T-odd quark-gluon-quark correlation function in the light-front quark-diquark model

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July 30, 2021

Proceedings for the XXVIII International Workshop on Deep-Inelastic Scattering and Related Subjects, Stony Brook University, New York, USA, 12-16 April 2021
doi:10.21468/SciPostPhysProc.?

Abstract

We have scrutinized the transverse momentum dependent quark–gluon–quark correlation function. We have utilized the light-front quark-diquark model to study the time-reversal-odd interaction-dependent twist-3 gluon distributions which we have obtained from the disintegration of the transverse momentum dependent quark–gluon–quark correlator. Specifically, we have studied the behavior of $\tilde{e}_L$ and $\tilde{e}_T$ while considering our diquark to be an axial-vector.

1 Introduction

Theoretically, in the description of hadron containing (semi-)inclusive high energy processes, the cross-sections are generally written in the powers of $1/Q$ with Q being the large momentum transfer of the collision. The convolution of the hard scattering coefficients and leading-twist distribution functions is used to communicate the contribution at the leading power. In the $1/Q$ expansion, the first sub leading power of twist-3 distribution and/or fragmentation functions are added to the cross-section [1].

Twist-3 distribution functions are not illuminated in the form of probability which is obvious and contrary to the interpretation of the twist-2 distributions. They are rather used to describe the parton densities in the core the nucleon. Information about the nucleon's parton structure is obtained from twist-3 distribution functions [2], particularly when the parton transverse momenta exist. The appeal on the twist-3 contributions also comes from the fact that they are related to the multi-parton correlation in the interior of nucleon [3, 4].

In this paper, we have utilized quark-diquark model to examine the quark distributions of twist-3 level which are ciphered in the quark-gluon-quark correlation. We have emphasized on the time-reversal-odd (T-odd) transverse momentum dependent distributions (TMDs). In the single-spin asymmetries (SSAs) measured in semi-inclusive deeply inelastic scattering (SIDIS) [5–7] the leading twist T-odd TMDs [8] play important roles in the TMD factorization approach [9–11]. In the TMD factorization approach at the twist-3 level there are eight T-odd distributions that can contribute to various azimuthal asymmetries in the SIDIS [2] and Drell–Yan [12] processes. Even though the twist-3 contributions are suppressed due to $1/Q$, still these experimental observables have potential and may be accessible in the kinematical regime where Q is not so large. The ideal experiments for exploring this kinematical region are at PAX [13] and Jefferson Lab [14,15].

2 Light-Front Quark-Diquark Model

We contemplate on our problem by considering the light-front quark-diquark model [16], where the proton has a spin-flavor $SU(4)$ structure and is written as a combination of isoscalar-scalar diquark singlet $\left|uS^0\right\rangle$, isoscalarvector diquark $\left|uA^0\right\rangle$ and isovector-vector diquark $\left|dA^1\right\rangle$ states [17,18]. We have

$$|P;\pm\rangle = C_S\left|uS^0\right\rangle^{\pm} + C_V\left|uA^0\right\rangle^{\pm} + C_{VV}\left|dA^1\right\rangle^{\pm}, \tag{1}$$

where S and A denote the scalar and axial-vector diquark and their superscripts are used to depict the isospin of that diquark. Here, we have used the light-cone convention $x^{\pm} = x^0 \pm x^3$ and the frame is chosen such that the proton has no transverse momentum, i.e., $P \equiv \left(P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\right)$; where the struck quark and diquark have momentum $p \equiv \left(xP^+, \frac{p^2+|\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right)$ and $P_X \equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right)$ respectively. $x = p^+/P^+$ is used to denote the longitudinal momentum fraction carried by the struck quark.

The two particle Fock-state expansion for axial-vector diquark is given as [19]

$$\begin{aligned}
|\nu A\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \times \Bigg[& \psi_{++}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}+1; xP^+, \mathbf{p}_\perp\right\rangle \\
& + \psi_{-+}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}+1; xP^+, \mathbf{p}_\perp\right\rangle + \psi_{+0}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}0; xP^+, \mathbf{p}_\perp\right\rangle \\
& + \psi_{-0}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}0; xP^+, \mathbf{p}_\perp\right\rangle + \psi_{+-}^{r(\nu)}(x,\mathbf{p}_\perp)\left|+\frac{1}{2}-1; xP^+, \mathbf{p}_\perp\right\rangle \\
& + \psi_{-}^{r(\nu)}(x,\mathbf{p}_\perp)\left|-\frac{1}{2}-1; xP^+, \mathbf{p}_\perp\right\rangle \Bigg],
\end{aligned} \tag{2}$$

where $\left|\lambda_q\lambda_D; xP^+, \mathbf{p}_\perp\right\rangle$ represents the two-particle state with quark helicity $\lambda_q = \pm\frac{1}{2}$, the helicity vector diquark is $\lambda_D = \pm1, 0$ (triplet). The superscript of ψ, r denotes the helicity of nucleon and the flavor index for the flavors u and d is denoted by ν.

3 Transverse Momentum Dependent Distributions (TMDs)

3.1 Quark-gluon-quark correlation function

We start our calculations with the transverse momentum dependent quark-gluon-quark correlation function which is defined in [20]

$$\begin{aligned}
\left(\tilde{\Phi}_A^{[\pm]\alpha}\right)_{ij}(x,p_T) \equiv \int \frac{d^2\xi_T\, d\xi^-}{(2\pi)^3} e^{ip\xi} \times \langle P,S|\bar{\psi}_j(0) g \int_{\pm\infty}^{\xi^-} d\eta^- \mathcal{L}^{[\pm]}(0,\eta^-) \\
F^{+\alpha}(\eta)\, \mathcal{L}^{\xi_T,\xi^+}(\eta^-,\xi^-)\, \psi_i(\xi)|P,S\rangle_c \Big|_{\eta^+=\xi^+=0, \eta_T=\xi_T, p^+=xP^+},
\end{aligned} \tag{3}$$

where $F^{\mu\nu}$ is the gluon antisymmetric field strength tensor. By definition, gauge-invariance is certified by the gauge-links $\mathcal{L}^{[\pm]}$ and $\mathcal{L}^{\xi_T,\xi^+}$. The sign " $\pm$ " in the subscript or superscript represents the future/past-pointing [10] nature gauge-link between the quark and gluon in the SIDIS/Drell-Yan processes, respectively.

The correlator can be rewritten further as [21]

$$\begin{aligned}\left(\tilde{\Phi}_A^{[\pm]\alpha}\right)_{ij}(x,p_T) = ig\int \frac{d^2\xi_T d\xi^- d\eta^-}{(2\pi)^4}\int dx' \frac{e^{ix'p+\eta^-}}{(x'\mp i\epsilon)} \quad \times e^{i[(x-x')p^+\cdot\xi^- - p_T\cdot\xi_T]} \\ \langle P,S|\bar{\psi}_j(0)\mathcal{L}^{[\pm]}\left(0,\eta^-\right)F^{+\alpha}(\eta) \quad \times \mathcal{L}^{\xi_T,\xi^+}\left(\eta^-,\xi^-\right)\psi_i(\xi)|P,S\rangle\Big|_{\eta^+=\xi^+=0,\eta_T=\xi_T},\end{aligned} \tag{4}$$

where the factor $1/\left(x'\mp i\epsilon\right)$ in Eq. (4) can be written as

$$\frac{1}{(x'\mp i\epsilon)} = \mathrm{P}\left(\frac{1}{x'}\right) \pm i\delta\left(x'\right). \tag{5}$$

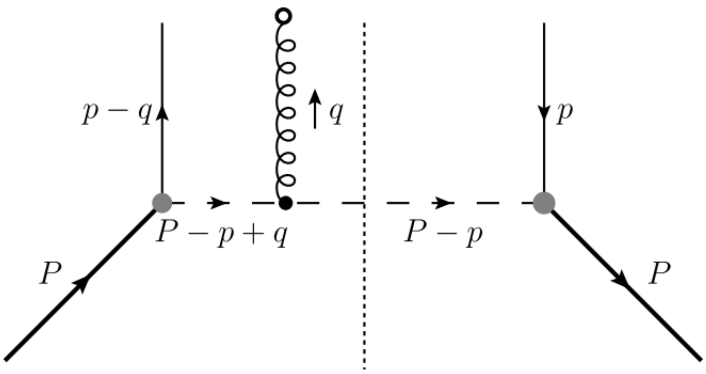

Figure 1: Diagram corresponding to quark-gluon-quark correlator. P, p and q denote the nucleon, quark and diquark momenta respectively [21, 22].

3.2 T-Odd Twist-3 Distributions

We can break the quark-gluon-quark correlator as [2]

$$\begin{aligned}\tilde{\Phi}_A^{\alpha}(x,p_T) = \frac{xM}{2}\Bigg\{\Bigg[\left(\tilde{f}^{\perp}-i\tilde{g}^{\perp}\right)\frac{p_{T\rho}}{M} - \left(\tilde{f}'_T+i\tilde{g}'_T\right)\epsilon_{T\rho\sigma}S_T^{\sigma} - \left(\tilde{f}_s^{\perp}+i\tilde{g}_s^{\perp}\right)\frac{\epsilon_{T\rho\sigma}p_T^{\sigma}}{M}\Bigg] \\ \left(g_T^{\alpha\rho}-i\epsilon_T^{\alpha\rho}\gamma_5\right) - \left(\tilde{h}_s+i\tilde{e}_s\right)\gamma_T^{\alpha}\gamma_5 + [(\tilde{h}+i\tilde{e}) + \left(\tilde{h}_T^{\perp}-i\tilde{e}_T^{\perp}\right)\frac{\epsilon_T^{\rho\sigma}p_{T\rho}S_{T\sigma}}{M}\Bigg]i\gamma_T^{\alpha}\Bigg\}\frac{\not{n}_+}{2}.\end{aligned} \tag{6}$$

This equation contains interaction-dependent twist-3 quark distributions, which is in turn dependent on the longitudinal momentum fraction and the transverse momentum denoted by x and p_T respectively. These are denoted by the functions appearing with a tilde. Out of these, $\tilde{g}^{\perp}$, $\tilde{f}_T$ (or $\tilde{f}'_T$), $\tilde{e}_L$, $\tilde{e}_T$, $\tilde{f}_T^{\perp}$, $\tilde{f}_L^{\perp}$, $\tilde{h}$ $\tilde{e}_T^{\perp}$ are T-odd; and $\tilde{f}^{\perp}$, $\tilde{g}_T$ (or $\tilde{g}'_T$), $\tilde{g}_T^{\perp}$, $\tilde{g}_L^{\perp}$, $\tilde{h}_L$, $\tilde{h}_T$, $\tilde{e}$, $\tilde{h}_T^{\perp}$ are T-even.

These TMDs can be projected out by using disparate Dirac matrices. In the right-hand side of Eq. (5), if one takes the real part then they can derive the traces of the T-even TMDs. On the other hand, if one uses the imaginary part, T-odd TMDs are obtained. Here we deal specifically with $\tilde{e}_L$ and $\tilde{e}_T$ [21] by considering the real part of Eq. (5) and using Dirac matrix $i\sigma^{\alpha+}\gamma_5$ as

$$\frac{1}{2Mx}\mathrm{Tr}\left[\tilde{\Phi}_{A\alpha}i\sigma^{\alpha+}\gamma_5\right] = S_L\left(\tilde{h}_L+i\tilde{e}_L\right) - \frac{p_T\cdot S_T}{M}\left(\tilde{h}_T+i\tilde{e}_T\right), \tag{7}$$

where S_T and S_L are the transverse and longitudinal polarization vector of the nucleon respectively.

While applying the approximation of the lowest order, we neglect every gauge-link in the correlator (Eq. (4)) and choose the model which has been used extensively in the calculation of TMD distributions [17,18]. Before all else, diquark model shows that the T-odd distributions are non-vanishing. We consider the case in which the diquark is an axial-vector.

Here, we calculate the T-odd TMDs emerging in the DIS process (Drell-Yan process comes with a minus sign). Above expression is proportional to the sum of the terms $\psi^{*r}_{-+}(x,p_T)\psi^{r}_{++}(x,p_T)$, $\psi^{*r}_{-0}(x,p_T)\psi^{r}_{+0}(x,p_T)$, $\psi^{*r}_{--}(x,p_T)\psi^{r}_{+-}(x,p_T)$, $\psi^{*r}_{++}(x,p_T)\psi^{r}_{-+}(x,p_T)$, $\psi^{*r}_{+0}(x,p_T)\psi^{r}_{-0}(x,p_T)$ and $\psi^{*r}_{+-}(x,p_T)\psi^{r}_{--}(x,p_T)$. We can solve this further and get the value of desired TMDs by using suitable light-front wave functions [9,23,24]. Also in a specific Feynman rule [21,22,25], the field strength tensor has been used in the form $F^{+\alpha} : -i\left(q^{+}g^{\alpha\rho} - q^{\alpha}g^{+\rho}\right)$. While using this model some divergences are found which appear to be emerging for a few T-odd TMDs when the integrations are performed over transverse momentum. In the quark-diquark model these kind of divergences are explicitly found very often [21,26]. So, it can be deduced that T-odd twist-3 distributions has this general feature that when on the nucleon-quark-diquark interaction vertex the point-like coupling is applied the divergences appear. To derive the finite results, one can choose a dipole form factor instead of a point-like coupling constant for the nucleon-quark-diquark coupling [17].

4 Conclusion

We have investigated the prospect to calculate the T-odd interaction-dependent twist-3 quark distributions in the quark diquark model. In the approxmation of lowest order we find that abandoning the gauge-links in the correlator can give results which are not equal to zero for the eight T-odd interaction-dependent quark TMDs of twist-3. Particularly, we find that the projection of T-odd twist-3 correlator with this specific Dirac matrix $i\sigma^{\alpha+}\gamma_5$ in the form of TMDs $\tilde{e}_L$ and $\tilde{e}_T$ while considering the case of a axial-vector diquark, leads to an equation proportional to the the light-front wave functions and the field strength tensor. From this one can get the expression of each TMD individually by specifying the nucleon helicity.

Acknowledgements

The work of HD is supported by the Science and Engineering Research Board, Government of India, under MATRICS (Ref No. MTR/2019/000003).

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