

# Towards a study of the effects of dynamical factorization breaking at LHCb

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## Abstract

The factorization of short-distance partonic cross-sections from universal long-distance kinematic distributions is fundamental to phenomenology at hadron colliders. It has been predicted however that observables sensitive to momenta transverse to the direction of an energetic parton cannot be factorized in the usual way, even at high energies. It should be possible to study this factorization breaking using Z+jet production in high-energy proton-proton collisions by studying azimuthal correlations between a Z boson and associated charged hadrons. A plan to perform this measurement with data collected by LHCb will be discussed, along with related work.

## 1 Introduction

Factorization theorems are useful because they allow the computation of cross-sections for processes with non-perturbative components. Beyond their phenomenological utility, however, factorization theorems make a formally rigorous connection between the partonic picture and the hadronic picture of Quantum Chromodynamics (QCD). This rigorous connection can be contrasted with the more *ad-hoc* connection established by the hadron formation models included in many Monte Carlo event generators. These models are equally useful in a phenomenological sense, but they are not formally derived from QCD: they are often inspired by specific features of QCD, but it is difficult or impossible to derive higher-order corrections to these models or to clearly identify the kinematic regions in which they fail. In regions where standard factorization theorems start to fail, certain techniques have already been developed to correct for effects that are typically neglected [1–3]. Hadron mass corrections in particular have sometimes allowed analysts to use universal parton distribution functions and fragmentation functions to fit cross-sections that are characterized by hard scales of only a few GeV at colliders and fixed-target experiments with center-of-mass energies of a few tens of GeVs. It has also been possible to develop special-purpose factorization frameworks for certain processes, like the decay of heavy hadrons [4], that allow for connections to perturbation theory in specific kinematic limits.

In Section 2 we will introduce the idea of transverse-momentum-dependent (TMD) factorization and provide a motivation for the study of TMD factorization breaking. In Section 3 we detail the measurement that we plan to make, and in Section 4 we present similar measurements that have already been made. We present our conclusions in Section 5.

## 33 2 TMD factorization breaking

34 The TMD factorization framework [5] is used to compute cross-sections that depend on the  
 35 component of a hadron momentum that is transverse to the direction of one of its constituent  
 36 partons, in the limit where this transverse momentum is much smaller than the largest energy  
 37 scale that is relevant to the scattering process. This transverse momentum is often called  $q_T$ ,  
 38 and the large energy scale is called  $Q$ : the TMD framework was established to compute  $q_T$   
 39 spectra in the limit that  $q_T \ll Q$ . In certain scattering channels, however, TMD factorization is  
 40 expected to break down in certain kinematic regions, even at very high energies. In particular:  
 41 a proof [6] has been written to show that it is not possible to use the TMD framework to  
 42 factorize the cross-section for the production of a pair of hadrons from a proton-proton collision  
 43 in the kinematic region where the two final-state hadrons are produced nearly back-to-back in  
 44 azimuth, which is a region where  $q_T$  tends to be very small. This breakage of factorization is  
 45 also expected to apply in back-to-back dijet production and  $Z$ +jet production, where a colored  
 46 parton coming out of the hard process can interact with the beam remnants and absorb virtual  
 47 emissions from initial-state partons.

48 Unlike most well-known effects that complicate the factorized picture, TMD factorization  
 49 breaking is not suppressed at high energies. Therefore, at a high-energy collider, it should be  
 50 easy to isolate TMD factorization breaking effects from any other type of factorization breaking.  
 51 This breakage of TMD factorization is also interesting because it does not generalize easily:  
 52 that is, there are certain factorizable observables that look very similar to observables that  
 53 break factorization. For example: proofs have shown that TMD factorization can be used to  
 54 compute  $q_T$  spectra in back-to-back hadron pair production in electron-positron annihilation  
 55 and also back-to-back lepton pair production in proton-proton collisions [5]. These processes  
 56 look superficially similar to back-to-back hadron pair production in proton-proton collisions.  
 57 In these processes, it may also be possible to use TMD factorization to compute a wide variety  
 58 of single-differential cross-sections, in addition to  $q_T$  spectra [7]. It is also expected to be  
 59 possible to use the collinear factorization framework to compute  $q_T$  spectra for hadron pair  
 60 production in proton-proton collisions in the wide-angle kinematic region where  $q_T$  is similar  
 61 in size to  $Q$  [8], in which case a change of only the kinematic region would distinguish a  
 62 factorizable observable and a factorization-breaking observable.

63 Cross-sections that do not factorize often tend to share certain characteristics: it seems to  
 64 become more difficult to factorize a cross-section as the number of hadrons involved in the  
 65 measurement increases, or as the observable becomes less inclusive or more differential. But,  
 66 there is not yet any set of rules that is both strict and generally applicable that can describe  
 67 which observables factorize under which conditions [9]. For now, a new proof must be written  
 68 for more or less each observable that needs to be factorized: factorization is handled on a case-  
 69 by-case basis. Because back-to-back production of a hadron pair in proton-proton collisions  
 70 breaks factorization and is also similar to processes that do not break factorization, it might  
 71 be used to bring attention to specific criteria that prohibit factorization.

## 72 3 Plan for measurement at LHCb

73 We plan to measure a differential cross-section for unidentified charged hadrons produced in  
 74 association with a  $Z$  boson and a jet. The same reasons that motivate the study of dihadron  
 75 production also motivate the study of  $Z$ +hadron production, which is also expected to break  
 76 TMD factorization. It is also experimentally easier to extract clean  $Z$ -hadron correlations be-  
 77 cause it is easy to reconstruct a  $Z$  boson via its decay to  $\mu^+\mu^-$ : LHCb in particular has a proven  
 78 ability to precisely measure  $Z$ +jet cross sections [10, 11], and the spectra of hadrons associated

79 to a  $Z$ +jet pair [12]. If the measurement is precise enough then a  $Z$ +hadron measurement  
 80 can be compared to a dihadron measurement to test if the number of colored partons coming  
 81 out of the hard vertex has an effect on the size of the factorization breaking.

### 82 3.1 Data sample and detector description

83 To make this measurement, we will use data that was collected by LHCb during 2016 with a  
 84  $p + p$  center-of-mass energy of 13 TeV. The LHCb detector [13] is a single-arm forward spec-  
 85 trometer covering the pseudorapidity range  $2 < \eta < 5$ . The detector includes a high-precision  
 86 tracking system consisting of a silicon-strip vertex detector around the  $p + p$  interaction region,  
 87 a large area silicon-strip detector located upstream of a dipole magnet, and three stations of  
 88 silicon-strip detectors and straw drift tubes placed downstream of the magnet. The tracking  
 89 system provides for momentum measurements with relative uncertainties that vary from 0.5%  
 90 at low momentum to 1% at 200 GeV/ $c$ . We will take hadron candidates from tracks that pass  
 91 through all layers of the tracking system. The detector also has electromagnetic and hadronic  
 92 calorimeters: both tracks and calorimeter clusters are used as input to a particle flow algorithm  
 93 that is used as part of the jet reconstruction. Muons are identified by a system composed of  
 94 alternating layers of iron and multiwire proportional chambers, and  $Z$  boson candidates will  
 95 be reconstructed from high-mass muon pairs. We will require both the jet and the  $Z$  boson to  
 96 have large transverse momenta: above about 15 or 20 GeV/ $c$ . Only one jet will be used from  
 97 each event: the jet with largest transverse momentum. Hadrons and jets associated with the  
 98  $Z$  boson should generally recoil against the  $Z$  boson, so an azimuthal window of width  $\pi/3$  or  
 99  $\pi/4$  on the away-side of the  $Z$  boson will be established as a signal region: the rest of azimuth  
 100 will be used for background estimation.

### 101 3.2 Kinematic variables

102 The hadronic cross-section will be binned in three kinematic variables:

$$p_{\text{out}} \equiv p_T^{h^\pm} \sin(\phi^{h^\pm} - \phi^{\text{jet}}), \quad (1)$$

$$103 \quad Q \equiv (p^Z + p^{\text{jet}})^2, \quad (2)$$

104 and

$$105 \quad z \equiv \frac{\mathbf{p}^{h^\pm} \cdot \mathbf{p}^{\text{jet}}}{\mathbf{p}^{\text{jet}} \cdot \mathbf{p}^{\text{jet}}}. \quad (3)$$

106 Here,  $p_T^{h^\pm}$  is the transverse momentum of the charged hadron, and  $\phi^{h^\pm}$  and  $\phi^Z$  are the az-  
 107 imuthal coordinates of the hadron and the  $Z$  boson. The four-momenta of the  $Z$  boson and  
 108 the jet are written as  $p^Z$  and  $p^{\text{jet}}$ , and the three-momenta of the jet and the hadron are written  
 109 as  $\mathbf{p}^{\text{jet}}$  and  $\mathbf{p}^{h^\pm}$ . In order to mitigate uncertainties that are associated with the beam luminos-  
 110 ity and  $Z$ +jet reconstruction, we normalize the hadronic cross-section by the cross-section for  
 111  $Z$ +jet production:

$$\frac{dN^{h^\pm}}{dp_{\text{out}} dz dQ} \bigg/ \frac{dN^{Z+\text{jet}}}{dQ}. \quad (4)$$

112 The variable  $p_{\text{out}}$  is used to probe transverse momenta generated by the parton shower and  
 113 by long-range dynamics in both the initial state and final state. To a first approximation, none  
 114 of  $p_{\text{out}}$  comes from transverse momentum generated at the hardest scales: in the partonic  
 115 center-of-mass frame, the  $Z$  boson and the outgoing parton come out exactly back-to-back.  
 116 Any transverse momentum imbalance is generated from processes characterized by smaller

117 energy scales. Some of the imbalance comes from transverse motion of the initial-state partons  
 118 inside the protons, much of the imbalance is generated by splittings in the parton showers in  
 119 the initial and final state, and some comes from the hadron formation process where partons  
 120 that are separated in azimuth exert forces on each other. Note that  $p_{\text{out}}$  ignores components  
 121 of the transverse momentum imbalance along the axis determined by the  $Z$  boson direction  
 122 of motion, which is roughly the same as the jet axis. The imbalance along this axis should  
 123 be determined mostly by collinear aspects of the parton splitting and fragmentation process:  
 124 with a focus on the off-axis component of the imbalance, we hope that we can improve our  
 125 sensitivity to uniquely transverse-momentum-dependent effects. This is important because  
 126 factorization is not expected to break in the collinear framework.

127 The  $Z$ +jet mass  $Q$  is a proxy for the hard scale that characterizes the scattering. The way  
 128 that the  $p_{\text{out}}$  distribution changes with the hard scale is described by Collins-Soper-Sterman  
 129 (CSS) evolution [14, 15]. CSS evolution is similar to a TMD variant of Dokshitzer-Gribov-  
 130 Lipatov-Altarelli-Parisi (DGLAP) evolution [16–18], with the notable exception that the CSS  
 131 evolution kernels have both perturbative and universal non-perturbative components while  
 132 DGLAP’s evolution kernels can be computed entirely in perturbation theory. CSS evolution  
 133 takes as input the set of kinematic distributions defined in the TMD framework, probed at one  
 134 hard scale, and describes how those distributions look at another hard scale. In particular: the  
 135 results of CSS evolution are only valid when the technique is applied to a factorizable distribu-  
 136 tion defined in the TMD framework. Hence: if we can test whether or not CSS evolution can  
 137 correctly model the relationship between  $p_{\text{out}}$  distributions measured at different values of the  
 138 hard scale  $Q$ , then we can test whether or not TMD factorization holds. The alternative is to  
 139 compute  $p_{\text{out}}$  distributions using TMD distributions that have already been extracted from fits  
 140 to  $e^+e^-$  and  $ep$  scatterings: but, as of now, no such fits have been extracted with very good  
 141 precision.

142 We also want to bin the hadronic cross-section in the fragmentation variable  $z$  in order to  
 143 allow calculations that compare to this measurement to exclude the low- $z$  and high- $z$  regions,  
 144 where calculation is difficult. The  $z$  variable will also provide a more complete or differential  
 145 picture of the hadron formation process, which might improve the power of the analysis if  
 146 factorization breaks most strongly in a sub-region of the full phase space.

147 In addition to measuring the differential cross-section, we plan to fit the  $p_{\text{out}}$  distribution in  
 148 each bin of  $Q \otimes z$  in order to show clearly how the shapes of the distributions evolve with  $Q$  in  
 149 each bin of  $z$ . These fits will hopefully make it easier to make a computation that determines  
 150 whether or not this measurement is consistent with the predictions of CSS evolution.

## 151 4 Prior measurements

152 Qualitatively, we have a good idea of what to expect from this measurement because similar  
 153 measurements have been made at PHENIX [19–21]. Because PHENIX uses  $p+p$  collisions with  
 154 lower beam energies than LHCb, which allow less phase space for  $Z$  boson production, they  
 155 measured  $\pi^0$ -hadron correlations and direct photon-hadron correlations instead of  $Z$ -hadron  
 156 correlations. Those measurements also used a slightly different set of kinematic variables,  
 157 since jet reconstruction was not feasible at PHENIX due to a limited acceptance. In order to  
 158 characterize the hard scale of the scatterings, PHENIX used the transverse momentum of the  
 159  $\pi^0$  or photon instead of an invariant mass-type variable like  $Q$ , and in order to estimate a  
 160 fragmentation variable like  $z$  they used  $x_E \equiv p_T^{h^\pm} / p_T^{\pi^0, \gamma} \cdot \cos(\phi^{\pi^0, \gamma} - \phi^{h^\pm})$ . Some of their re-  
 161 sults are shown in Figure 1. The cores of their  $p_{\text{out}}$  distributions can be fit to Gaussian shapes,  
 162 but further out in their wings the distributions fall off too slowly to fit an exponential: this is  
 163 consistent with expectations from perturbation theory for the power-law fall-off of energetic

164 gluon radiation densities. Qualitatively, the fitted Gaussian widths of the  $p_{\text{out}}$  distributions  
 165 increased with the hard scale. This result matches the predictions of CSS evolution: as more  
 166 energy becomes available to the particles involved in the scattering, more transverse momen-  
 167 tum is generated in the parton showers. No calculation has yet been published that compares  
 168 the PHENIX measurement to the quantitative predictions of CSS evolution.

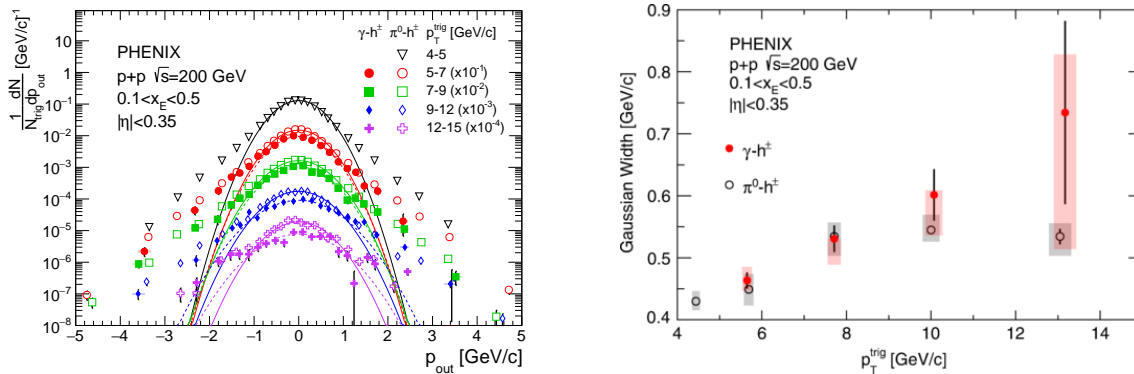


Figure 1: On the left are shown some of the  $p_{\text{out}}$  distributions extracted by the PHENIX collaboration. The solid lines show Gaussian fits to the data points. On the right, the Gaussian widths are plotted against the hard scale. Qualitatively, the widths increase with the hard scale. Both plots are from Reference [20].

## 169 5 Conclusion

170 We want to quantify the breakdown of transverse-momentum-dependent factorization, which  
 171 is a fundamental prediction of QCD that has not yet been verified. We hope that investigation  
 172 into the regions where factorization fails might inspire techniques that extend a type of fac-  
 173 torization to observables that cannot currently be factorized. In addition, these investigations  
 174 might help to develop a more general set of rules to determine which processes do and do  
 175 not factorize. Measurements that might be sensitive to the breakdown of factorization have  
 176 already been made by the PHENIX collaboration, and we plan to make another set of mea-  
 177 surements with data from LHCb. These measurements allow us to test for the breakdown of  
 178 factorization via the breakdown of CSS evolution. We hope that the increased availability of  
 179 measurements from a variety of energy and rapidity ranges will encourage the calculation of  
 180 a quantitative comparison between these measurements and the predictions of CSS evolution,  
 181 especially with the improved array of kinematic variables that the LHCb measurement will use  
 182 to parameterize the scattering and fragmentation process.

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## 185 References

- 186 [1] J. V. Guerrero, A. Accardi, *Gauge Invariance and Kaon Production in Deep Inelastic Scatter-*  
 187 *ing at Low Scales*, Phys. Rev. D **97** 114012 (2018), doi:[10.1103/PhysRevD.97.114012](https://doi.org/10.1103/PhysRevD.97.114012).
- 188 [2] T. B. Liu, J. W. Qiu, *Power Corrections in Semi-Inclusive Deep Inelastic Scatterings at Fixed-*  
 189 *Target Energies*, Phys. Rev. D **101** 014008 (2020), doi:[10.1103/PhysRevD.101.014008](https://doi.org/10.1103/PhysRevD.101.014008).

- 190 [3] B. Z. Kopeliovich, R. Pasechnik, I. K. Potashnikova, *Diffraction Dijet Pro-*  
191 *duction: Breakdown of Factorization*, Phys. Rev. D **98** 114021 (2018),  
192 doi:[10.1103/PhysRevD.98.114021](https://doi.org/10.1103/PhysRevD.98.114021).
- 193 [4] M. Neubert, *Heavy Quark Symmetry*, Phys. Rept. **245** 259 (1994), doi:[10.1016/0370-](https://doi.org/10.1016/0370-1573(94)90091-4)  
194 [1573\(94\)90091-4](https://doi.org/10.1016/0370-1573(94)90091-4).
- 195 [5] J. Collins, *Foundations of Perturbative QCD*, Cambridge Monographs on Particle Physics,  
196 Nuclear Physics and Cosmology; Cambridge University Press: Cambridge (2011),  
197 doi:[10.1017/CBO9780511975592](https://doi.org/10.1017/CBO9780511975592).
- 198 [6] T. Rogers, P. Mulders, *No Generalized TMD-Factorization in the Hadro-Production*  
199 *of High Transverse Momentum Hadrons*, Phys. Rev. D **81** 094006 (2010),  
200 doi:[10.1103/PhysRevD.81.094006](https://doi.org/10.1103/PhysRevD.81.094006).
- 201 [7] M. Schwartz, K. Yan, H. X. Zhu, *Factorization Violation and Scale Invariance*, Phys. Rev.  
202 D **97** 096017 (2018), doi:[10.1103/PhysRevD.97.096017](https://doi.org/10.1103/PhysRevD.97.096017).
- 203 [8] J. Collins, J. W. Qiu,  *$k_T$ -Factorization is Violated in Production of High-Transverse-*  
204 *Momentum Particles in Hadron-Hadron Collisions*, Phys. Rev. D **75** 114014 (2007),  
205 doi:[10.1103/PhysRevD.75.114014](https://doi.org/10.1103/PhysRevD.75.114014).
- 206 [9] S. Catani, D. de Florian, G. Rodrigo, *Space-Like (vs. Time-Like) Collinear Limits in QCD:*  
207 *is Factorization Violated?*, JHEP **07** 026 (2012), doi:[10.1007/JHEP07\(2012\)026](https://doi.org/10.1007/JHEP07(2012)026).
- 208 [10] LHCb Collaboration, *Study of Forward Z+Jet Production in pp Collisions at  $\sqrt{s} = 7$  TeV*,  
209 JHEP **01** 33 (2014), doi:[10.1007/JHEP01\(2014\)033](https://doi.org/10.1007/JHEP01(2014)033).
- 210 [11] LHCb Collaboration, *Measurement of Forward W and Z Boson Production in Associ-*  
211 *ation with Jets in Proton-Proton Collisions at  $\sqrt{s} = 8$  TeV*, JHEP **05** 131 (2016),  
212 doi:[10.1007/JHEP05\(2016\)131](https://doi.org/10.1007/JHEP05(2016)131).
- 213 [12] LHCb Collaboration, *Measurement of Charged Hadron Production in Z-Tagged Jets*  
214 *in Proton-Proton Collisions at  $\sqrt{s} = 8$  TeV*, Phys. Rev. Lett. **123** 232001 (2019),  
215 doi:[10.1103/PhysRevLett.123.232001](https://doi.org/10.1103/PhysRevLett.123.232001).
- 216 [13] LHCb Collaboration, *The LHCb Detector at the LHC*, JINST **3** S08005 (2008),  
217 doi:[10.1088/1748-0221/3/08/S08005](https://doi.org/10.1088/1748-0221/3/08/S08005).
- 218 [14] J. C. Collins, D. E. Soper, *Back-to-Back Jets in QCD*, Nucl. Phys. B **193** 381 (1981), Erra-  
219 tum: Nucl. Phys. B **213** 545 (1983), doi:[10.1016/0550-3213\(81\)90339-4](https://doi.org/10.1016/0550-3213(81)90339-4).
- 220 [15] J. C. Collins, D. E. Soper, *Parton Distribution and Decay Functions*, Nucl. Phys. B **194** 445  
221 (1982), doi:[10.1016/0550-3213\(82\)90021-9](https://doi.org/10.1016/0550-3213(82)90021-9).
- 222 [16] V. N. Gribov, L. N. Lipatov, *Deep Inelastic e p Scattering in Perturbation Theory*, Sov. J.  
223 Nucl. Phys. **15** 438 (1972), Yad. Fiz. **15** 781 (1972).
- 224 [17] G. Altarelli, G. Parisi, *Asymptotic Freedom in Parton Language*, Nucl. Phys. B **126** 298  
225 (1977), doi:[10.1016/0550-3213\(77\)90384-4](https://doi.org/10.1016/0550-3213(77)90384-4).
- 226 [18] Y. L. Dokshitzer, *Calculation of the Structure Functions for Deep Inelastic Scattering and*  
227  *$e^+e^-$  Annihilation by Perturbation Theory in Quantum Chromodynamics*, Sov. Phys. JETP  
228 **46** 641 (1977), Zh. Eksp. Teor. Fiz. **73** 1216 (1977).



- 229 [19] PHENIX Collaboration, *Nonperturbative-Transverse-Momentum Effects and Evolution*  
230 *in Dihadron and Direct Photon-Hadron Angular Correlations in  $p + p$  Collisions at*  
231  $\sqrt{s} = 510$  GeV, Phys. Rev. D **95** 072002 (2017), doi:[10.1103/PhysRevD.95.072002](https://doi.org/10.1103/PhysRevD.95.072002).
- 232 [20] PHENIX Collaboration, *Nonperturbative Transverse-Momentum-Dependent Effects in*  
233 *Dihadron and Direct Photon-Hadron Angular Correlations in  $p + p$  Collisions at*  
234  $\sqrt{s} = 200$  GeV, Phys. Rev. D **98** 072004 (2018), doi:[10.1103/PhysRevD.98.072004](https://doi.org/10.1103/PhysRevD.98.072004).
- 235 [21] PHENIX Collaboration, *Nonperturbative Transverse Momentum Broadening in Dihadron*  
236 *Angular Correlations in  $\sqrt{s} = 200$  GeV Proton-Nucleus Collisions*, Phys. Rev. C **99** 044912  
237 (2019), doi:[10.1103/PhysRevC.99.044912](https://doi.org/10.1103/PhysRevC.99.044912).