box} \text{Tr}[c_{\phi ud} y^{d\dagger} y^u] + 6\text{Tr}[(c_{d\phi})^\dagger y^d c_{\phi d}] + 2\text{Tr}[(c_{e\phi})^\dagger y^e c_{\phi e}] - 6\text{Tr}[(c_{u\phi})^\dagger y^u c_{\phi u}] \\ & + 6\text{Tr}[c_{\phi d} y^{d\dagger} y^d c_{\phi d}] + 2\text{Tr}[c_{\phi e} y^{e\dagger} y^e c_{\phi e}] + 2\text{Tr}[c_{\phi l}^{(1)} y^e y^{e\dagger} c_{\phi l}^{(1)}] + 2\text{Tr}[c_{\phi l}^{(1)} y^e y^{e\dagger} c_{\phi l}^{(3)}] \\ & - 4\text{Tr}[c_{\phi l}^{(1)} y^e c_{\phi e} y^{e\dagger}] + 2\text{Tr}[c_{\phi l}^{(3)} y^e y^{e\dagger} c_{\phi l}^{(1)}] + 2\text{Tr}[c_{\phi l}^{(3)} y^e y^{e\dagger} c_{\phi l}^{(3)}] - 4\text{Tr}[c_{\phi l}^{(3)} y^e c_{\phi e} y^{e\dagger}] \\ & + 6\text{Tr}[c_{\phi q}^{(1)} y^d y^{d\dagger} c_{\phi q}^{(1)}] + 6\text{Tr}[c_{\phi q}^{(1)} y^d y^{d\dagger} c_{\phi q}^{(3)}] - 12\text{Tr}[c_{\phi q}^{(1)} y^d c_{\phi d} y^{d\dagger}] + 6\text{Tr}[c_{\phi q}^{(1)} y^u y^{u\dagger} c_{\phi q}^{(1)}] \end{aligned}$$

$$\begin{aligned}
& -6\text{Tr}[c_{\phi q}^{(1)} y^u y^{u\dagger} c_{\phi q}^{(3)}] - 12\text{Tr}[c_{\phi q}^{(1)} y^u c_{\phi u} y^{u\dagger}] + 6\text{Tr}[c_{\phi q}^{(3)} y^d y^{d\dagger} c_{\phi q}^{(1)}] + 6\text{Tr}[c_{\phi q}^{(3)} y^d y^{d\dagger} c_{\phi q}^{(3)}] \\
& - 12\text{Tr}[c_{\phi q}^{(3)} y^d c_{\phi d} y^{d\dagger}] - 6\text{Tr}[c_{\phi q}^{(3)} y^u y^{u\dagger} c_{\phi q}^{(1)}] + 6\text{Tr}[c_{\phi q}^{(3)} y^u y^{u\dagger} c_{\phi q}^{(3)}] + 12\text{Tr}[c_{\phi q}^{(3)} y^u c_{\phi u} y^{u\dagger}] \\
& + 6\text{Tr}[c_{\phi u} y^{u\dagger} y^u c_{\phi u}] + \frac{32}{3} c_{\phi\Box} g_2^2 \text{Tr}[c_{\phi l}^{(3)}] + 32 c_{\phi\Box} g_2^2 \text{Tr}[c_{\phi q}^{(3)}] \Big\} \frac{\mu^2}{\Lambda^2},
\end{aligned}$$

$$\dot{c}_{\phi D} \supset (-10c_{\phi D}^2 + 4c_{\phi D} c_{\phi\Box}) \frac{\mu^2}{\Lambda^2}, \quad (80)$$

$$\dot{c}_{\phi\Box} \supset \left(-\frac{3}{2} c_{\phi D}^2 - 14c_{\phi D} c_{\phi\Box} + 36c_{\phi\Box}^2 \right) \frac{\mu^2}{\Lambda^2}. \quad (81)$$

312

$$\dot{c}_{\phi 8} = -126c_{\phi}^2 - 60c_{\phi D}^2 \lambda^2 - 864c_{\phi\Box}^2 \lambda^2 - 4(c_{\phi D}^2 - 10c_{\phi\Box} c_{\phi D} + 32c_{\phi\Box}^2) \lambda^2 \quad (82)$$

$$\begin{aligned}
& + 48c_{\phi}(15c_{\phi D} + 18\lambda c_{\phi\Box} - 4c_{\phi\Box})\lambda + \frac{1}{6}g_2^2 \left(c_{\phi D}^2 - 12c_{\phi\Box} c_{\phi D} - 24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] \right. \\
& - 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] - 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] - 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] - 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] \\
& \left. + 12\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] \right) \lambda + \frac{1}{12}g_1^2 \left(3c_{\phi D}^2 - 4c_{\phi\Box} c_{\phi D} - 16c_{\phi\Box}^2 - 64\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] \right. \\
& - 192\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] \Big) \lambda + \frac{1}{12}g_2^2 \left(-5c_{\phi D}^2 + 20c_{\phi\Box} c_{\phi D} - 16c_{\phi\Box}^2 \right. \\
& + 32\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 96\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] - 48\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] \Big) \lambda + \frac{1}{2} \left\{ -3(g_2^2 + 4\lambda)c_{\phi D}^2 \right. \\
& - 4c_{\phi\Box}(3g_1^2 + 3g_2^2 - 112\lambda)c_{\phi D} - 48c_{\phi}(c_{\phi D} - 9c_{\phi\Box}) - 8c_{\phi\Box}^2(128\lambda + 3g_1^2 + 3g_2^2) \\
& + 8 \left(-6\text{Tr}[(c_{d\phi})^\dagger c_{d\phi}] - 2\text{Tr}[(c_{e\phi})^\dagger c_{e\phi}] - 6\text{Tr}[(c_{u\phi})^\dagger c_{u\phi}] - 3\text{Tr}[y^d (c_{d\phi})^\dagger c_{\phi q}^{(1)}] \right. \\
& - 3\text{Tr}[y^d (c_{d\phi})^\dagger c_{\phi q}^{(3)}] + 3\text{Tr}[y^{d\dagger} c_{d\phi} c_{\phi d}] - \text{Tr}[y^e (c_{e\phi})^\dagger c_{\phi l}^{(1)}] - \text{Tr}[y^e (c_{e\phi})^\dagger c_{\phi l}^{(3)}] + \text{Tr}[y^{e\dagger} c_{e\phi} c_{\phi e}] \\
& - 3\text{Tr}[y^u c_{\phi u} (c_{u\phi})^\dagger] + 3\text{Tr}[y^{u\dagger} c_{\phi q}^{(1)} c_{u\phi}] - 3\text{Tr}[y^{u\dagger} c_{\phi q}^{(3)} c_{u\phi}] - 3\text{Tr}[c_{u\phi} c_{\phi u} y^{u\dagger}] + 3\text{Tr}[c_{\phi d} (c_{d\phi})^\dagger y^d] \\
& + \text{Tr}[c_{\phi e} (c_{e\phi})^\dagger y^e] - \text{Tr}[c_{\phi l}^{(1)} c_{e\phi} y^{e\dagger}] - \text{Tr}[c_{\phi l}^{(3)} c_{e\phi} y^{e\dagger}] - 3\text{Tr}[c_{\phi q}^{(1)} c_{d\phi} y^{d\dagger}] - 3\text{Tr}[c_{\phi q}^{(3)} c_{d\phi} y^{d\dagger}] \\
& + 3\text{Tr}[(c_{u\phi})^\dagger c_{\phi q}^{(1)} y^u] - 3\text{Tr}[(c_{u\phi})^\dagger c_{\phi q}^{(3)} y^u] - 6\text{Tr}[y^d c_{\phi d} y^{d\dagger} c_{\phi q}^{(1)}] - 6\text{Tr}[y^d c_{\phi d} y^{d\dagger} c_{\phi q}^{(3)}] \\
& + 3\text{Tr}[y^d c_{\phi d} c_{\phi d} y^{d\dagger}] - 2\text{Tr}[y^e c_{\phi e} y^{e\dagger} c_{\phi l}^{(1)}] - 2\text{Tr}[y^e c_{\phi e} y^{e\dagger} c_{\phi l}^{(3)}] + \text{Tr}[y^e c_{\phi e} c_{\phi e} y^{e\dagger}] \\
& + \text{Tr}[y^{e\dagger} c_{\phi l}^{(1)} c_{\phi l}^{(1)} y^e] + \text{Tr}[y^{e\dagger} c_{\phi l}^{(1)} c_{\phi l}^{(3)} y^e] + \text{Tr}[y^{e\dagger} c_{\phi l}^{(3)} c_{\phi l}^{(1)} y^e] + \text{Tr}[y^{e\dagger} c_{\phi l}^{(3)} c_{\phi l}^{(3)} y^e] \\
& + 3(\text{Tr}[y^{d\dagger} c_{\phi q}^{(1)} c_{\phi q}^{(1)} y^d] + \text{Tr}[y^{d\dagger} c_{\phi q}^{(1)} c_{\phi q}^{(3)} y^d] + \text{Tr}[y^{d\dagger} c_{\phi q}^{(3)} c_{\phi q}^{(1)} y^d] + \text{Tr}[y^{d\dagger} c_{\phi q}^{(3)} c_{\phi q}^{(3)} y^d] \\
& + \text{Tr}[y^u c_{\phi u} c_{\phi u} y^{u\dagger}] - 2\text{Tr}[y^{u\dagger} c_{\phi q}^{(1)} y^u c_{\phi u}] + \text{Tr}[y^{u\dagger} c_{\phi q}^{(1)} c_{\phi q}^{(1)} y^u] - \text{Tr}[y^{u\dagger} c_{\phi q}^{(1)} c_{\phi q}^{(3)} y^u] \\
& + 2\text{Tr}[y^{u\dagger} c_{\phi q}^{(3)} y^u c_{\phi u}] - \text{Tr}[y^{u\dagger} c_{\phi q}^{(3)} c_{\phi q}^{(1)} y^u] + \text{Tr}[y^{u\dagger} c_{\phi q}^{(3)} c_{\phi q}^{(3)} y^u] \Big) \Big\} \lambda \\
& - \frac{2}{3} c_{\phi D} g_2^2 (c_{\phi\Box} + 4\text{Tr}[c_{\phi l}^{(3)}] + 12\text{Tr}[c_{\phi q}^{(3)}]) \lambda - \frac{1}{3} c_{\phi D} g_1^2 \left(c_{\phi D} + c_{\phi\Box} - 4\text{Tr}[c_{\phi d}] - 4\text{Tr}[c_{\phi e}] \right. \\
& - 4\text{Tr}[c_{\phi l}^{(1)}] + 4\text{Tr}[c_{\phi q}^{(1)}] + 8\text{Tr}[c_{\phi u}] \Big) \lambda - \frac{3}{8} c_{\phi D}^2 (g_1^2 + g_2^2)^2 + 6 \left(\text{Tr}[y^d (c_{d\phi})^\dagger y^d (c_{d\phi})^\dagger] \right. \\
& + 2\text{Tr}[y^d (c_{d\phi})^\dagger c_{d\phi} y^{d\dagger}] + \text{Tr}[y^{d\dagger} c_{d\phi} y^{d\dagger} c_{d\phi}] + 2\text{Tr}[y^{d\dagger} c_{d\phi} (c_{d\phi})^\dagger y^d] \Big) + 2\text{Tr}[y^e (c_{e\phi})^\dagger y^e (c_{e\phi})^\dagger] \\
& + 4\text{Tr}[y^e (c_{e\phi})^\dagger c_{e\phi} y^{e\dagger}] + 2\text{Tr}[y^{e\dagger} c_{e\phi} y^{e\dagger} c_{e\phi}] + 4\text{Tr}[y^{e\dagger} c_{e\phi} (c_{e\phi})^\dagger y^e] + 6 \left(\text{Tr}[y^u (c_{u\phi})^\dagger y^u (c_{u\phi})^\dagger] \right. \\
& \left. + 2\text{Tr}[y^u (c_{u\phi})^\dagger c_{u\phi} y^{u\dagger}] + \text{Tr}[y^{u\dagger} c_{u\phi} y^{u\dagger} c_{u\phi}] + 2\text{Tr}[y^{u\dagger} c_{u\phi} (c_{u\phi})^\dagger y^u] \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{1}{3}\left(3c_\phi + \lambda(c_{\phi D} - 8c_{\phi\Box})\right)\left(9c_{\phi D}g_1^2 - 9c_{\phi D}g_2^2 + 20c_{\phi\Box}g_2^2 + 12\lambda(c_{\phi D} - 2c_{\phi\Box})\right. \\
& + 18\text{Tr}[y^d(c_{d\phi})^\dagger] + 18\text{Tr}[y^{d\dagger}c_{d\phi}] + 6\text{Tr}[y^e(c_{e\phi})^\dagger] + 6\text{Tr}[y^{e\dagger}c_{e\phi}] + 18\text{Tr}[y^u(c_{u\phi})^\dagger] \\
& + 18\text{Tr}[y^{u\dagger}c_{u\phi}] + 36\text{Tr}[y^d(c_{\phi ud})^\dagger y^{u\dagger}] - 72\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^d] - 24\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}y^e] + 36\text{Tr}[y^u c_{\phi ud}y^{d\dagger}] \\
& \left. - 72\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^u] + 8g_2^2(\text{Tr}[c_{\phi l}^{(3)}] + 3\text{Tr}[c_{\phi q}^{(3)}])\right), \\
\dot{c}_{\phi^6}^{(1)} = & -12c_\phi(c_{\phi D} + 8c_{\phi\Box}) + \frac{1}{24}(3g_1^2 + 157g_2^2 - 552\lambda)c_{\phi D}^2 - 3\text{Tr}[y^d(c_{d\phi})^\dagger]c_{\phi D} \quad (83) \\
& - 3\text{Tr}[y^{d\dagger}c_{d\phi}]c_{\phi D} - \text{Tr}[y^e(c_{e\phi})^\dagger]c_{\phi D} - \text{Tr}[y^{e\dagger}c_{e\phi}]c_{\phi D} - 3\text{Tr}[y^u(c_{u\phi})^\dagger]c_{\phi D} \\
& - 3\text{Tr}[y^{u\dagger}c_{u\phi}]c_{\phi D} - 6\text{Tr}[y^d(c_{\phi ud})^\dagger y^{u\dagger}]c_{\phi D} + 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^d]c_{\phi D} + 4\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}y^e]c_{\phi D} \\
& - 6\text{Tr}[y^u c_{\phi ud}y^{d\dagger}]c_{\phi D} - 2g_2^2\text{Tr}[c_{\phi d}]c_{\phi D} - 2g_1^2\text{Tr}[c_{\phi e}]c_{\phi D} - 2g_2^2\text{Tr}[c_{\phi l}^{(1)}]c_{\phi D} + \frac{8}{3}g_2^2\text{Tr}[c_{\phi l}^{(3)}]c_{\phi D} \\
& + 2g_1^2\text{Tr}[c_{\phi q}^{(1)}]c_{\phi D} + 8g_2^2\text{Tr}[c_{\phi q}^{(3)}]c_{\phi D} + 4g_1^2\text{Tr}[c_{\phi u}]c_{\phi D} + \left(-2g_1^2 + \frac{29g_2^2}{2} - 64\lambda\right)c_{\phi\Box}c_{\phi D} \\
& + \frac{4}{3}c_{\phi\Box}^2(15g_1^2 + 5g_2^2 + 264\lambda) - 24c_{\phi\Box}\text{Tr}[y^d(c_{d\phi})^\dagger] - 24c_{\phi\Box}\text{Tr}[y^{d\dagger}c_{d\phi}] - 8c_{\phi\Box}\text{Tr}[y^e(c_{e\phi})^\dagger] \\
& - 8c_{\phi\Box}\text{Tr}[y^{e\dagger}c_{e\phi}] - 24c_{\phi\Box}\text{Tr}[y^u(c_{u\phi})^\dagger] - 24c_{\phi\Box}\text{Tr}[y^{u\dagger}c_{u\phi}] + 6\text{Tr}[(c_{d\phi})^\dagger c_{d\phi}] + 2\text{Tr}[(c_{e\phi})^\dagger c_{e\phi}] \\
& + 6\text{Tr}[(c_{u\phi})^\dagger c_{u\phi}] + 12g_2^2\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] + 4g_2^2\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] + 8g_2^2\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] + 8g_2^2\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] \\
& - \frac{20}{3}g_2^2\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 24g_2^2\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] + 24g_1^2\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] - 20g_2^2\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] \\
& + 12g_2^2\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] - 3g_1^2\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 4g_2^2\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 18\text{Tr}[y^d(c_{d\phi})^\dagger c_{\phi q}^{(1)}] \\
& + 30\text{Tr}[y^d(c_{d\phi})^\dagger c_{\phi q}^{(3)}] - 48c_{\phi\Box}\text{Tr}[y^d(c_{\phi ud})^\dagger y^{u\dagger}] - 6\text{Tr}[y^d(c_{\phi ud})^\dagger(c_{u\phi})^\dagger] - 18\text{Tr}[y^{d\dagger}c_{d\phi}c_{\phi d}] \\
& + 96c_{\phi\Box}\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^d] + 6\text{Tr}[y^e(c_{e\phi})^\dagger c_{\phi l}^{(1)}] + 10\text{Tr}[y^e(c_{e\phi})^\dagger c_{\phi l}^{(3)}] - 6\text{Tr}[y^{e\dagger}c_{e\phi}c_{\phi e}] \\
& + 32c_{\phi\Box}\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}y^e] + 18\text{Tr}[y^u c_{\phi u}(c_{u\phi})^\dagger] - 48c_{\phi\Box}\text{Tr}[y^u c_{\phi ud}y^{d\dagger}] - 6\text{Tr}[y^{u\dagger}c_{d\phi}(c_{\phi ud})^\dagger] \\
& - 18\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{u\phi}] + 12(c_{\phi D} + 8c_{\phi\Box})\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^u] + 30\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}c_{u\phi}] + 18\text{Tr}[c_{u\phi}c_{\phi u}y^{u\dagger}] \\
& - 6\text{Tr}[c_{u\phi}c_{\phi ud}y^{d\dagger}] - 18\text{Tr}[c_{\phi d}(c_{d\phi})^\dagger y^d] - 6\text{Tr}[c_{\phi e}(c_{e\phi})^\dagger y^e] + 6\text{Tr}[c_{\phi l}^{(1)}c_{e\phi}y^{e\dagger}] + 10\text{Tr}[c_{\phi l}^{(3)}c_{e\phi}y^{e\dagger}] \\
& + 18\text{Tr}[c_{\phi q}^{(1)}c_{d\phi}y^{d\dagger}] + 30\text{Tr}[c_{\phi q}^{(3)}c_{d\phi}y^{d\dagger}] - 6\text{Tr}[c_{\phi ud}(c_{d\phi})^\dagger y^u] - 18\text{Tr}[(c_{u\phi})^\dagger c_{\phi q}^{(1)}y^u] \\
& + 30\text{Tr}[(c_{u\phi})^\dagger c_{\phi q}^{(3)}y^u] + 36\text{Tr}[y^d c_{\phi d}y^{d\dagger}c_{\phi q}^{(1)}] + 36\text{Tr}[y^d c_{\phi d}y^{d\dagger}c_{\phi q}^{(3)}] - 18\text{Tr}[y^d c_{\phi d}c_{\phi d}y^{d\dagger}] \\
& - 3\text{Tr}[y^d(c_{\phi ud})^\dagger c_{\phi ud}y^{d\dagger}] - 18\text{Tr}[y^{d\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(1)}y^d] - 18\text{Tr}[y^{d\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(3)}y^d] + 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^u c_{\phi ud}] \\
& - 18\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(1)}y^d] - 30\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(3)}y^d] + 12\text{Tr}[y^e c_{\phi e}y^{e\dagger}c_{\phi l}^{(1)}] + 12\text{Tr}[y^e c_{\phi e}y^{e\dagger}c_{\phi l}^{(3)}] \\
& - 6\text{Tr}[y^e c_{\phi e}c_{\phi e}y^{e\dagger}] - 6\text{Tr}[y^{e\dagger}c_{\phi l}^{(1)}c_{\phi l}^{(1)}y^e] - 6\text{Tr}[y^{e\dagger}c_{\phi l}^{(1)}c_{\phi l}^{(3)}y^e] - 6\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}c_{\phi l}^{(1)}y^e] \\
& - 10\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}c_{\phi l}^{(3)}y^e] - 18\text{Tr}[y^u c_{\phi u}c_{\phi u}y^{u\dagger}] - 3\text{Tr}[y^u c_{\phi ud}(c_{\phi ud})^\dagger y^{u\dagger}] + 36\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}y^u c_{\phi u}] \\
& - 18\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(1)}y^u] + 18\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(3)}y^u] + 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^d(c_{\phi ud})^\dagger] - 36\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^u c_{\phi u}] \\
& + 18\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(1)}y^u] - 30\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(3)}y^u] - \frac{32}{3}c_{\phi\Box}g_2^2\text{Tr}[c_{\phi l}^{(3)}] - 32c_{\phi\Box}g_2^2\text{Tr}[c_{\phi q}^{(3)}], \\
\dot{c}_{\phi^6}^{(2)} = & -\frac{2}{3}\left(3\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] + \text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] + 2\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] + 6\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}]\right)g_2^2 \quad (84) \\
& - 18c_\phi c_{\phi D} + \frac{1}{12}\left((-35g_1^2 + 46g_2^2 - 312\lambda)c_{\phi D}^2 + 4c_{\phi\Box}(20g_1^2 + 15g_2^2 + 192\lambda)c_{\phi D} + 160c_{\phi\Box}^2g_1^2\right) \\
& + 12\text{Tr}[y^d(c_{d\phi})^\dagger c_{\phi q}^{(1)}] + 6\text{Tr}[y^d(c_{\phi ud})^\dagger(c_{u\phi})^\dagger] - 12\text{Tr}[y^{d\dagger}c_{d\phi}c_{\phi d}] + 4\text{Tr}[y^e(c_{e\phi})^\dagger c_{\phi l}^{(1)}]
\end{aligned}$$

$$\begin{aligned}
& -4\text{Tr}[y^{e\dagger}c_{e\phi}c_{\phi e}] + 12\text{Tr}[y^uc_{\phi u}(c_{u\phi})^\dagger] + 6\text{Tr}[y^{u\dagger}c_{d\phi}(c_{\phi ud})^\dagger] - 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{u\phi}] \\
& + 12\text{Tr}[c_{u\phi}c_{\phi u}y^{u\dagger}] + 6\text{Tr}[c_{u\phi}c_{\phi ud}y^{d\dagger}] - 12\text{Tr}[c_{\phi d}(c_{d\phi})^\dagger y^d] - 4\text{Tr}[c_{\phi e}(c_{e\phi})^\dagger y^e] + 4\text{Tr}[c_{\phi l}^{(1)}c_{e\phi}y^{e\dagger}] \\
& + 12\text{Tr}[c_{\phi q}^{(1)}c_{d\phi}y^{d\dagger}] + 6\text{Tr}[c_{\phi ud}(c_{d\phi})^\dagger y^u] - 12\text{Tr}[(c_{u\phi})^\dagger c_{\phi q}^{(1)}y^u] + 24\text{Tr}[y^dc_{\phi d}y^{d\dagger}c_{\phi q}^{(1)}] \\
& + 24\text{Tr}[y^dc_{\phi d}y^{d\dagger}c_{\phi q}^{(3)}] - 12\text{Tr}[y^dc_{\phi d}c_{\phi d}y^{d\dagger}] + 3\text{Tr}[y^d(c_{\phi ud})^\dagger c_{\phi ud}y^{d\dagger}] - 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(1)}y^d] \\
& - 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(3)}y^d] - 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^uc_{\phi ud}] - 12\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(1)}y^d] + 8\text{Tr}[y^ec_{\phi e}y^{e\dagger}c_{\phi l}^{(1)}] \\
& + 8\text{Tr}[y^ec_{\phi e}y^{e\dagger}c_{\phi l}^{(3)}] - 4\text{Tr}[y^ec_{\phi e}c_{\phi e}y^{e\dagger}] - 4\text{Tr}[y^{e\dagger}c_{\phi l}^{(1)}c_{\phi l}^{(1)}y^e] - 4\text{Tr}[y^{e\dagger}c_{\phi l}^{(1)}c_{\phi l}^{(3)}y^e] \\
& - 4\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}c_{\phi l}^{(1)}y^e] - 12\text{Tr}[y^uc_{\phi u}c_{\phi u}y^{u\dagger}] + 3\text{Tr}[y^uc_{\phi ud}(c_{\phi ud})^\dagger y^{u\dagger}] + 24\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}y^uc_{\phi u}] \\
& - 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(1)}y^u] + 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(1)}c_{\phi q}^{(3)}y^u] - 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^d(c_{\phi ud})^\dagger] - 24\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^uc_{\phi u}] \\
& + 12\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}c_{\phi q}^{(1)}y^u] + \frac{1}{3}\left(16\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}]g_1^2 + 48\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}]g_1^2 - 6\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}]g_1^2\right. \\
& - 4c_{\phi D}\text{Tr}[c_{\phi d}]g_1^2 - 4c_{\phi D}\text{Tr}[c_{\phi e}]g_1^2 - 4c_{\phi D}\text{Tr}[c_{\phi l}^{(1)}]g_1^2 + 4c_{\phi D}\text{Tr}[c_{\phi q}^{(1)}]g_1^2 + 8c_{\phi D}\text{Tr}[c_{\phi u}]g_1^2 \\
& - 18c_{\phi D}\text{Tr}[y^d(c_{d\phi})^\dagger] - 18c_{\phi D}\text{Tr}[y^{d\dagger}c_{d\phi}] - 6c_{\phi D}\text{Tr}[y^e(c_{e\phi})^\dagger] - 6c_{\phi D}\text{Tr}[y^{e\dagger}c_{e\phi}] \\
& - 18c_{\phi D}\text{Tr}[y^u(c_{u\phi})^\dagger] - 18c_{\phi D}\text{Tr}[y^{u\dagger}c_{u\phi}] - 6g_2^2\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + 3g_2^2\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] \\
& - 36c_{\phi D}\text{Tr}[y^d(c_{\phi ud})^\dagger y^{u\dagger}] + 72c_{\phi D}\text{Tr}[y^{d\dagger}c_{\phi q}^{(3)}y^d] + 24c_{\phi D}\text{Tr}[y^{e\dagger}c_{\phi l}^{(3)}y^e] - 36c_{\phi D}\text{Tr}[y^uc_{\phi ud}y^{d\dagger}] \\
& \left. + 72c_{\phi D}\text{Tr}[y^{u\dagger}c_{\phi q}^{(3)}y^u]\right),
\end{aligned}$$

$$\begin{aligned}
\dot{c}_{\phi^4}^{(1)} &= \frac{1}{3}\left(24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] + 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] + 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] - 16\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] \quad (85) \right. \\
& + 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] - 48\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] - 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 32c_{\phi D}c_{\phi\Box} \\
& \left. - 11c_{\phi D}^2 - 16c_{\phi\Box}^2\right),
\end{aligned}$$

$$\begin{aligned}
\dot{c}_{\phi^4}^{(2)} &= \frac{1}{3}\left(-8(3\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] + \text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] + 2\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] + 2\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] \quad (86) \right. \\
& + 6\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] + 6\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 3\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] - 16c_{\phi D}c_{\phi\Box} - 5c_{\phi D}^2 - 16c_{\phi\Box}^2) \left. \right),
\end{aligned}$$

$$\begin{aligned}
\dot{c}_{\phi^4}^{(3)} &= \frac{1}{3}\left(32\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 96\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] - 16c_{\phi D}c_{\phi\Box} \quad (87) \right. \\
& \left. + 7c_{\phi D}^2 - 40c_{\phi\Box}^2\right),
\end{aligned}$$

$$\begin{aligned}
\dot{c}_{W^2\phi^4}^{(1)} &= -\frac{1}{24}g_2^2\left(3c_{\phi D}^2 + 24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] - 48\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] \quad (88) \right. \\
& - 144\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] \\
& \left. + 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}]\right),
\end{aligned}$$

$$\begin{aligned}
\dot{c}_{W^2\phi^4 D^2}^{(1)} &= -\frac{1}{3}g_2\left(3c_{\phi D}^2 + 24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] - 48\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] \quad (89) \right. \\
& - 144\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] \\
& \left. + 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}]\right),
\end{aligned}$$

$$\begin{aligned} \dot{c}_{B^2\phi^4}^{(1)} = & \frac{1}{24}g_1^2 \left(3c_{\phi D}^2 + 24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] - 48\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] \right. \\ & - 144\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] \\ & \left. + 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] \right), \end{aligned} \quad (90)$$

$$\begin{aligned} \dot{c}_{B\phi^4 D^2}^{(1)} = & \frac{1}{3}g_1 \left(3c_{\phi D}^2 + 24\text{Tr}[(c_{\phi d})^\dagger c_{\phi d}] - 48\text{Tr}[(c_{\phi l}^{(3)})^\dagger c_{\phi l}^{(3)}] + 48\text{Tr}[(c_{\phi q}^{(1)})^\dagger c_{\phi q}^{(1)}] \right. \\ & - 144\text{Tr}[(c_{\phi q}^{(3)})^\dagger c_{\phi q}^{(3)}] + 24\text{Tr}[(c_{\phi u})^\dagger c_{\phi u}] + 24\text{Tr}[(c_{\phi ud})^\dagger c_{\phi ud}] + 8\text{Tr}[(c_{\phi e})^\dagger c_{\phi e}] \\ & \left. + 16\text{Tr}[(c_{\phi l}^{(1)})^\dagger c_{\phi l}^{(1)}] \right). \end{aligned} \quad (91)$$

313 C Ultraviolet completion of the Standard Model

314 The purpose of this appendix is proving that there exists at least one UV completion of
315 the SM that induces arbitrary values of c_ϕ , $c_{\phi D}$ and $c_{\phi\Box}$. To this aim, let us extend the SM
316 (for $\mu^2 = 0$) with three colorless scalars: $\mathcal{S} \sim (1, 1)_0$, $\Xi_0 \sim (1, 3)_0$ and $\Xi_1 \sim (1, 3)_1$. The
317 numbers in parentheses and the subscript indicate the representations of $SU(3)_c$, $SU(2)_L$
318 and $U(1)_Y$, respectively.

319 Let us assume that they all have mass M much larger than the EW scale, and that
320 the new physics interaction Lagrangian is:

$$\mathcal{L}_{\text{NP}} = \kappa_{\mathcal{S}} \mathcal{S} \phi^\dagger \phi + \lambda_{\mathcal{S}} \mathcal{S}^2 \phi^\dagger \phi + \kappa_{\Xi_0} \phi^\dagger \Xi_0^a \sigma_a \phi + \left(\kappa_{\Xi_1} \Xi_1^{a\dagger} \tilde{\phi}^\dagger \sigma_a \phi + \text{h.c.} \right). \quad (92)$$

321 (Other triple and quartic terms are allowed, but we just ignored them for simplicity.)
322 Then, by integrating out the heavy modes at tree level at the scale M , we obtain [46]:

$$\begin{aligned} \frac{c_\phi}{\Lambda^2} &= -\lambda_{\mathcal{S}} \frac{\kappa_{\mathcal{S}}^2}{M^4}, \\ \frac{c_{\phi D}}{\Lambda^2} &= \frac{2}{M^4} (2\kappa_{\Xi_1}^2 - \kappa_{\Xi_0}^2), \\ \frac{c_{\phi\Box}}{\Lambda^2} &= \frac{1}{2M^4} (4\kappa_{\Xi_1}^2 + \kappa_{\Xi_0}^2 - \kappa_{\mathcal{S}}^2). \end{aligned} \quad (93)$$

323 Obviously, c_ϕ can have arbitrary sign by just tuning $\lambda_{\mathcal{S}}$. Likewise, $c_{\phi D}$ can be made
324 arbitrarily negative provided $\kappa_{\Xi_1}/\kappa_{\Xi_0} \ll 1$, and positive otherwise. Notwithstanding this
325 later choice, $c_{\phi\Box}$ will be positive for small enough $\kappa_{\mathcal{S}}$ and negative for large values of this
326 parameter. In summary, the signs of the three tree-level generated dimension-six operators
327 are arbitrary and uncorrelated.

328 In the process of integrating out the fields of mass M , dimension-eight operators arise
329 too. With the help of `MatchingTools` [74], we find that (see also Ref. [33]):

$$\frac{c_{\phi^4}^{(1)}}{\Lambda^4} = 4 \frac{\kappa_{\Xi_0}^2}{M^6}, \quad \frac{c_{\phi^4}^{(2)}}{\Lambda^4} = 8 \frac{\kappa_{\Xi_1}^2}{M^6}, \quad \frac{c_{\phi^4}^{(3)}}{\Lambda^4} = \frac{2}{M^6} (\kappa_{\mathcal{S}}^2 - \kappa_{\Xi_0}^2). \quad (94)$$

330 Contrary to the dimension-six Wilson coefficients above, these couplings fulfill the posi-
331 tivity bounds $c_{\phi^4}^{(2)} \geq 0$, $c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} \geq 0$ and $c_{\phi^4}^{(1)} + c_{\phi^4}^{(2)} + c_{\phi^4}^{(3)} \geq 0$ obtained in Ref. [33] for
332 arbitrary values of the κ s.

References

- 333
- 334 [1] I. Brivio and M. Trott, *The Standard Model as an Effective Field Theory*, Phys.
335 Rept. **793**, 1 (2019), doi:10.1016/j.physrep.2018.11.002, 1706.08945.
- 336 [2] L. Yuan, *Exotics and BSM in ATLAS and CMS (Non dark matter searches)*, PoS
337 **CORFU2019**, 051 (2020), doi:10.22323/1.376.0051.
- 338 [3] J. Ellis, M. Madigan, K. Mimasu, V. Sanz and T. You, *Top, Higgs, Diboson and*
339 *Electroweak Fit to the Standard Model Effective Field Theory*, JHEP **04**, 279 (2021),
340 doi:10.1007/JHEP04(2021)279, 2012.02779.
- 341 [4] C. Grojean, E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group*
342 *Scaling of Higgs Operators and $\Gamma(h \rightarrow \gamma\gamma)$* , JHEP **04**, 016
343 (2013), doi:10.1007/JHEP04(2013)016, 1301.2588.
- 344 [5] J. Elias-Miró, J. R. Espinosa, E. Masso and A. Pomarol, *Renormalization of*
345 *dimension-six operators relevant for the Higgs decays $h \rightarrow \gamma\gamma, \gamma Z$* , JHEP **08**, 033
346 (2013), doi:10.1007/JHEP08(2013)033, 1302.5661.
- 347 [6] J. Elias-Miro, J. R. Espinosa, E. Masso and A. Pomarol, *Higgs windows to new physics*
348 *through $d=6$ operators: constraints and one-loop anomalous dimensions*, JHEP **11**,
349 066 (2013), doi:10.1007/JHEP11(2013)066, 1308.1879.
- 350 [7] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of*
351 *the Standard Model Dimension Six Operators I: Formalism and λ Dependence*,
352 JHEP **10**, 087 (2013), doi:10.1007/JHEP10(2013)087, 1308.2627.
- 353 [8] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of*
354 *the Standard Model Dimension Six Operators II: Yukawa Dependence*, JHEP **01**, 035
355 (2014), doi:10.1007/JHEP01(2014)035, 1310.4838.
- 356 [9] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group*
357 *Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling De-*
358 *pendence and Phenomenology*, JHEP **04**, 159 (2014), doi:10.1007/JHEP04(2014)159,
359 1312.2014.
- 360 [10] G. Buchalla, A. Celis, C. Krause and J.-N. Toelstede, *Master Formula for One-Loop*
361 *Renormalization of Bosonic SMEFT Operators* (2019), 1904.07840.
- 362 [11] S. Descotes-Genon, A. Falkowski, M. Fedele, M. González-Alonso and
363 J. Virto, *The CKM parameters in the SMEFT*, JHEP **05**, 172 (2019),
364 doi:10.1007/JHEP05(2019)172, 1812.08163.
- 365 [12] S. Bißmann, J. Erdmann, C. Grunwald, G. Hiller and K. Kröninger, *Constraining*
366 *top-quark couplings combining top-quark and B decay observables*, Eur. Phys. J. C
367 **80**(2), 136 (2020), doi:10.1140/epjc/s10052-020-7680-9, 1909.13632.
- 368 [13] J. Terol-Calvo, M. Tórtola and A. Vicente, *High-energy constraints from low-*
369 *energy neutrino nonstandard interactions*, Phys. Rev. D **101**(9), 095010 (2020),
370 doi:10.1103/PhysRevD.101.095010, 1912.09131.
- 371 [14] C. Degrande, G. Durieux, F. Maltoni, K. Mimasu, E. Vryonidou and C. Zhang,
372 *Automated one-loop computations in the standard model effective field theory*, Phys.
373 Rev. D **103**(9), 096024 (2021), doi:10.1103/PhysRevD.103.096024, 2008.11743.

- 374 [15] A. Azatov, R. Contino, C. S. Machado and F. Riva, *Helicity selection rules*
375 *and noninterference for BSM amplitudes*, Phys. Rev. D **95**(6), 065014 (2017),
376 doi:10.1103/PhysRevD.95.065014, 1607.05236.
- 377 [16] M. Chala, C. Krause and G. Nardini, *Signals of the electroweak phase tran-*
378 *sition at colliders and gravitational wave observatories*, JHEP **07**, 062 (2018),
379 doi:10.1007/JHEP07(2018)062, 1802.02168.
- 380 [17] C. W. Murphy, *Dimension-8 operators in the Standard Model Effective Field Theory*,
381 JHEP **10**, 174 (2020), doi:10.1007/JHEP10(2020)174, 2005.00059.
- 382 [18] T. Corbett, A. Helset, A. Martin and M. Trott, *EWPD in the SMEFT to dimension*
383 *eight* (2021), 2102.02819.
- 384 [19] G. Panico, A. Pomarol and M. Riembau, *EFT approach to the electron Electric Dipole*
385 *Moment at the two-loop level*, JHEP **04**, 090 (2019), doi:10.1007/JHEP04(2019)090,
386 1810.09413.
- 387 [20] M. Ardu and S. Davidson, *What is Leading Order for LFV in SMEFT?* (2021),
388 2103.07212.
- 389 [21] L. Lehman, *Extending the Standard Model Effective Field Theory with the*
390 *Complete Set of Dimension-7 Operators*, Phys. Rev. D **90**(12), 125023 (2014),
391 doi:10.1103/PhysRevD.90.125023, 1410.4193.
- 392 [22] Y. Liao and X.-D. Ma, *Renormalization Group Evolution of Dimension-*
393 *seven Baryon- and Lepton-number-violating Operators*, JHEP **11**, 043 (2016),
394 doi:10.1007/JHEP11(2016)043, 1607.07309.
- 395 [23] C. Hays, A. Martin, V. Sanz and J. Setford, *On the impact of dimension-*
396 *eight SMEFT operators on Higgs measurements*, JHEP **02**, 123 (2019),
397 doi:10.1007/JHEP02(2019)123, 1808.00442.
- 398 [24] J. Ellis, S.-F. Ge, H.-J. He and R.-Q. Xiao, *Probing the scale of new physics*
399 *in the $ZZ\gamma$ coupling at e^+e^- colliders*, Chin. Phys. C **44**(6), 063106 (2020),
400 doi:10.1088/1674-1137/44/6/063106, 1902.06631.
- 401 [25] S. Alioli, R. Boughezal, E. Mereghetti and F. Petriello, *Novel angular dependence in*
402 *Drell-Yan lepton production via dimension-8 operators*, Phys. Lett. B **809**, 135703
403 (2020), doi:10.1016/j.physletb.2020.135703, 2003.11615.
- 404 [26] J. Ellis, H.-J. He and R.-Q. Xiao, *Probing new physics in dimension-8 neutral gauge*
405 *couplings at e^+e^- colliders*, Sci. China Phys. Mech. Astron. **64**(2), 221062 (2021),
406 doi:10.1007/s11433-020-1617-3, 2008.04298.
- 407 [27] C. Hays, A. Helset, A. Martin and M. Trott, *Exact SMEFT formulation and expansion*
408 *to $\mathcal{O}(v^4/\Lambda^4)$* , JHEP **11**, 087 (2020), doi:10.1007/JHEP11(2020)087, 2007.00565.
- 409 [28] J. Gu, L.-T. Wang and C. Zhang, *An unambiguous test of positivity at lepton colliders*
410 (2020), 2011.03055.
- 411 [29] N. Craig, M. Jiang, Y.-Y. Li and D. Sutherland, *Loops and Trees in Generic EFTs*,
412 JHEP **08**, 086 (2020), doi:10.1007/JHEP08(2020)086, 2001.00017.
- 413 [30] S. Davidson, M. Gorbahn and M. Leak, *Majorana neutrino masses in the renor-*
414 *malization group equations for lepton flavor violation*, Phys. Rev. D **98**(9), 095014
415 (2018), doi:10.1103/PhysRevD.98.095014, 1807.04283.

- 416 [31] C. Zhang and S.-Y. Zhou, *Positivity bounds on vector boson scattering at the LHC*,
417 Phys. Rev. D **100**(9), 095003 (2019), doi:10.1103/PhysRevD.100.095003, 1808.
418 00010.
- 419 [32] Q. Bi, C. Zhang and S.-Y. Zhou, *Positivity constraints on aQGC: carving out the*
420 *physical parameter space*, JHEP **06**, 137 (2019), doi:10.1007/JHEP06(2019)137,
421 1902.08977.
- 422 [33] G. N. Remmen and N. L. Rodd, *Consistency of the Standard Model Effective Field*
423 *Theory*, JHEP **12**, 032 (2019), doi:10.1007/JHEP12(2019)032, 1908.09845.
- 424 [34] G. N. Remmen and N. L. Rodd, *Flavor Constraints from Unitarity and Analyticity*,
425 Phys. Rev. Lett. **125**(8), 081601 (2020), doi:10.1103/PhysRevLett.125.081601, 2004.
426 02885.
- 427 [35] G. N. Remmen and N. L. Rodd, *Signs, Spin, SMEFT: Positivity at Dimension Six*
428 (2020), 2010.04723.
- 429 [36] Q. Bonnefoy, E. Gendy and C. Grojean, *Positivity bounds on Minimal Flavor Viola-*
430 *tion*, JHEP **04**, 115 (2021), doi:10.1007/JHEP04(2021)115, 2011.12855.
- 431 [37] B. Bellazzini, J. Elias Miró, R. Rattazzi, M. Riembau and F. Riva, *Positive Moments*
432 *for Scattering Amplitudes* (2020), 2011.00037.
- 433 [38] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, *Dimension-Six Terms in the*
434 *Standard Model Lagrangian*, JHEP **10**, 085 (2010), doi:10.1007/JHEP10(2010)085,
435 1008.4884.
- 436 [39] H.-L. Li, Z. Ren, J. Shu, M.-L. Xiao, J.-H. Yu and Y.-H. Zheng, *Complete Set of*
437 *Dimension-8 Operators in the Standard Model Effective Field Theory* (2020), 2005.
438 00008.
- 439 [40] J. Elias-Miro, J. R. Espinosa and A. Pomarol, *One-loop non-renormalization results in*
440 *EFTs*, Phys. Lett. B **747**, 272 (2015), doi:10.1016/j.physletb.2015.05.056, 1412.7151.
- 441 [41] C. Cheung and C.-H. Shen, *Nonrenormalization Theorems without Supersymmetry*,
442 Phys. Rev. Lett. **115**(7), 071601 (2015), doi:10.1103/PhysRevLett.115.071601, 1505.
443 01844.
- 444 [42] Z. Bern, J. Parra-Martinez and E. Sawyer, *Nonrenormalization and Opera-*
445 *tor Mixing via On-Shell Methods*, Phys. Rev. Lett. **124**(5), 051601 (2020),
446 doi:10.1103/PhysRevLett.124.051601, 1910.05831.
- 447 [43] F. del Aguila, M. Perez-Victoria and J. Santiago, *Observable contributions of*
448 *new exotic quarks to quark mixing*, JHEP **09**, 011 (2000), doi:10.1088/1126-
449 6708/2000/09/011, hep-ph/0007316.
- 450 [44] F. del Aguila, J. de Blas and M. Perez-Victoria, *Effects of new leptons in Electroweak*
451 *Precision Data*, Phys. Rev. D **78**, 013010 (2008), doi:10.1103/PhysRevD.78.013010,
452 0803.4008.
- 453 [45] F. del Aguila, J. de Blas and M. Perez-Victoria, *Electroweak Limits on General New*
454 *Vector Bosons*, JHEP **09**, 033 (2010), doi:10.1007/JHEP09(2010)033, 1005.3998.
- 455 [46] J. de Blas, M. Chala, M. Perez-Victoria and J. Santiago, *Observable Effects of General*
456 *New Scalar Particles*, JHEP **04**, 078 (2015), doi:10.1007/JHEP04(2015)078, 1412.
457 8480.

- 458 [47] J. de Blas, J. C. Criado, M. Perez-Victoria and J. Santiago, *Effective description of*
459 *general extensions of the Standard Model: the complete tree-level dictionary*, JHEP
460 **03**, 109 (2018), doi:10.1007/JHEP03(2018)109, 1711.10391.
- 461 [48] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 - A*
462 *complete toolbox for tree-level phenomenology*, Comput. Phys. Commun. **185**, 2250
463 (2014), doi:10.1016/j.cpc.2014.04.012, 1310.1921.
- 464 [49] T. Hahn, *Generating Feynman diagrams and amplitudes with FeynArts 3*, Com-
465 put. Phys. Commun. **140**, 418 (2001), doi:10.1016/S0010-4655(01)00290-9, hep-ph/
466 0012260.
- 467 [50] T. Hahn and M. Perez-Victoria, *Automatized one loop calculations in four-dimensions*
468 *and D-dimensions*, Comput. Phys. Commun. **118**, 153 (1999), doi:10.1016/S0010-
469 4655(98)00173-8, hep-ph/9807565.
- 470 [51] A. Carmona, A. Lazopoulos, P. Olgoso and J. Santiago, *MatchMaker: automated*
471 *one-loop matching*, (to appear) .
- 472 [52] V. Gherardi, D. Marzocca and E. Venturini, *Matching scalar leptoquarks to the*
473 *SMEFT at one loop*, JHEP **07**, 225 (2020), doi:10.1007/JHEP07(2020)225, [Erratum:
474 JHEP 01, 006 (2021)], 2003.12525.
- 475 [53] J. C. Criado and M. Pérez-Victoria, *Field redefinitions in effective theories at higher*
476 *orders*, JHEP **03**, 038 (2019), doi:10.1007/JHEP03(2019)038, 1811.09413.
- 477 [54] J. Elias Miró, J. Ingoldby and M. Riembau, *EFT anomalous dimensions from the*
478 *S-matrix*, JHEP **09**, 163 (2020), doi:10.1007/JHEP09(2020)163, 2005.06983.
- 479 [55] P. Baratella, C. Fernandez and A. Pomarol, *Renormalization of Higher-Dimensional*
480 *Operators from On-shell Amplitudes*, Nucl. Phys. B **959**, 115155 (2020),
481 doi:10.1016/j.nuclphysb.2020.115155, 2005.07129.
- 482 [56] M. E. Peskin and T. Takeuchi, *A New constraint on a strongly interacting Higgs*
483 *sector*, Phys. Rev. Lett. **65**, 964 (1990), doi:10.1103/PhysRevLett.65.964.
- 484 [57] J. de Blas, M. Ciuchini, E. Franco, S. Mishima, M. Pierini, L. Reina and L. Silvestrini,
485 *Electroweak precision constraints at present and future colliders*, PoS ICHEP2016,
486 690 (2017), doi:10.22323/1.282.0690, 1611.05354.
- 487 [58] F. Maltoni, L. Mantani and K. Mimasu, *Top-quark electroweak interactions at high*
488 *energy*, JHEP **10**, 004 (2019), doi:10.1007/JHEP10(2019)004, 1904.05637.
- 489 [59] X.-m. Zhang, *Operators analysis for Higgs potential and cosmological bound on Higgs*
490 *mass*, Phys. Rev. D **47**, 3065 (1993), doi:10.1103/PhysRevD.47.3065, hep-ph/
491 9301277.
- 492 [60] C. Grojean, G. Servant and J. D. Wells, *First-order electroweak phase transi-*
493 *tion in the standard model with a low cutoff*, Phys. Rev. D **71**, 036001 (2005),
494 doi:10.1103/PhysRevD.71.036001, hep-ph/0407019.
- 495 [61] D. Bodeker, L. Fromme, S. J. Huber and M. Seniuch, *The Baryon asymmetry in*
496 *the standard model with a low cut-off*, JHEP **02**, 026 (2005), doi:10.1088/1126-
497 6708/2005/02/026, hep-ph/0412366.

- 498 [62] C. Delaunay, C. Grojean and J. D. Wells, *Dynamics of Non-renormalizable*
499 *Electroweak Symmetry Breaking*, JHEP **04**, 029 (2008), doi:10.1088/1126-
500 6708/2008/04/029, 0711.2511.
- 501 [63] J. de Vries, M. Postma, J. van de Vis and G. White, *Electroweak Baryoge-*
502 *nesis and the Standard Model Effective Field Theory*, JHEP **01**, 089 (2018),
503 doi:10.1007/JHEP01(2018)089, 1710.04061.
- 504 [64] C. Caprini *et al.*, *Detecting gravitational waves from cosmological phase transitions*
505 *with LISA: an update*, JCAP **03**, 024 (2020), doi:10.1088/1475-7516/2020/03/024,
506 1910.13125.
- 507 [65] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, *On the Anomalous Elec-*
508 *troweak Baryon Number Nonconservation in the Early Universe*, Phys. Lett. B **155**,
509 36 (1985), doi:10.1016/0370-2693(85)91028-7.
- 510 [66] P. H. Chankowski and Z. Pluciennik, *Renormalization group equations for seesaw*
511 *neutrino masses*, Phys. Lett. B **316**, 312 (1993), doi:10.1016/0370-2693(93)90330-K,
512 hep-ph/9306333.
- 513 [67] K. S. Babu, C. N. Leung and J. T. Pantaleone, *Renormalization of the neutrino*
514 *mass operator*, Phys. Lett. B **319**, 191 (1993), doi:10.1016/0370-2693(93)90801-N,
515 hep-ph/9309223.
- 516 [68] S. Antusch, M. Drees, J. Kersten, M. Lindner and M. Ratz, *Neutrino mass oper-*
517 *ator renormalization revisited*, Phys. Lett. B **519**, 238 (2001), doi:10.1016/S0370-
518 2693(01)01127-3, hep-ph/0108005.
- 519 [69] R. Alonso, H.-M. Chang, E. E. Jenkins, A. V. Manohar and B. Shotwell, *Renormal-*
520 *ization group evolution of dimension-six baryon number violating operators*, Phys.
521 Lett. B **734**, 302 (2014), doi:10.1016/j.physletb.2014.05.065, 1405.0486.
- 522 [70] Y. Liao and X.-D. Ma, *Renormalization Group Evolution of Dimension-seven Oper-*
523 *ators in Standard Model Effective Field Theory and Relevant Phenomenology*, JHEP
524 **03**, 179 (2019), doi:10.1007/JHEP03(2019)179, 1901.10302.
- 525 [71] M. Chala and A. Titov, *Neutrino masses in the Standard Model effective field theory*
526 (2021), 2104.08248.
- 527 [72] M. Jiang, T. Ma and J. Shu, *Renormalization Group Evolution from On-shell SMEFT*,
528 JHEP **01**, 101 (2021), doi:10.1007/JHEP01(2021)101, 2005.10261.
- 529 [73] A. Barzinji, M. Trott and A. Vasudevan, *Equations of Motion for the Standard Model*
530 *Effective Field Theory: Theory and Applications*, Phys. Rev. D **98**(11), 116005 (2018),
531 doi:10.1103/PhysRevD.98.116005, 1806.06354.
- 532 [74] J. C. Criado, *MatchingTools: a Python library for symbolic effective field theory*
533 *calculations*, Comput. Phys. Commun. **227**, 42 (2018), doi:10.1016/j.cpc.2018.02.016,
534 1710.06445.