

Eikonal Dressed Gluon Exponentiation to study power corrections to Thrust, C -parameter, and Angularity

Neelima Agarwal¹, Ayan Mukhopadhyay², Sourav Pal^{2*}, and Anurag Tripathi²

¹ Department of Physics, Chaitanya Bharathi Institute of Technology, Gandipet, Hyderabad, Telangana State 500075, India

² Department of Physics, Indian Institute of Technology Hyderabad, Kandi, Sangareddy, Telangana State 502285, India

* Corresponding Author, spalexam@gmail.com

October 29, 2021



*15th International Symposium on Radiative Corrections:
Applications of Quantum Field Theory to Phenomenology,
FSU, Tallahassee, FL, USA, 17-21 May 2021
doi:10.21468/SciPostPhysProc.?*

Abstract

Dressed Gluon Exponentiation is a well known method to study power corrections. We present here its eikonalized version – Eikonal Dressed Gluon Exponentiation (EDGE) [1] to determine the dominant power corrections to shape variables such as Thrust, C -parameter, and Angularity. Our method remarkably simplifies the calculations.

Contents

1 Introduction	1
2 Dressed gluon exponentiation	2
3 Eikonal Dressed gluon exponentiation	4
3.1 EDGE in energy fractions	4
3.2 EDGE in light-cone variables	5
4 Exponentiation	6
5 Conclusion	7
References	7

1 Introduction

Event shape variables are classical tools for precise determination of the strong coupling constant [2–6] from collider data, and are useful while constructing the models for hadronization.

Due to the infrared safety of the event shape variables, they can be calculated in the perturbation theory. The state of the art is NNLO calculations at fixed orders [7–11], while NLL [12–15], and NNLL resummation [16–27] frameworks have been developed over several years. If one denotes the event shape variable by e , then its corresponding distribution $d\sigma/de$ peaks in the two jet limit ($e \rightarrow 0$). In this particular limit the distributions are affected by the non-perturbative power corrections of order $(\Lambda/eQ)^n$ and $(\Lambda^2/eQ^2)^n$, arising from the soft gluon and the collinear gluon corrections respectively. Here, we discuss the analytic non-perturbative corrections, which are suppressed by powers of (Λ/Q) as compared to the perturbative corrections.

A generic observable σ in perturbative QCD is a sum of perturbative corrections and the power corrections,

$$\sigma\left(\frac{Q}{\mu}, \alpha_s\right) = \sigma_{\text{pert}}\left(\frac{Q}{\mu_f}, \frac{\mu_f}{\mu}, \alpha_s\right) + \sum_n \sigma_n\left(\frac{\mu_f}{\mu}, \alpha_s\right) \left(\frac{\mu_f}{Q}\right)^n, \quad (1)$$

where μ_f is the factorization scale, and μ is the renormalization scale. The perturbative part of the observable shows a factorial growth order by order in the strong coupling constant α_s , which implies that the perturbative series is divergent. One of the known techniques to sum a series with factorial growth is Borel summation, and upon performing Borel summation the factorial divergences appear as singularities in the Borel plane. We can avoid these singularities by contour deformation, however, the ambiguity in choosing the direction of the contour deformation results in ambiguous perturbative results. These ambiguities can be studied efficiently by studying the power corrections of these observables [28]. For event shape distributions at LEP, the soft power corrections (corrections due to soft gluons) are dominant over the collinear power corrections (corrections due to collinear gluons). These soft power corrections are so dominant at the scale $e \approx \Lambda/Q$ that we require resummation in order to make a stable prediction. However, the collinear corrections become relevant at smaller values of $e \approx (\Lambda/Q)^2$, which are generally out of experimental range. Thus, in this paper, we only discuss the soft power corrections.

An elegant and efficient way to deal with the perturbative logarithms and the non perturbative power corrections is Dressed gluon exponentiation (DGE) [29]. In this paper, we aim to calculate only the dominant soft power corrections using the Dressed gluon exponentiation. We merge the eikonal approximation for the emission of the soft gluons with the well known Dressed gluon exponentiation, and we call the resulting method eikonal Dressed gluon exponentiation (EDGE) [1].

The paper is structured as follows: section 2 reviews the basic concepts about Dressed gluon exponentiation, section 3 shows how EDGE can be used in terms of two different kinematic variables such as energy fractions and light cone co-ordinates to calculate the soft power corrections, and section 4 describes the calculation of Sudakov exponent by taking the Laplace transformation of the Borel function derived in section 3.

2 Dressed gluon exponentiation

The event shape distribution in the single dressed gluon approximation serves as the starting point for DGE, which is built from the one-loop real emission contribution to the event shape for a gluon with virtuality $k^2 \neq 0$. The renormalon resummed event shape distribution, which dominates in the large N_f limit, is given by [28],

$$\frac{1}{\sigma} \frac{d\sigma}{de}(e, Q^2) = -\frac{C_F}{2\beta_0} \int_0^1 d\xi \frac{d\mathcal{F}(e, \xi)}{d\xi} A(\xi Q^2), \quad (2)$$

where $\beta_0 = \frac{11}{12}C_A - \frac{1}{6}N_f$, $\xi = k^2/Q^2$, and $A(\xi Q^2)$ is the large- β_0 running coupling ($A = \beta_0 \alpha_s / \pi$) on the time-like axis. In the $\overline{\text{MS}}$ scheme, the Borel representation of $A(\xi Q^2)$ has the form,

$$A(\xi Q^2) = \int_0^\infty du (Q^2/\Lambda^2)^{-u} \frac{\sin \pi u}{\pi u} e^{\frac{5}{3}u} \xi^{-u}. \quad (3)$$

Now, if we denote the squared matrix element as \mathcal{M} , then we can write down the characteristic function $\mathcal{F}(e, \xi)$ as [30, 31],

$$\mathcal{F}(e, \xi) = \int dx_1 dx_2 \mathcal{M}(x_1, x_2, \xi) \delta(e - \bar{e}(x_1, x_2, \xi)), \quad (4)$$

where x_i 's are the energy fractions, and \bar{e} is the expression of the event shape in terms of the kinematic variables. The Borel representation of cross-section distribution is obtained by interchanging the order of integration of eq. (2), and it takes the form,

$$\frac{1}{\sigma} \frac{d\sigma}{de}(e, Q^2) = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B(e, u), \quad (5)$$

where the Borel function for single dressed gluon $B(e, u)$ is defined by,

$$B(e, u) = -\frac{\sin \pi u}{\pi u} e^{\frac{5}{3}u} \int_0^\infty d\xi \xi^{-u} \frac{d\mathcal{F}(e, \xi)}{d\xi}. \quad (6)$$

This Borel function $B(e, u)$ is free from any renormalon singularities in the u plane. However, when the Borel function for single dressed gluon is exponentiated via a Laplace transform, it generates the renormalon poles in the Borel plane, which we will discuss in section 4.

The exponentiation of the logarithmically enhanced terms in the Laplace space is guaranteed by the additive nature of the event shapes and the factorization of matrix elements for soft and collinear radiations. Then, the resummed cross-section in the Laplace space has the form,

$$\frac{1}{\sigma} \frac{d\sigma(e, Q^2)}{de} = \int_{C-i\infty}^{C+i\infty} \frac{d\nu}{2\pi i} e^{\nu e} \exp[S(\nu, Q^2)], \quad (7)$$

where C lies to the right of the singularities of the integrand. This exponentiation in the Laplace space effectively resums the Sudakov logarithms and the power correction terms in the two-jet region. The Sudakov exponent has the form [32],

$$S(\nu, Q^2) = \int_0^1 de \frac{1}{\sigma} \frac{d\sigma(e, Q^2)}{de} (e^{-\nu e} - 1). \quad (8)$$

This integral is dominant in $\nu \rightarrow \infty$, which corresponds to the Sudakov region $e \rightarrow 0$. Using Eq. (5), the Sudakov exponent is reduced to,

$$S(\nu, Q^2) = \frac{C_F}{2\beta_0} \int_0^\infty du (Q^2/\Lambda^2)^{-u} B_\nu^e(u), \quad (9)$$

where

$$B_\nu^e(u) = \int_0^1 de B(e, u) (e^{-\nu e} - 1). \quad (10)$$

3 Eikonal Dressed gluon exponentiation

In this section we describe Eikonal Dressed gluon exponentiation (EDGE) [1] in two well known kinematic variables: energy fractions, and the light-cone variables. The basic idea of EDGE is to calculate the Borel function by considering the eikonal approximated matrix element and the eikonal approximated version of the relevant event shape variable.

3.1 EDGE in energy fractions

In this subsection, we describe EDGE in the energy fraction variables x_i . The colour and coupling stripped off matrix element in the eikonal limit for the process $\gamma^* \rightarrow q\bar{q}g$ with a massive gluon is given by,

$$\mathcal{M}_{\text{soft}}(x_1, x_2, \xi) = \frac{2}{(1-x_1)(1-x_2)} \quad (11)$$

where,

$$x_1 = \frac{2p_1 \cdot Q}{Q^2}, \quad x_2 = \frac{2p_2 \cdot Q}{Q^2}, \quad x_3 = \frac{2k \cdot Q}{Q^2}, \quad \xi = \frac{k^2}{Q^2}. \quad (12)$$

Here p_1, p_2 , and k denote the momenta of the quark, anti-quark, and the gluon respectively, and $Q = p_1 + p_2 + k$, which fixes $x_3 = 2 - x_1 - x_2$.

The eikonal approximated versions of thrust (T), C -parameter (c), and the angularities (τ_a) are given by [1],

$$T = \text{Max} \left\{ x_1, x_2, \sqrt{x_3^2 - 4\xi} \right\}, \quad c_{\text{eik}}(x_1, x_2) = \frac{(1-x_1)(1-x_2)}{(1-x_1) + (1-x_2)},$$

$$\tau_a^{\text{eik}}(x_1, x_2, \xi) = (1-x_1)^{1-a/2} (1-x_2)^{a/2}. \quad (13)$$

Note that, the definition of thrust is so simple that it does not require any approximation in the eikonal limit, and we can use the full definition of thrust in EDGE. Also note that, we have used the modified definition [33] of C -parameter, which is given by $c = C/6$. Using the eikonal approximated event shapes and performing the integration over x_1, x_2 of eq. (4) by using the soft boundary of the phase space, we get the characteristic functions for thrust, C -parameter and angularities as,

$$\mathcal{F}(t, \xi) = -\frac{4}{t} \log \left(\frac{\xi}{t(q-t)} \right),$$

$$\mathcal{F}(c, \xi) = -\frac{4}{c} \log \left(\frac{\xi}{c} \right) - 8 \log \left(1 + \sqrt{1 - \frac{4c^2}{\xi}} \right) - 4 \log \left(\frac{1}{2} \right),$$

$$\mathcal{F}(\tau_a, \xi) = -\frac{4}{\tau_a} \frac{1}{1-a} \log \xi. \quad (14)$$

where $t = 1 - T$, and $q = \sqrt{T^2 + 4\xi}$. Now, using the results for characteristic functions and eq. (6), we proceed for the calculation of the Borel function for the emission of one gluon. The upper and the lower limit of the integration are determined from the collinear

($x_1 = 1 - \xi, x_2 = 0$) and the soft ($x_1 = x_2 = 1 - \sqrt{\xi}$) gluon boundary conditions respectively. The Borel functions for thrust, C -parameter, and the angularities are given by [1],

$$\begin{aligned}
B(t, u) &= \frac{\sin \pi u}{\pi u} e^{\frac{5u}{3}} \frac{4}{u} \frac{1}{t} \left(\frac{1}{t^{2u}} - \frac{1}{t^u} \right), \\
B(c, u) &= 4 \frac{\sin \pi u}{\pi u} e^{\frac{5u}{3}} \frac{1}{c} \left[\frac{1}{(2c)^{2u}} \frac{\sqrt{\pi} \Gamma(u)}{\Gamma(u + \frac{1}{2})} - \frac{1}{uc^u} \right], \\
B(\tau_a, u) &= \frac{\sin \pi u}{\pi u} e^{\frac{5}{3}u} \frac{4}{1-a} \frac{1}{u} \frac{1}{\tau_a} \left[\frac{1}{\tau_a^{2u}} - \frac{1}{\tau_a^{\frac{2u}{2-a}}} \right]. \tag{15}
\end{aligned}$$

The results for the characteristics functions and the Borel functions for thrust, C -parameter and angularities match with the leading singular terms of the same function presented in [32] for thrust, [33] for C -parameter, and [34] for angularities.

Thus, we have obtained the correct leading singular terms using the eikonal definitions of the event shape variables and the soft approximated version of the squared matrix element, and as mentioned in [1], the computation of these functions is substantially less complex in EDGE as compared to the full calculation using the traditional Dressed gluon exponentiation.

3.2 EDGE in light-cone variables

In this subsection, we will calculate the Borel function for thrust, C -parameter, and the angularities following the same steps of the previous subsection using the transverse momentum k_\perp and rapidity y of the massive gluon. In these variables, the eikonal approximated version of the three event shape variables considered in the previous section can be written as,

$$\bar{e}(k, Q) = \sqrt{\frac{k_\perp^2 + k^2}{Q^2}} h_e(y), \tag{16}$$

where k_\perp and y denote transverse momentum of the gluon and pseudo-rapidity measured with respect to the thrust axis respectively. For thrust (t), C -parameter (c), and angularities (τ_a), $h_e(y)$ takes the form,

$$h_t(y) = e^{-|y|}, \quad h_c(y) = \frac{1}{2 \cosh y}, \quad h_{\tau_a}(y) = e^{-|y|(1-a)}. \tag{17}$$

The matrix element for the emission of soft gluons can be computed easily by applying the eikonal Feynman rules. The fact that the soft phase space factorizes from the hard part reduces the soft cross-section to a very simple universal form,

$$\frac{d\sigma}{\sigma} = \frac{1}{3} \frac{4}{k^2 + k_\perp^2} dk_\perp^2 dy. \tag{18}$$

Now, using eq. (4) and (18) and integrating over the transverse momentum k_\perp , the characteristics function takes a very simple form [1],

$$\mathcal{F}(e, \xi) = \frac{8}{e} \int_{y_{\min}} dy. \tag{19}$$

Here, only the lower limit is relevant for soft power corrections. The lower limit for this integration is calculated using eq. (16) and (17), and for three event shape variables they are given by,

$$y_{\min}(t) = \log\left(\frac{1}{t}\sqrt{\xi}\right), \quad y_{\min}(c) = \cosh^{-1}\left(\sqrt{\xi}/2c\right),$$

$$y_{\min}(\tau_a) = \frac{1}{1-a} \log\left(\frac{1}{\tau_a}\sqrt{\xi}\right). \quad (20)$$

Using eq. (19) and (20), we get the characteristics functions for the thrust, C-parameter, and Angularities as,

$$\mathcal{F}(t, \xi) = -\frac{8}{t} \log\left(\frac{\sqrt{\xi}}{t}\right), \quad \mathcal{F}(c, \xi) = -\frac{8}{c} \cosh^{-1}\left(\frac{\sqrt{\xi}}{2c}\right),$$

$$\mathcal{F}(\tau_a, \xi) = -\frac{8}{\tau_a(1-a)} \log\left(\frac{\sqrt{\xi}}{\tau_a}\right), \quad (21)$$

which again agrees with the known results presented in [32–34] for the three event shape variables. Now, using eq. (6) one can calculate the Borel functions, which will be same as those presented in eq. (15).

4 Exponentiation

In this section, we will compute the Sudakov exponent for thrust. Similar conclusions can be made for the other two shape variables mentioned in the article. In the Sudakov region ($\nu \rightarrow \infty$), using $B(t, u)$ of eq. (15), and keeping only the logarithmic enhanced terms, the Borel function for thrust in Laplace space can be written as,

$$B_{\nu}^{t, \text{eik}}(u) = 2e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) \left(\nu^{2u} - 1\right) \frac{2}{u} - \Gamma(-u) \left(\nu^u - 1\right) \frac{2}{u} \right]. \quad (22)$$

Note that, this is free from any $u = 0$ singularities. The coefficient of $\Gamma(-2u)$ corresponds to large-angle soft gluon emissions, while the coefficient of $\Gamma(-u)$ corresponds to the collinear gluon emissions. However, this expression contains two types of poles: $\Gamma(-2u)$ has poles for all positive integers and half-integers, and $\Gamma(-u)$ has poles for all positive integers. The prefactor $\sin \pi u$ regulates the poles for the integer values of u .

We will now compare our result with the full result for B_{ν}^t presented in [33, 34] which is given by,

$$B_{\nu}^t(u) = 2e^{\frac{5}{3}u} \frac{\sin \pi u}{\pi u} \left[\Gamma(-2u) \left(\nu^{2u} - 1\right) \frac{2}{u} - \Gamma(-u) \left(\nu^u - 1\right) \left(\frac{2}{u} + \frac{1}{1-u} + \frac{1}{2-u}\right) \right]. \quad (23)$$

We note that $B_{\nu}^{t, \text{eik}}(u)$ does not contain $u = 1$ and $u = 2$ poles as compared to the full $B_{\nu}^t(u)$. We note that the poles related to the large angle soft gluon emissions are same in the approximated and the full result, thus leading logarithm terms are same for the full $B_{\nu}^t(u)$ and $B_{\nu}^{t, \text{eik}}(u)$. A detailed comparison of the leading and the sub-leading logarithms are discussed in [1]. It is important to note that EDGE does not produce any new spurious renormalon poles.

The poles in $B_{\nu}^t(u)$ in the real u -axis implies that the integral over u to determine the Sudakov exponent of eq. (9) is ill-defined and one needs to deform the contour to evaluate

this integral. The deformation of the contour around the real u -axis introduces an ambiguity in the result, which can be determined by calculating the residue of $B_{\nu}^t(u)$ at the poles. The sizes of these residues determine the amount of the power corrections. It is evident from the expression of $B_{\nu}^t(u)$, that the power corrections due to soft gluons will arise from the poles at $u = m/2$, whereas the collinear power corrections will arise from the poles at $u = 1$, and $u = 2$. It was discussed in [1, 32–34] that the soft power corrections are dominant over the collinear corrections. For example, at LEP, the collinear power corrections are 1000 times smaller than soft corrections [1]. As EDGE correctly reproduces the poles related to the emission of large angle soft gluons, thus we can determine the soft power corrections correctly using EDGE.

The dominance of the soft power corrections is true for all the event shapes considered in this paper. Thus the dominant soft power correction for other variables considered in this paper can be calculated using the same steps.

5 Conclusion

In this paper, we have discussed EDGE, developed in [1], based on the well known Dressed gluon exponentiation. We have demonstrated that the soft power corrections can be calculated using the eikonal squared matrix element, together with the eikonal version of the relevant event shape variable. We have also observed that EDGE does not generate any new spurious renormalon poles in the Sudakov exponent. We believe that, as compared to traditional Dressed Gluon Exponentiation, EDGE is simple enough and can be extended to the hadronic event shape variables, which are relevant at the LHC.

Acknowledgements

SP would like to thank the organisers of RADCOR-LoopFest 2021. SP and AM would like to thank MHRD Govt. of India for an SRF fellowship. AT would like to thank Lorenzo Magnea for suggesting this project and for very fruitful discussions, Einan Gardi for very useful discussions, and the University of Turin and INFN Turin for warm hospitality during the course of this work.

References

- [1] N. Agarwal, A. Mukhopadhyay, S. Pal and A. Tripathi, *Power corrections to event shapes using eikonal dressed gluon exponentiation*, JHEP **03**, 155 (2021), doi:[10.1007/JHEP03\(2021\)155](https://doi.org/10.1007/JHEP03(2021)155), [2012.06842](https://arxiv.org/abs/2012.06842).
- [2] R. Abbate, M. Fickinger, A. H. Hoang, V. Mateu and I. W. Stewart, *Thrust at N^3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$* , Phys.Rev. **D83**, 074021 (2011), doi:[10.1103/PhysRevD.83.074021](https://doi.org/10.1103/PhysRevD.83.074021), [1006.3080](https://arxiv.org/abs/1006.3080).
- [3] A. H. Hoang, D. W. Kolodrubetz, V. Mateu and I. W. Stewart, *Precise determination of α_s from the C -parameter distribution*, Phys. Rev. D **91**(9), 094018 (2015), doi:[10.1103/PhysRevD.91.094018](https://doi.org/10.1103/PhysRevD.91.094018), [1501.04111](https://arxiv.org/abs/1501.04111).
- [4] S.-Q. Wang, S. J. Brodsky, X.-G. Wu, J.-M. Shen and L. Di Giustino, *Novel method for the precise determination of the QCD running coupling from event shape distributions in electron-positron annihilation*, Phys. Rev. D **100**(9), 094010 (2019), doi:[10.1103/PhysRevD.100.094010](https://doi.org/10.1103/PhysRevD.100.094010), [1908.00060](https://arxiv.org/abs/1908.00060).

- [5] S. Marzani, D. Reichelt, S. Schumann, G. Soyez and V. Theeuwes, *Fitting the Strong Coupling Constant with Soft-Drop Thrust*, JHEP **11**, 179 (2019), doi:[10.1007/JHEP11\(2019\)179](https://doi.org/10.1007/JHEP11(2019)179), [1906.10504](https://arxiv.org/abs/1906.10504).
- [6] T. Gehrmann, G. Luisoni and P. F. Monni, *Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution*, Eur. Phys. J. C **73**(1), 2265 (2013), doi:[10.1140/epjc/s10052-012-2265-x](https://doi.org/10.1140/epjc/s10052-012-2265-x), [1210.6945](https://arxiv.org/abs/1210.6945).
- [7] A. Gehrmann-De Ridder, T. Gehrmann, E. Glover and G. Heinrich, *NNLO corrections to event shapes in e^+e^- annihilation*, JHEP **0712**, 094 (2007), doi:[10.1088/1126-6708/2007/12/094](https://doi.org/10.1088/1126-6708/2007/12/094), [0711.4711](https://arxiv.org/abs/hep-th/0711.4711).
- [8] S. Weinzierl, *Event shapes and jet rates in electron-positron annihilation at NNLO*, JHEP **0906**, 041 (2009), doi:[10.1088/1126-6708/2009/06/041](https://doi.org/10.1088/1126-6708/2009/06/041), [0904.1077](https://arxiv.org/abs/hep-th/0904.1077).
- [9] T. Gehrmann, A. Huss, J. Mo and J. Niehues, *Second-order QCD corrections to event shape distributions in deep inelastic scattering*, Eur. Phys. J. C **79**(12), 1022 (2019), doi:[10.1140/epjc/s10052-019-7528-3](https://doi.org/10.1140/epjc/s10052-019-7528-3), [1909.02760](https://arxiv.org/abs/1909.02760).
- [10] A. Kardos, G. Somogyi and A. Verbytskyi, *Determination of α_s beyond NNLO using event shape moments* (2020), [2009.00281](https://arxiv.org/abs/2009.00281).
- [11] A. Gehrmann-De Ridder, T. Gehrmann, E. Glover and G. Heinrich, *NNLO moments of event shapes in e^+e^- annihilation*, JHEP **05**, 106 (2009), doi:[10.1088/1126-6708/2009/05/106](https://doi.org/10.1088/1126-6708/2009/05/106), [0903.4658](https://arxiv.org/abs/hep-th/0903.4658).
- [12] S. Catani, L. Trentadue, G. Turnock and B. Webber, *Resummation of large logarithms in e^+e^- event shape distributions*, Nucl.Phys. **B407**, 3 (1993), doi:[10.1016/0550-3213\(93\)90271-P](https://doi.org/10.1016/0550-3213(93)90271-P).
- [13] A. Banfi, G. P. Salam and G. Zanderighi, *Phenomenology of event shapes at hadron colliders*, JHEP **06**, 038 (2010), doi:[10.1007/JHEP06\(2010\)038](https://doi.org/10.1007/JHEP06(2010)038), [1001.4082](https://arxiv.org/abs/hep-th/1001.4082).
- [14] A. Banfi, G. P. Salam and G. Zanderighi, *Principles of general final-state resummation and automated implementation*, JHEP **03**, 073 (2005), doi:[10.1088/1126-6708/2005/03/073](https://doi.org/10.1088/1126-6708/2005/03/073), [hep-ph/0407286](https://arxiv.org/abs/hep-ph/0407286).
- [15] A. Banfi, G. Salam and G. Zanderighi, *Semi-numerical resummation of event shapes*, JHEP **0201**, 018 (2002), [hep-ph/0112156](https://arxiv.org/abs/hep-ph/0112156).
- [16] A. Banfi, H. McAslan, P. F. Monni and G. Zanderighi, *The two-jet rate in e^+e^- at next-to-next-to-leading-logarithmic order*, Phys. Rev. Lett. **117**(17), 172001 (2016), doi:[10.1103/PhysRevLett.117.172001](https://doi.org/10.1103/PhysRevLett.117.172001), [1607.03111](https://arxiv.org/abs/1607.03111).
- [17] T. Becher and G. Bell, *NNLL Resummation for Jet Broadening*, JHEP **1211**, 126 (2012), doi:[10.1007/JHEP11\(2012\)126](https://doi.org/10.1007/JHEP11(2012)126), [1210.0580](https://arxiv.org/abs/1210.0580).
- [18] A. H. Hoang, D. W. Kolodrubetz, V. Mateu and I. W. Stewart, *C-parameter distribution at N^3LL including power corrections*, Phys. Rev. D **91**(9), 094017 (2015), doi:[10.1103/PhysRevD.91.094017](https://doi.org/10.1103/PhysRevD.91.094017), [1411.6633](https://arxiv.org/abs/1411.6633).
- [19] A. Budhraj, A. Jain and M. Procura, *One-loop angularity distributions with recoil using Soft-Collinear Effective Theory*, JHEP **08**, 144 (2019), doi:[10.1007/JHEP08\(2019\)144](https://doi.org/10.1007/JHEP08(2019)144), [1903.11087](https://arxiv.org/abs/1903.11087).

- [20] G. Bell, A. Hornig, C. Lee and J. Talbert, *e^+e^- angularity distributions at NNLL' accuracy*, JHEP **01**, 147 (2019), doi:[10.1007/JHEP01\(2019\)147](https://doi.org/10.1007/JHEP01(2019)147), [1808.07867](https://arxiv.org/abs/1808.07867).
- [21] A. H. Hoang, D. W. Kolodrubetz, V. Mateu and I. W. Stewart, *State-of-the-art predictions for C -parameter and a determination of α_s* , Nucl. Part. Phys. Proc. **273-275**, 2015 (2016), doi:[10.1063/1.4938630](https://doi.org/10.1063/1.4938630), [1501.04753](https://arxiv.org/abs/1501.04753).
- [22] D. W. Kolodrubetz, *Accuracy and Precision in Collider Event Shapes*, Ph.D. thesis, MIT, Cambridge, CTP (2016), [1605.06435](https://arxiv.org/abs/1605.06435).
- [23] C. Lepenik and V. Mateu, *NLO Massive Event-Shape Differential and Cumulative Distributions*, JHEP **03**, 024 (2020), doi:[10.1007/JHEP03\(2020\)024](https://doi.org/10.1007/JHEP03(2020)024), [1912.08211](https://arxiv.org/abs/1912.08211).
- [24] C. Lee, A. Hornig and G. Ovanessian, *Probing the Structure of Jets: Factorized and Resummed Angularity Distributions in SCET*, PoS **EFT09**, 010 (2009), doi:[10.22323/1.069.0010](https://doi.org/10.22323/1.069.0010), [0905.0168](https://arxiv.org/abs/0905.0168).
- [25] A. Hornig, C. Lee and G. Ovanessian, *Effective Predictions of Event Shapes: Factorized, Resummed, and Gapped Angularity Distributions*, JHEP **05**, 122 (2009), doi:[10.1088/1126-6708/2009/05/122](https://doi.org/10.1088/1126-6708/2009/05/122), [0901.3780](https://arxiv.org/abs/0901.3780).
- [26] A. Banfi, H. McAslan, P. F. Monni and G. Zanderighi, *A general method for resummation of event-shape distributions in e^+e^- annihilation*, JHEP **05**, 102 (2015), doi:[10.1007/JHEP05\(2015\)102](https://doi.org/10.1007/JHEP05(2015)102), [1412.2126](https://arxiv.org/abs/1412.2126).
- [27] J. Zhu, D. Kang and T. Maji, *Angularity in DIS at next-to-next-to-leading log accuracy* (2021), [2106.14429](https://arxiv.org/abs/2106.14429).
- [28] M. Beneke and V. M. Braun, *Heavy quark effective theory beyond perturbation theory: Renormalons, the pole mass and the residual mass term*, Nucl.Phys. **B426**, 301 (1994), doi:[10.1016/0550-3213\(94\)90314-X](https://doi.org/10.1016/0550-3213(94)90314-X), [hep-ph/9402364](https://arxiv.org/abs/hep-ph/9402364).
- [29] E. Gardi, *Dressed gluon exponentiation*, Nucl.Phys. **B622**, 365 (2002), doi:[10.1016/S0550-3213\(01\)00594-6](https://doi.org/10.1016/S0550-3213(01)00594-6), [hep-ph/0108222](https://arxiv.org/abs/hep-ph/0108222).
- [30] E. Gardi, *Perturbative and nonperturbative aspects of moments of the thrust distribution in e^+e^- annihilation*, JHEP **04**, 030 (2000), doi:[10.1088/1126-6708/2000/04/030](https://doi.org/10.1088/1126-6708/2000/04/030), [hep-ph/0003179](https://arxiv.org/abs/hep-ph/0003179).
- [31] Y. L. Dokshitzer, G. Marchesini and B. Webber, *Dispersive approach to power behaved contributions in QCD hard processes*, Nucl.Phys. **B469**, 93 (1996), doi:[10.1016/0550-3213\(96\)00155-1](https://doi.org/10.1016/0550-3213(96)00155-1), [hep-ph/9512336](https://arxiv.org/abs/hep-ph/9512336).
- [32] E. Gardi and J. Rathsmann, *Renormalon resummation and exponentiation of soft and collinear gluon radiation in the thrust distribution*, Nucl.Phys. **B609**, 123 (2001), doi:[10.1016/S0550-3213\(01\)00284-X](https://doi.org/10.1016/S0550-3213(01)00284-X), [hep-ph/0103217](https://arxiv.org/abs/hep-ph/0103217).
- [33] E. Gardi and L. Magnea, *The C parameter distribution in e^+e^- annihilation*, JHEP **0308**, 030 (2003), [hep-ph/0306094](https://arxiv.org/abs/hep-ph/0306094).
- [34] C. F. Berger and L. Magnea, *Scaling of power corrections for angularities from dressed gluon exponentiation*, Phys.Rev. **D70**, 094010 (2004), doi:[10.1103/PhysRevD.70.094010](https://doi.org/10.1103/PhysRevD.70.094010), [hep-ph/0407024](https://arxiv.org/abs/hep-ph/0407024).