

# Transverse-momentum-dependent parton distribution functions for spin-1 hadrons

S. Kumano<sup>1,2\*</sup> and Qin-Tao Song<sup>3</sup>

**1** KEK Theory Center, Institute of Particle and Nuclear Studies, KEK,  
Oho 1-1, Tsukuba, Ibaraki, 305-0801, Japan

**2** J-PARC Branch, KEK Theory Center, Institute of Particle and Nuclear Studies, KEK,  
and Theory Group, Particle and Nuclear Physics Division, J-PARC Center,  
Shirakata 203-1, Tokai, Ibaraki, 319-1106, Japan

**3** School of Physics and Microelectronics, Zhengzhou University,  
Zhengzhou, Henan 450001, China

\* shunzo.kumano@kek.jp, songqintao@zzu.edu.cn

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## Abstract

We show transverse-momentum-dependent parton distribution functions (TMDs) for spin-1 hadrons including twist-3 and 4 functions by taking the decomposition of a quark correlation function in the Lorentz-invariant way with the conditions of Hermiticity and parity invariance. We found 30 new TMDs in the tensor-polarized spin-1 hadron at twists 3 and 4 in addition to 10 TMDs at twist 2. Since time-reversal-odd terms of the collinear correlation function should vanish after integrals over the partonic transverse momentum, we obtained new sum rules for the time-reversal-odd structure functions,  $\int d^2k_T g_{LT} = \int d^2k_T h_{LL} = \int d^2k_T h_{3LL} = 0$ , at twists 3 and 4. We also indicated that transverse-momentum-dependent fragmentation functions exist in tensor-polarized spin-1 hadrons. The TMDs can probe color degrees of freedom, so that they are valuable in providing unique opportunities for creating interdisciplinary physics fields such as gluon condensate, color Aharonov-Bohm effect, and color entanglement. We also found three new collinear PDFs at twists 3 and 4, and a twist-2 relation and a sum rule were derived in analogy to the Wandzura-Wilczek relation and the Burkhardt-Cottingham sum rule on the structure function  $g_2$ .

## 1 Introduction

The field of transverse-momentum-dependent parton distribution functions (TMDs) is one of rapidly-developing fields in hadron physics. It is interesting because the explicit color degrees of freedom can be probed by the TMDs. Depending on the color flow in hadrons, the TMDs have opposite signs, for example, in semi-inclusive deep inelastic scattering and Drell-Yan processes. The TMD physics is related to fundamental aspects of quantum physics, such as color Aharonov-Bohm effect and color entanglement, and the TMDs are also valuable for understanding the color glass condensate. The color is confined in hadrons and it does not appear easily in experimental observables. The TMDs provide a unique opportunity to shed light on new hadron phenomena involving the color.

The TMDs have been investigated so far for the spin-1/2 nucleons. On the collinear structure function  $b_1$  of the spin-1 deuteron, there was a measurement by the HERMES collaboration in 2005 [1]. In future, there are experimental projects to investigate structure functions of spin-1 hadrons, especially the spin-1 deuteron, at the Jefferson laboratory (JLab) [2], Fermilab [3], and Nuclotron-based Ion Collider fAcility (NICA) [4], and electron-ion colliders in US and China. We also have been investigating structure functions of spin-1 hadrons theoretically by considering these experimental projects [5–8]. Therefore, it is a good opportunity and timely to investigate TMDs also for the spin-1 hadrons. However, only the twist-2 theoretical formalism was available [9]. The twist-3 and 4 parts were investigated recently in Ref. [7], where new structure functions and sum rules were proposed at twists 3 and 4. In addition, a twist-2 relation and a sum rule were derived for the twist-3 collinear structure function  $f_{LT}$  [8]. We explain these works in this paper.

## 2 Correlation functions and polarizations for spin-1 hadrons

The TMDs and collinear parton distribution functions (PDFs) are defined from the correlation function given by the matrix element as

$$\Phi_{ij}^{[c]}(k, P, T | n) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P, T | \bar{\psi}_j(0) W^{[c]}(0, \xi) \psi_i(\xi) | P, T \rangle. \quad (1)$$

Here,  $k$  and  $P$  are quark and hadron momenta,  $T$  indicates the tensor polarization of the hadron,  $\xi$  is the space-time coordinate of the quark,  $\psi$  is the quark field,  $W^{[c]}(0, \xi)$  is the gauge link,  $c$  indicates the integral path, and  $n$  is the lighcone vector  $n^\mu = (1, 0, 0, -1)/\sqrt{2}$ . Since only the tensor polarization is considered in this work, the vector polarization  $S$  is not explicitly written. The TMD and collinear correlation functions are defined by integrating the quark transverse momenta as

$$\Phi^{[c]}(x, k_T, P, T) = \int dk^+ dk^- \Phi^{[c]}(k, P, T | n) \delta(k^+ - xP^+), \quad (2)$$

$$\Phi(x, P, T) = \int d^2k_T \Phi^{[c]}(x, k_T, P, T). \quad (3)$$

The covariant form of the tensor polarization  $T^{\mu\nu}$  of a spin-1 hadron is expressed by the tensor-polarization parameters  $S_T^x, S_T^y, S_L, S_{LL}, S_{TT}^{xx}, S_{TT}^{xy}, S_{LT}^x, S_{LT}^y$  as

$$T^{\mu\nu} = \frac{1}{2} \left[ \frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu - \frac{2}{3} S_{LL} (\bar{n}^{\{\mu} n^{\nu\}} - g_T^{\mu\nu}) + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu + \frac{P^+}{M} \bar{n}^{\{\mu} S_{LT}^{\nu\}} - \frac{M}{2P^+} n^{\{\mu} S_{LT}^{\nu\}} + S_{TT}^{\mu\nu} \right]. \quad (4)$$

Here,  $a^{\{\mu} b^{\nu\}}$  indicates the symmetrized combination  $a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ ,  $M$  is the hadron mass, and  $P^+$  is the lightcone momentum given by  $P^+ = (P^0 + P^3)/\sqrt{2}$ . Using this tensor together with available Lorentz vectors  $P, k$ , and  $n$ , we wrote the general form of the correlation function by considering the condition of Hermiticity and parity invariance as [7]

$$\begin{aligned} \Phi(k, P, T | n) = & \frac{A_{13}}{M} T_{kk} + \frac{A_{14}}{M^2} T_{kk} \not{P} + \dots + \frac{A_{20}}{M^2} \varepsilon^{\mu\nu\rho k} \gamma_\mu \gamma_5 T_{\nu k} \\ & + \frac{B_{21}M}{P \cdot n} T_{kn} + \frac{B_{22}M^3}{(P \cdot n)^2} T_{nn} + \dots + \frac{B_{52}M}{P \cdot n} \sigma_{\mu k} T^{\mu n}, \end{aligned} \quad (5)$$

where the notation  $X_{\mu k} \equiv X_{\mu\nu} k^\nu$  is used for the contraction of the tensor  $X_{\mu\nu}$  with  $k^\nu$ . The important point of this expression is that the terms with the lightcone vector  $n$  are included for finding the twist-3 and 4 structure functions, as investigated in finding twist-3 and 4 structure functions in the spin-1/2 nucleons [10]. Integrating this expression as given in Eq. (2), we obtained possible tensor-polarized structure functions up to twist 4.

### 3 TMDs and PDFs of spin-1 hadrons up to twist 4

#### TMDs for tensor-polarized spin-1 hadrons

The TMDs are defined from the correlation functions of Eqs. (2) and (5) by taking traces with various  $\gamma$  matrices as  $\Phi^{[\Gamma]}(x, k_T, T) \equiv \frac{1}{2} \text{Tr}[\Phi(x, k_T, T)\Gamma]$ , and possible TMDs are listed in Tables 1, 2, and 3. The upper half sections (unpolarized U, longitudinally polarized L, transversely polarized T) of these tables are the same as the ones of the spin-1/2 nucleons. In this work, the tensor-polarized sections (LL, LT, TT) are studied. The square brackets [ ] indicate chiral-odd distributions and the others are chiral-even ones. The T-even (T: time reversal) and T-odd distributions are separately written. The twist-2 part was studied in Ref. [9] by calculating  $\Phi^{[\gamma^+]}$ ,  $\Phi^{[\gamma^+\gamma_5]}$ , and  $\Phi^{[i\sigma^i\gamma_5]}$  ( $\Phi^{[\sigma^{i+}]}$ ). Our new results were for the twist-3 and twist-4 functions in Tables 2 and 3 [7].

Since the full expressions are lengthy, we explain the outline by taking an example on the correlation function  $\Phi^{[\gamma^i]}$  and related twist-3 TMDs. The twist-3 TMDs were obtained by calculating  $\Phi^{[\gamma^i]}$ ,  $\Phi^{[1]}$ ,  $\Phi^{[i\gamma_5]}$ ,  $\Phi^{[\gamma^i\gamma_5]}$ ,  $\Phi^{[\sigma^{ij}]}$ , and  $\Phi^{[\sigma^{i+}]}$ . Among them,  $\Phi^{[\gamma^i]}$  is expressed by the polarization parameters and TMDs as

$$\begin{aligned} \Phi^{[\gamma^i]}(x, k_T, T) = & \frac{M}{P^+} \left[ f_{LL}^\perp(x, k_T^2) S_{LL} \frac{k_T^i}{M} + f_{LT}'(x, k_T^2) S_{LT}^i \right. \\ & - f_{LT}^\perp(x, k_T^2) \frac{k_T^i S_{LT} \cdot k_T}{M^2} - f_{TT}'(x, k_T^2) \frac{S_{TT}^{ij} k_{Tj}}{M} \\ & \left. + f_{TT}^\perp(x, k_T^2) \frac{k_T \cdot S_{TT} \cdot k_T}{M^2} \frac{k_T^i}{M} \right], \quad (6) \end{aligned}$$

where the TMDs  $f_{LL}^\perp, \dots, f_{TT}^\perp$  are expressed by the expansion coefficients  $A_i$  and  $B_i$  of Eq. (5). The guideline for assigning the TMD names  $f_{LL}^\perp, \dots, f_{TT}^\perp$  is explained in Ref. [7] depending on polarizations and transverse-momentum factors. In this way, we obtained five TMDs in  $\Phi^{[\gamma^i]}$  and they are listed in Table 2. In listing the TMDs in the tables, we redefined the TMDs by  $F(x, k_T^2) \equiv F'(x, k_T^2) - (k_T^2/(2M^2))F^\perp(x, k_T^2)$  where  $k_T^2 = -\vec{k}_T^2$ , and the functions without ' are shown in the tables. The other twist-3 TMDs are also listed in Table 2. In the same way, the twist-4 TMDs are obtained by calculating  $\Phi^{[\gamma^-]}$ ,  $\Phi^{[\gamma^-\gamma_5]}$ , and  $\Phi^{[\sigma^{i-}]}$  and they are listed in Table 3. There are 40 TMDs in the tensor-polarized spin-1 hadron as shown in Tables 1, 2, and 3. Among them, we found that 30 new TMDs exist at twist-3 and 4 as

$$\begin{aligned} \text{Twist-3 TMD: } & f_{LL}^\perp, e_{LL}, f_{LT}, f_{LT}^\perp, e_{1T}, e_{1T}^\perp, f_{TT}, f_{TT}^\perp, e_{TT}, e_{TT}^\perp, g_{LL}^\perp, g_{LT}, g_{LT}^\perp, \\ & g_{TT}, g_{TT}^\perp, h_{1L}, h_{LT}, h_{LT}^\perp, h_{TT}, h_{TT}^\perp, \\ \text{Twist-4 TMD: } & f_{3LL}, f_{3LT}, f_{3TT}, g_{3LT}, g_{3TT}, h_{3LL}^\perp, h_{3LT}, h_{3LT}^\perp, h_{3TT}, h_{3TT}^\perp. \quad (7) \end{aligned}$$

They are classified by the time-reversal and chirality properties. Since the T-odd collinear PDFs should vanish  $f(x)_{\text{T-odd}} = 0$  due to the time-reversal invariance, we have the following sum rules at twist-3 and 4

$$\int d^2k_T g_{LT}(x, k_T^2) = \int d^2k_T h_{LL}(x, k_T^2) = \int d^2k_T h_{3LT}(x, k_T^2) = 0. \quad (8)$$

Quark	$U(\gamma^+)$		$L(\gamma^+\gamma_5)$		$T(i\sigma^i\gamma_5/\sigma^{i+})$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_1$					$[h_1^+]$
L			$g_{1L}$		$[h_{1L}^+]$	
T		$f_{1T}^+$	$g_{1T}$		$[h_{1T}, [h_{1T}^+]$	
LL	$f_{1LL}$					$[h_{1LL}^+]$
LT	$f_{1LT}$			$g_{1LT}$		$[h_{1LT}, [h_{1LT}^+]$
TT	$f_{1TT}$			$g_{1TT}$		$[h_{1TT}, [h_{1TT}^+]$

Table 1: Twist-2 TMDs.

Quark	$\gamma^+, i\gamma_5$		$\gamma^+\gamma_5$		$\sigma^i, \sigma^{i+}$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_1^+$ [ $e_1^+$ ]			$g_1^+$		$[h_1^+]$
L		$f_{1L}^+$ [ $e_{1L}^+$ ]	$g_{1L}^+$			$[h_{1L}^+]$
T		$f_{1T}^+, f_{1T}^+$ [ $e_{1T}^+, e_{1T}^+$ ]	$g_{1T}, g_{1T}^+$			$[h_{1T}, [h_{1T}^+]$
LL	$f_{1LL}^+$ [ $e_{1LL}^+$ ]			$g_{1LL}^+$		$[h_{1LL}^+]$
LT	$f_{1LT}^+, f_{1LT}^+$ [ $e_{1LT}^+, e_{1LT}^+$ ]			$g_{1LT}, g_{1LT}^+$		$[h_{1LT}, [h_{1LT}^+]$
TT	$f_{1TT}^+, f_{1TT}^+$ [ $e_{1TT}^+, e_{1TT}^+$ ]			$g_{1TT}, g_{1TT}^+$		$[h_{1TT}, [h_{1TT}^+]$

Table 2: Twist-3 TMDs.

Quark	$\gamma^-$		$\gamma^-\gamma_5$		$\sigma^{i-}$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_3$					$[h_3^+]$
L			$g_{3L}$			$[h_{3L}^+]$
T		$f_{3T}^+$	$g_{3T}$			$[h_{3T}, [h_{3T}^+]$
LL	$f_{3LL}$					$[h_{3LL}^+]$
LT	$f_{3LT}$			$g_{3LT}$		$[h_{3LT}, [h_{3LT}^+]$
TT	$f_{3TT}$			$g_{3TT}$		$[h_{3TT}, [h_{3TT}^+]$

Table 3: Twist-4 TMDs.

## TMD fragmentation functions of spin-1 hadrons

New TMD fragmentation functions also exist for the spin-1 hadrons [7], and they are obtained simply by changing the kinematical variables and function names as

$$\text{Kinematical variables: } x, k_T, S, T, M, n, \gamma^+, \sigma^{i+} \Rightarrow z, k_T, S_h, T_h, M_h, \bar{n}, \gamma^-, \sigma^{i-},$$

$$\text{Distribution functions: } f, g, h, e \Rightarrow \text{Fragmentation functions: } D, G, H, E. \quad (9)$$

## PDFs for tensor-polarized spin-1 hadrons

The collinear PDFs of the tensor-polarized hadrons [11] are obtained by integrating the TMDs over the transverse momentum as  $f(x) = \int d^2k_T f(x, k_T^2)$ . Many functions vanish by this integral and the remaining PDFs are shown in Tables 4, 5, and 6. Therefore, the new twist-3 and 4 functions, which we found, are [7]

$$\text{Twist-3 PDF: } e_{LL}, f_{LT}, \quad \text{Twist-4 PDF: } f_{3LL}. \quad (10)$$

The asterisks \*1, \*2, \*3, \*4 indicate the collinear PDFs  $h_{1LT}(x), g_{LT}(x), h_{LL}(x), h_{3LT}(x)$  vanish, respectively, because of the time-reversal invariance. However, since the time-reversal invariance cannot be imposed in the fragmentation functions, the corresponding fragmentation functions  $H_{1LT}(z), G_{LT}(z), H_{LL}(z), H_{3LT}(z)$ , as indicated by the replacements of Eq. (9), should exist as collinear fragmentation functions [7, 12].

## Twist-2 relation and sum rule for $f_{LT}$

Since we obtained the new PDFs for spin-1 hadrons, it is possible to investigate useful relations for them in the similar way to the Wandzura-Wilczek (WW) relation and the Burkhardt-Cottingham (BC) sum rule. For finding twist-2 relations, it is important to specify higher-twist effects. Twist-3 effects are investigated by the non-local operator  $\bar{\psi}(0)(\partial^\mu \gamma^\alpha - \partial^\alpha \gamma^\mu)\psi(\xi)$ , whose matrix element between tensor-polarized spin-1 hadron states is expressed by  $f_{1LL}$  and  $f_{LT}$  up to twist 3. For the tensor-polarized spin-1 hadron, twist-3 multiparton distribution functions

$$F_{LT}(x, y), G_{LT}(x, y), H_{LL}^\perp(x, y), H_{TT}(x, y), \quad (11)$$

exist in general [8]. The twist-3 matrix element is expressed by  $F_{G,LT}$  and  $G_{G,LT}$  defined with the gluon field tensor  $G^{\mu\nu}$ . Specifying the twist-3 terms, we obtained [8]

$$f_{LT}(x) = \frac{3}{2} \int_x^{\epsilon(x)} dy \frac{f_{1LL}(y)}{y} + \int_x^{\epsilon(x)} dy \frac{f_{LT}^{(HT)}(y)}{y}, \quad (12)$$

where the higher-twist function  $f_{LT}^{(HT)}(x)$  is expressed by the integral of  $F_{G,LT}$  and  $G_{G,LT}$ . Here,  $\epsilon(x) = 1$  ( $-1$ ) for  $x > 0$  ( $x < 0$ ). In the positive (negative)  $x$  region,  $f(x)$  is a quark (antiquark) distribution. We define  $f^+$  distribution by  $f^+(x) \equiv f(x) + \bar{f}(x) = f(x) - f(-x)$  for  $f = f_{1LL}, f_{LT}, f_{LT}^{(HT)}$  at  $x > 0$ , the relation becomes  $f_{LT}^+(x) = \frac{3}{2} \int_x^1 dy f_{1LL}^+(y)/y$  if the higher-twist terms are neglected. Furthermore, if the function  $f_{2LT}$  is defined by  $f_{2LT}(x) \equiv (2/3)f_{LT}(x) - f_{1LL}(x)$ , this integral relation becomes

$$f_{2LT}^+(x) = -f_{1LL}^+(x) + \int_x^1 \frac{dy}{y} f_{1LL}^+(y). \quad (13)$$

Quark	U ( $\gamma^+$ )		L ( $\gamma^+\gamma_s$ )		T ( $i\sigma^+\gamma_s / \sigma^+$ )	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_1$					
L			$g_{1L}(g_1)$			
T					$[h_1]$	
LL	$f_{1LL}(h_1)$					
LT						*1
TT						

Table 4: Twist-2 PDFs.

Quark	$\gamma^-, 1, i\gamma_s$		$\gamma^-\gamma_s$		$\sigma^-, \sigma^+$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$[e]$					
L					$[h_1]$	
T			$g_T$			
LL	$[e_{1L}]$					*3
LT	$f_{LT}$			*2		
TT						

Table 5: Twist-3 PDFs.

Quark	$\gamma^-$		$\gamma^-\gamma_s$		$\sigma^-$	
	T-even	T-odd	T-even	T-odd	T-even	T-odd
U	$f_3$					
L			$g_{3L}$			
T					$[h_{3T}]$	
LL	$f_{3LL}$					
LT						*4
TT						

Table 6: Twist-4 PDFs.

We obtain a sum rule by integrating this twist-2 relation over  $x$  as

$$\int_0^1 dx f_{2LT}^+(x) = 0. \quad (14)$$

These are analogous equations to the WW relation and BC sum rule. The function  $f_{1LL}$  is given by the tensor-polarized structure function  $b_1$  as  $-(3/2)f_{1LL}^+ = b_1^q + b_1^{\bar{q}}$ . If the tensor-polarized antiquark distributions vanish, the parton-model sum rule  $\int dx b_1 = 0$  exists [5]. Then, there is a sum rule for the  $f_{LT}$  itself as  $\int_0^1 dx f_{LT}^+(x) = 0$ .

## 4 Conclusion

We proposed new TMD structure functions at twist 3 and 4 by decomposing the TMD correlation function in the Lorentz invariant way with constraints of Hermiticity and parity invariance. There are 40 TMDs for the tensor-polarized spin-1 hadron up to twist 4, and we found new 30 TMDs at twist 3 and 4. Then, we showed sum rules for the time-reversal-odd TMDs. There exist also corresponding fragmentation functions simply changing kinematical variables and function names. Integrating the TMDs over the transverse momentum, we obtained new collinear PDFs  $e_{LL}$ ,  $f_{LT}$ , and  $f_{3LL}$  at twist 3 and 4, in addition to the twist-2 function  $f_{1LL}$  ( $b_1$ ). We also indicated the twist-2 relation and the sum rule for  $f_{LT}$  in analogy to the Wandzura-Wilczek relation and the Burkhardt-Cottingham sum rule.

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