Two-Loop renormalization of High-Dimensional QCD operators and Higgs EFT amplitudes

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Abstract

We consider two-loop renormalization of high-dimensional operators in QCD and the relevant Higgs EFT amplitudes. Efficient unitarity-IBP strategy is used to compute the two-loop minimal form factors of length-3 operators up to dimension sixteen. From the ultraviolet divergences of form factor results, we extract the renormalization matrices and compute anomalous dimensions. We also obtain the analytic finite remainder functions which exhibit several universal transcendentality structures. The form factors we compute are equivalent to Higgs plus three-gluon amplitudes that capture high-order top mass corrections in Higgs effective field theory.

Contents

1 Introduction

Gauge invariant operators play important roles in QFT. For example, they correspond to composite states such as color-singlet hadrons in QCD and also appear as effective interaction vertices in effective field theories (EFT). At classical level, an important problem is to find a set of independent basis for the operators of a certain canonical dimension. At quantum level, the operators receive quantum loop corrections and it is important to perform renormalization, where the canonical dimensions are modified by anomalous dimensions. Further problems include computing scattering amplitudes in EFTs where the operators need to be taken into account as well.

In this report, we will address all these problems by considering gauge invariant local operators which are composed of field strength $F_{\mu\nu}$ and covariant derivatives D_μ. The field strength carries a color index as $F_{\mu\nu} = F_{\mu\nu}^a T^a$, where T^a are the adjoint generators of gauge group and satisfy

$$[T^a, T^b] = if^{abc}T^c.\tag{1}$$

The covariant derivative acts in the standard way as

$$D_\mu \star = \partial_\mu \star + ig[A_\mu, \star], \qquad [D_\mu, D_\nu] \star = ig[F_{\mu\nu}, \star].\tag{2}$$

A gauge invariant scalar operator can be written in the following general form:

$$\mathcal{O}(x) \sim c(a_1, ..., a_n)X(\eta^{\mu\nu})\big(D_{\mu_{11}}...D_{\mu_{1m_1}}F_{\nu_1\rho_1}\big)^{a_1}\cdots\big(D_{\mu_{n1}}...D_{\mu_{nm_n}}F_{\nu_n\rho_n}\big)^{a_n}(x),\tag{3}$$

where $c(a_1, ..., a_n)$ are color factors (*e.g.* given in terms of $\text{Tr}(..T^{a_i}..T^{a_j}..)$). All Lorentz indices $\{\mu_i, \nu_i, \rho_i\}$ are contracted in pairs by metric $\eta^{\mu\nu}$ contained in the function $X(\eta)$. These operators form composite color-singlet states in QCD, and they are also related to the Higgs EFT, which is obtained in the gluon fusion process by integrating the heavy top quark [1–4].

To study the aforementioned problems associated to these operators, a useful observable to consider is the form factor defined as (see *e.g.* [5] for an introduction):

$$\mathcal{F}_{\mathcal{O},n}(1, ..., n; q) \equiv \int d^D x e^{-iq\cdot x}\langle 1...n|\mathcal{O}(x)|0\rangle,\tag{4}$$

where p_i are momenta for on-shell states and $q = \sum_i p_i$ is an off-shell momentum associated to the operator. The form factors allow to apply modern on-shell amplitude techniques to study "off-shell" operators, for both constructing (classical) operator bases and for computing high-loop quantum corrections. In the remaining sections, we would like to report some recent progress on these problems, mainly based on [6, 7].

2 Operator basis

The operators at a given canonical dimension in general are not independent with each other, because they can be related to each other through equations of motion (EoM) or Bianchi identities (BI):

$$\text{EoM}: \quad D_\mu F^{\mu\nu} = 0, \qquad\qquad \text{BI}: \quad D_\mu F_{\nu\rho} + D_\nu F_{\rho\mu} + D_\rho F_{\mu\nu} = 0.\tag{5}$$

Our goal in this section is to find a set of independent operators in the sense that there are no above relations among the operators.

For simplicity, we will focus on length-2 and length-3 operators, in which there are only 2 and 3 $F_{\mu\nu}$ fields respectively, plus arbitrary insertion of covariant derivatives. Two operators

operator	D_μ	$F_{\mu\nu}$
kinematics	p_μ	$p_\mu\varepsilon_\nu - p_\nu\varepsilon_\mu$

(a) *D-dimensional dictionary*

operator	$D_{\dot{\alpha}\alpha}$	$f_{\alpha\beta}$	$\bar{f}_{\dot{\alpha}\dot{\beta}}$
spinor	$\tilde{\lambda}_{\dot{\alpha}}\lambda_\alpha$	$\lambda_\alpha\lambda_\beta$	$-\tilde{\lambda}_{\dot{\alpha}}\tilde{\lambda}_{\dot{\beta}}$

(b) *Four-dimensional dictionary*

Table 1: Dictionary between operators and form factors in both *D* and 4 dimensions.

will be set equivalent if their difference can be written in terms of high length operators. For example, one can show that $D_{12...}F_{34}D_{12...}F_{34}$ and $\frac{1}{2}\partial_1^2\big(D_{2...}F_{34}D_{2...}F_{34}\big)$ are equivalent up to length-3 operators. Given this criterion, there is only one independent length-2 operator at each even dimension: $(\partial)^{\Delta_0-4}\text{tr}(F^{\mu\nu}F_{\mu\nu})$.

For constructing the basis of length-3 operators, one can introduce two *primitive operators*:

$$\mathcal{O}^{\text{P1}} = \text{Tr}(D_1F_{23}D_4F_{23}F_{14}) = \frac{1}{2}f^{abc}(D_1F_{23})^a(D_4F_{23})^b(F_{14})^c,$$
$$\mathcal{O}^{\text{P2}} = \text{Tr}(F_{12}F_{13}F_{23}) = \frac{1}{2}f^{abc}(F_{12}^aF_{13}^bF_{23}^c), \tag{6}$$

where for short notations, we use integer numbers to represent Lorentz indices and also often abbreviate $D_iD_j....$ as $D_{ij...}$, for example, $F^{\mu_1\mu_2}D_{\mu_1}D_{\mu_5}F^{\mu_3\mu_4}D_{\mu_2}D^{\mu_5}F_{\mu_3\mu_4} \to F_{12}D_{15}F_{34}D_{25}F_{34}$. The "primitive" is in the following sense that: all higher dimensional operators can be constructed by inserting pairs of covariant derivatives $\{D^\mu, D_\mu\}$ in primitive operators. (Note that for the length-2 case there is a single primitive operator $\text{tr}(F^{\mu\nu}F_{\mu\nu})$.)

As a concrete example, let us consider operators of dimension-10, which will serve as major examples in the following discussion. One can construct basis operators by inserting *DD* pair in the primitive operators (6). There are five inequivalent ways of adding *DD* pairs which provide 5 independent length-3 operators:

<p style="text-align:center">Example: Dim-10 basis (Form I)</p>

$$\mathcal{O}''_{10;1} = \text{Tr}(D_{12}F_{34}D_{15}F_{34}F_{25}), \; \mathcal{O}''_{10;2} = \text{Tr}(D_{12}F_{34}D_5F_{34}D_1F_{25}), \; \mathcal{O}''_{10;3} = \text{Tr}(D_2F_{34}D_{15}F_{34}D_1F_{25}),$$
$$\mathcal{O}''_{10;4} = \text{Tr}(D_{12}F_{34}D_1F_{35}D_2F_{45}), \; \mathcal{O}''_{10;5} = \text{Tr}(D_{12}F_{34}D_{12}F_{35}F_{45}), \tag{7}$$

where $\mathcal{O}''_{10;1}$ to $\mathcal{O}''_{10;3}$ are obtained by adding a single *DD* pair to $\mathcal{O}^{\text{P1}}$, and $\mathcal{O}''_{10;4}, \mathcal{O}''_{10;5}$ are obtained by inserting two *DD* pairs to $\mathcal{O}^{\text{P2}}$.

For the convenience of loop computations, it is important to simplify the operator basis such that they can manifest certain symmetry properties. This can be achieved by considering the so-called *minimal* tree-level form factors, for which the number of external gluons is equal to the length of the operator. In particular, one can establish a dictionary from an operator to its tree-level minimal form factor [8–10]:

$$\mathcal{O}_L \iff \mathcal{F}_{\mathcal{O},L}(1,\dots,L), \tag{8}$$

where each *D* and *F* in the operator are mapped to the kinematics in form factor via Table 1.

With the help of minimal form factors, we can decompose operators according to the color structure and helicity structure. There are two color factors for length-3 minimal form factors:

$$f^{abc} = \text{Tr}(T^aT^bT^c) - \text{Tr}(T^aT^cT^b), \qquad d^{abc} = \text{Tr}(T^aT^bT^c) + \text{Tr}(T^aT^cT^b), \tag{9}$$

and the corresponding operators associated to these two factors will be said in *f*-sector and *d*-sector respectively. Furthermore, the minimal form factors have two helicity sectors:

$$\alpha\text{-sector}: \quad \mathcal{F}_{\mathcal{O}}^{(0),\text{min}} \neq 0 \text{ only for } (-,-,+),(+,+,-),$$
$$\beta\text{-sector}: \quad \mathcal{F}_{\mathcal{O}}^{(0),\text{min}} \neq 0 \text{ only for } (-,-,-),(+,+,+). \tag{10}$$

We summarize the operators in different sectors according the form factor structure as

Operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{\alpha;f}$	$\propto A_1$	0	f^{abc}
$\mathcal{O}_{\alpha;d}$	$\propto A_1$	0	d^{abc}
$\mathcal{O}_{\beta;f}$	0	$\propto A_2$	f^{abc}
$\mathcal{O}_{\beta;d}$	0	$\propto A_2$	d^{abc}

(11)

where the two helicity sectors are associated to two spinor factors:

$$A_1 = \langle 12 \rangle^3 [13][23], \qquad A_2 = \langle 12 \rangle \langle 13 \rangle \langle 23 \rangle. \tag{12}$$

As examples, let us consider again the dim-10 operators. Firstly, by properly (anti)symmetrizing the set of basis (7), one can write the operators in different color sectors as

<p align="center" style="color:blue">Example: Dim-10 basis (Form II)</p>

$$\mathcal{O}'_{10;1} = \frac{1}{2} f^{abc} (D_{12} F_{34})^a (D_{15} F_{34})^b (F_{25})^c, \quad \mathcal{O}'_{10;2} = f^{abc} (D_{12} F_{34})^a (D_5 F_{34})^b (D_1 F_{25})^c,$$

$$\mathcal{O}'_{10;3} = d^{abc} (D_{12} F_{34})^a (D_5 F_{34})^b (D_1 F_{25})^c, \tag{13}$$

$$\mathcal{O}'_{10;4} = \frac{1}{2} f^{abc} (D_{12} F_{34})^a (D_1 F_{35})^b (D_2 F_{45})^c, \quad \mathcal{O}'_{10;5} = \frac{1}{2} f^{abc} (D_{12} F_{34})^a (D_{12} F_{35})^b (F_{45})^c.$$

Furthermore, by investigating the spinor factors of minimal form factors, the operators can be put in different helicity sectors via certain linear combinations. We summarize the final dimension-10 length-3 basis operators that will be used for loop computation as

<p align="center" style="color:blue">Example: Dim-10 basis (Form III)</p>

Basis operator	$\mathcal{F}^{(0)}(-,-,+)$	$\mathcal{F}^{(0)}(-,-,-)$	color factor
$\mathcal{O}_{10;\alpha;f;1} = \frac{1}{2}\partial^2 \mathcal{O}_{P1} - \frac{1}{12}\partial^4 \mathcal{O}_{P2}$	$\frac{1}{2}s_{123}A_1$	0	f^{abc}
$\mathcal{O}_{10;\alpha;f;2} = \mathcal{O}'_{10;1} - \mathcal{O}'_{10;5}$	$\frac{1}{2}s_{12}A_1$	0	f^{abc}
$\mathcal{O}_{10;\alpha;d;1} = \mathcal{O}'_{10;3}$	$\frac{1}{2}(s_{13} - s_{23})A_1$	0	d^{abc}
$\mathcal{O}_{10;\beta;f;1} = \frac{1}{12}\partial^4 \mathcal{O}_{P2}$	0	$\frac{1}{4}s_{123}^2 A_2$	f^{abc}
$\mathcal{O}_{10;\beta;f;2} = \mathcal{O}'_{10;5}$	0	$\frac{1}{4}(s_{12}^2 + s_{23}^2 + s_{13}^2)A_2$	f^{abc}

(14)

3 Loop computation via unitarity-IBP

To compute the loop form factors for the basis operators, we apply the unitarity-IBP strategy that combines unitarity cut [11–13] and integration by parts (IBP) methods [14, 15] (using public packages *e.g.* [16–19]). The work flow of our strategy can be illustrated as follows:

$$\mathcal{F}^{(l)}\Big|_{\text{cut}} = \prod (\text{Tree blocks}) = \text{Cut integrand} \xrightarrow{\text{IBP with cuts}} \sum_i c_i \left(I_i \big|_{\text{cut}} \right). \tag{15}$$

This strategy has been used to study form factors and Higgs amplitudes in [6, 7, 20, 21] and for pure gluon amplitudes in [22, 23]. Similar strategy has been used in the numerical unitarity approach [24, 25], and the idea of applying cuts to simplify IBP has also been used in *e.g.* [26–29].

The complete set of two-loop master integrals for minimal length-3 form factors are given in Fig. 1. The two-loop color-ordered form factors, associated with color factor $\text{tr}(T^{a_1} T^{a_2} T^{a_3})$ via color decomposition, can be written as a sum of master integrals I_i as

$$F_{\mathcal{O}}^{(2)} = \Big[c_1 I_1 + c_2 I_2 + c_3 I_3 + c_4 I_4 + \big(c_5 I_5 + c_6 I_6 \big) + \big(c_7 I_7 + c_8 I_8 \big) + c_9 I_9 \Big] + \text{cyc.perm.}, \tag{16}$$
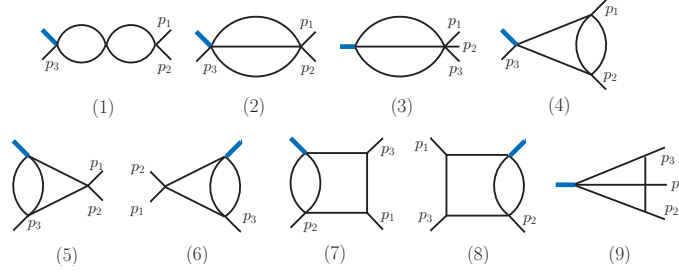
Figure 1: Master integrals for the planar two-loop minimal form factors.
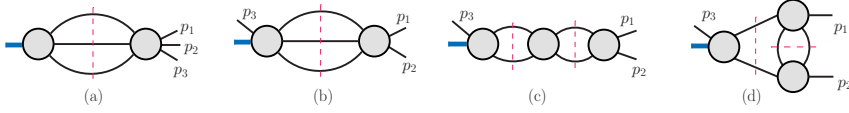

Figure 2: Complete set of unitarity cuts for the planar two-loop form factors.

where master integrals I_i correspond to the topology and labeling given in Fig. 1. The master coefficients c_i are what to be computed. The spanning set of cuts used to fix all coefficients are given in Fig. 2. Note that the two-loop minimal form factors of length-3 operators have no sub-leading-color contribution, thus the set of planar cuts are enough to fix the full results. More details can be found in [6].

4 Results and analysis

The master integrals in Fig. 1 are known in terms of 2d harmonic polylogarithms [30, 31]. Together with IBP coefficients, the form factors can be written in explicit functional form, from which one can extract the wanted physical information.

The bare form factors contain divergences and can be schematically expanded as:

$$\text{Loop form factor} = (\text{Universal IR div.}) + (\text{UV div.}) + (\text{Finite part}), \tag{17}$$

where the infrared (IR) divergences depend only on the configuration of external on-shell states, while the UV divergences are related to the operator and coupling renormalization.

Operator renormalization

In dimensional regularization, both IR and UV divergences are regularized by $\epsilon = (4-D)/2$, and it may seem non-trivial to disentangle the two divergences. Fortunately, this problem can be easily solved, thanks to the universal structure of IR divergences. In particular, the two-loop IR can be obtained by the Catani form [32], which is determined by the one-loop form factor together with some universal functions independent of operarors.

After subtracting IR divergences, the obtained UV divergences can be eliminated by performing operator renormalization. The renormalization constant Z in general takes a *matrix* form as:

$$\mathcal{O}_{R,i} = Z_i{}^j \mathcal{O}_{B,j}, \tag{18}$$

since different operators in the same basis can generally mix with each other under renormalization. From the renormalization constant, one can further define the dilation operator as

$$\mathbb{D} = -\frac{d \log Z}{d \log \mu}. \tag{19}$$

5

Finally, the eigenvalues of the dilatation operator give the anomalous dimensions.

We consider again the dimension-10 operators in (14). Separated in f and d sectors, the renormalization matrices at one loop are

$$
Z^{(1)}_{\mathcal{O}_{10,f}} = \frac{N_c}{\epsilon}
\begin{pmatrix}
-\frac{11}{3} & 0 & 0 & 0 & 0 \\
\hline
0 & \frac{7}{6} & 0 & 0 & 0 \\
0 & -\frac{3}{5} & \frac{71}{30} & 0 & 0 \\
\hline
0 & 0 & 0 & \frac{1}{2} & 0 \\
0 & 0 & 0 & -1 & \frac{17}{6}
\end{pmatrix}, \qquad
Z^{(1)}_{\mathcal{O}_{10;d}} = \frac{N_c}{\epsilon}\frac{13}{6},
\tag{20}
$$

where we have included the length-2 operator in the f-sector and the basis is chosen as $\{\partial^6 \mathrm{tr}(F^2), \mathcal{O}_{10;\alpha;f;1}, \mathcal{O}_{10;\alpha;f;2}, \mathcal{O}_{10;\beta;f;1}, \mathcal{O}_{10;\beta;f;2}\}$. For the two-loop renormalization matrices, the $1/\epsilon^2$ part is determined by one-loop results, and the intrinsic new information is contained in the $1/\epsilon$ part, given as

$$
Z^{(2)}_{\mathcal{O}_{10,f}}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{N_c^2}{\epsilon}
\begin{pmatrix}
-\frac{34}{3} & 0 & 0 & 0 & 0 \\
-\frac{1}{3} & \frac{269}{72} & 0 & \frac{5}{2} & 0 \\
-\frac{209}{900} & -\frac{5579}{18000} & \frac{712}{125} & \frac{1493}{1200} & \frac{5}{36} \\
\hline
-1 & 0 & 0 & \frac{25}{12} & 0 \\
-\frac{19}{36} & \frac{139}{2400} & \frac{499}{800} & -\frac{143}{288} & \frac{2195}{288}
\end{pmatrix},
\quad
Z^{(2)}_{\mathcal{O}_{10;d}}\Big|_{\frac{1}{\epsilon}\text{-part.}} = \frac{575}{144}\frac{N_c^2}{\epsilon}.
\tag{21}
$$

From them it is straightforward to compute the dilation operator as well as anomalous dimensions, see [6] for more detail and further results up to dimension 16. (See also some previous one-loop results on high dimensional operators in [33–36].)

Finite remainder

The finite remainder parts of the form factors capture important information for the Higgs-plus-three-gluon amplitudes. The two-loop form factor with the leading operator $\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu})$ was computed in [37], and the results with high dimension operators correspond to Higgs amplitudes with high order of top mass corrections [6, 7, 20, 21] and can be used to improved the precision for the cross section of Higgs plus a jet production. Below we briefly discuss some interested features of their analytic structure.

One can decompose the two-loop remainder functions according to their trancendentality degree as:

$$
\mathcal{R}^{(2)}_{\mathcal{O}} = \sum_{n=0}^{4} \mathcal{R}^{(2)}_{\mathcal{O}}\Big|_{\text{deg-}n},
\tag{22}
$$

where the maximal transcendentality degree is 4 at two loops. Here "transcendental degree" is a mathematical notation characterizing the algebraic complexity of functions and numbers. For example, the degree of rational functions is zero, π or $\log(x)$ has degree 1, and the Riemann zeta value ζ_n or polylogrithm $\mathrm{Li}_n(x)$ has degree n.

The maximally transcendental part turns out to be universally given by the function (see also [20, 21, 38–40]):

$$
\mathcal{R}^{(2)}_{\mathcal{O}}\Big|_{\text{deg-4}} = -\frac{3}{2}\mathrm{Li}_4(u) + \frac{3}{4}\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{4}\log(w)\left[\mathrm{Li}_3\left(-\frac{u}{v}\right) + \mathrm{Li}_3\left(-\frac{v}{u}\right)\right]
$$
$$
+ \frac{\log^2(u)}{32}\left[\log^2(u) + \log^2(v) + \log^2(w) - 4\log(v)\log(w)\right]
$$
$$
+ \frac{\zeta_2}{8}\left[5\log^2(u) - 2\log(v)\log(w)\right] - \frac{1}{4}\zeta_4 + \mathrm{perms}(u,v,w),
\tag{23}
$$

where the scalar ratios

$$u = \frac{s_{12}}{s_{123}}, \qquad v = \frac{s_{23}}{s_{123}}, \qquad w = \frac{s_{13}}{s_{123}} . \qquad (24)$$

This suggests that the two-loop minimal form factors of general length-3 operators obey the maximal transcendentality principle [41–43].

Moreover, lower degree parts also present universal structures. The degree-3 part are determined by the function $T_3(u, v, w)$ given as

$$\begin{aligned}
T_3(u, v, w) := &\Big[-\mathrm{Li}_3\Big(-\frac{u}{w}\Big) + \log(u)\mathrm{Li}_2\Big(\frac{v}{1-u}\Big) - \frac{1}{2}\log(u)\log(1-u)\log\Big(\frac{w^2}{1-u}\Big) \\
&+ \frac{1}{2}\mathrm{Li}_3\Big(-\frac{uv}{w}\Big) + \frac{1}{2}\log(u)\log(v)\log(w) + \frac{1}{12}\log^3(w) + (u \leftrightarrow v)\Big] \\
&+ \mathrm{Li}_3(1-v) - \mathrm{Li}_3(u) + \frac{1}{2}\log^2(v)\log\Big(\frac{1-v}{u}\Big) - \zeta_2 \log\Big(\frac{uv}{w}\Big),
\end{aligned} \qquad (25)$$

together with simple $\pi^2 \log$ and ζ_3. Building blocks of degree-2 part are $T_2(u, v)$:

$$T_2(u, v) := \mathrm{Li}_2(1-u) + \mathrm{Li}_2(1-v) + \log(u)\log(v) - \zeta_2, \qquad (26)$$

together with $\log^2$ and π^2. When expanding the remainders in these deg-3 and deg-2 building blocks, the coefficients in front of them are just rational functions of u, v, w. Similar transcendental functions have appeared in the $\mathcal{N} = 4$ form factors [38, 44, 45].

5   Conclusion

We construct the operator basis for high dimensional QCD operators which are also related to Higgs EFT amplitudes. The efficient on-shell unitarity-IBP method is applied to compute the loop form factors associated with these operators. Based on the loop form factor results, the two-loop renormalization are performed for operators up to dimension 16, and the finite remainder functions are also obtained. We can summarize the main steps and results in the following graph:
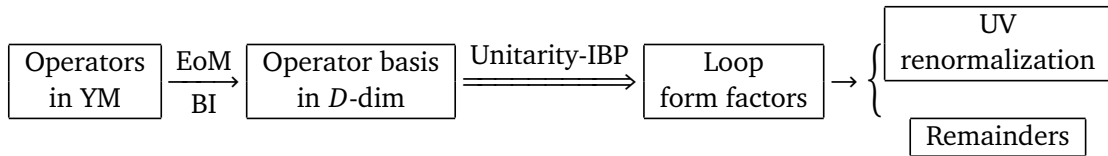
Although our discussion has mostly focused on the length-3 operators, we would like to stress that similar methods can be generalized to deal with more general high length operators. Since our method works in full D dimensions in dimensional regularization, the strategy can be also used to study the so-called evanescent operators. We hope to report these studies in the near future [46].

Acknowledgements

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