A new Wilson line-based classical action for gluodynamics

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Abstract

We develop a new classical action that in addition to MHV vertices contains also N^{k}MHV vertices, where $1 \le k \le n-4$ with n the number of external legs. The lowest order vertex is the four-point MHV vertex – there is no three point vertex and thus the amplitude calculation involves fewer vertices than in the CSW method. The action is obtained by a canonical transformation of the Yang-Mills action in the light-cone gauge, where the field transformations are based on Wilson line functionals.

Contents

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1 Introduction

The following work focuses on a description of pure gluonic scattering amplitudes in terms of a new action, currently developed at the classical level (thus suitable for tree amplitudes). Despite considered as fundamental, gluon fields are often not the most efficient degrees of freedom for computing amplitudes. Interestingly, in [1], the Maximally Helicity Violating (MHV) vertices used in the Cachazo-Svrcek-Witten (CSW) method [2] were found to be associated with straight infinite Wilson lines on certain complex plane (self-dual plane). These Wilson lines emerge upon transforming the positive helicity field in the light cone Yang-Mills action, to a new action (often called as the 'MHV action') where the MHV vertices are explicit [3–6]. A similar Wilson line-type structure was found in [7] for the negative helicity field. Moreover in the latter, we suggested that such Wilson lines should be a part of a larger structure that extends beyond the self-dual plane.

Indeed, in [8], we derived a new classical action for gluodynamics in which the fields are connected to Wilson line functionals spreading over the anti-self dual and self-dual planes. This new action can be directly derived from the Yang-Mills action. However, the easiest way is to start with the MHV action and canonically transform the anti-self-dual part. This transformation removes the triple gluon vertex in the MHV action as a result of which the new action does not have any triple-gluon vertices at all. These triple-gluon vertices are basically resummed inside the Wilson lines. Thus, starting from the four-point MHV vertex, the vertices in the new action consist of MHV, $\overline{\text{MHV}}$ and other helicity configurations. This reduced the number of diagrams required to calculate amplitudes. We explicitly calculated amplitudes up to 8 external gluons using this new action and found agreement with standard results.

2 MHV Lagrangian

The starting point is the full Yang-Mills action on the constant light-cone time x^+ in the light-cone gauge $\hat{A}^+ = 0$. We denote $\hat{A} = A_a t^a$, here t^a are color generators in the fundamental representation satisfying $\left[t^a, t^b\right] = i\sqrt{2} f^{abc} t^c$ and $\mathrm{Tr}(t^a t^b) = \delta^{ab}$. Integrating out the $\hat{A}^-$ fields (appearing quadratically), leaves only two complex fields $\hat{A}^\bullet, \hat{A}^\star$ that correspond to plus-helicity and minus-helicity gluon fields. We use the so-called 'double-null' coordinates defined as $v^+ = v \cdot \eta$, $v^- = v \cdot \tilde{\eta}$, $v^\bullet = v \cdot \varepsilon_\perp^+$, $v^\star = v \cdot \varepsilon_\perp^-$ with the two light-like basis four-vectors $\eta = (1,0,0,-1)/\sqrt{2}$, $\tilde{\eta} = (1,0,0,1)/\sqrt{2}$, and two space like complex four-vectors spanning the transverse plane $\varepsilon_\perp^\pm = \frac{1}{\sqrt{2}}(0,1,\pm i,0)$. The Yang-Mills action in this setup reads

$$S_{\mathrm{Y-M}}^{(\mathrm{LC})}\left[A^\bullet, A^\star\right] = \int dx^+ \int d^3\mathbf{x} \left\{ -\mathrm{Tr}\hat{A}^\bullet \square \hat{A}^\star - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\bullet \hat{A}^\bullet \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] \right.$$
$$\left. - 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\star \hat{A}^\star \left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] - 2g^2\,\mathrm{Tr}\left[\partial_- \hat{A}^\bullet, \hat{A}^\star\right] \partial_-^{-2} \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] \right\}, \quad (1)$$

where $\square = 2(\partial_+\partial_- - \partial_\bullet\partial_\star)$. Thus, we see there are $(++-)$, $(--+)$ and $(++--)$ vertices. Above, the bold position vector is defined as $\mathbf{x} \equiv \left(x^-, x^\bullet, x^\star\right)$.

The MHV action [3], implementing the CSW rules [2] is obtained from the Yang-Mills action Eq. (1) by canonically transforming the fields to a new pair $(\hat{B}^\bullet, \hat{B}^\star)$ with a constraint that the kinetic term and $(++-)$ vertex in Eq. (1) is mapped to the kinetic term in the new action:

$$\mathrm{Tr}\hat{A}^\bullet \square \hat{A}^\star + 2ig\,\mathrm{Tr}\,\partial_-^{-1}\partial_\bullet \hat{A}^\bullet \left[\partial_- \hat{A}^\star, \hat{A}^\bullet\right] \longrightarrow \mathrm{Tr}\,\hat{B}^\bullet \square \hat{B}^\star\,. \quad (2)$$

Solving the above transformation for $\hat{A}^\bullet$, $\hat{A}^\star$ and substituting it in Eq. (1) results in the MHV action consisting of an infinite set of MHV vertices

$$S^{(\mathrm{LC})}_{\mathrm{Y-M}}[B^{\bullet},B^{\star}] = \int dx^{+}\left(-\int d^{3}\mathbf{x}\,\mathrm{Tr}\,\hat{B}^{\bullet}\Box\hat{B}^{\star} + \mathcal{L}^{(\mathrm{LC})}_{--+} + \dots + \mathcal{L}^{(\mathrm{LC})}_{--+\dots+} + \dots\right), \tag{3}$$

where $\mathcal{L}^{(\mathrm{LC})}_{--+\dots+}$ represents a generic n-point MHV vertex in the action, which in our conventions has the following form in the momentum space

$$\mathcal{L}^{(\mathrm{LC})}_{--+\dots+} = \int d^{3}\mathbf{p}_{1}\dots d^{3}\mathbf{p}_{n}\delta^{3}\left(\mathbf{p}_{1}+\dots+\mathbf{p}_{n}\right)\widetilde{\mathcal{V}}^{b_1\dots b_n}_{--+\dots+}\left(\mathbf{p}_{1},\dots,\mathbf{p}_{n}\right) \\ \widetilde{B}^{\star}_{b_1}\left(x^{+};\mathbf{p}_{1}\right)\widetilde{B}^{\star}_{b_2}\left(x^{+};\mathbf{p}_{2}\right)\widetilde{B}^{\bullet}_{b_3}\left(x^{+};\mathbf{p}_{3}\right)\dots\widetilde{B}^{\bullet}_{b_n}\left(x^{+};\mathbf{p}_{n}\right), \tag{4}$$

with

$$\widetilde{\mathcal{V}}^{b_1\dots b_n}_{--+\dots+}\left(\mathbf{p}_{1},\dots,\mathbf{p}_{n}\right) = \sum \mathrm{Tr}\left(t^{b_1}\dots t^{b_n}\right)\frac{(-g)^{n-2}}{(n-2)!}\left(\frac{p_1^{+}}{p_2^{+}}\right)^{2}\frac{\widetilde{v}^{*4}_{21}}{\widetilde{v}^{*}_{1n}\widetilde{v}^{*}_{n(n-1)}\widetilde{v}^{*}_{(n-1)(n-2)}\dots\widetilde{v}^{*}_{21}}, \tag{5}$$

where the sum is over noncyclic permutations. Above, we introduced spinor-like variables

$$\tilde{v}_{ij} = p_i^{+}\left(\frac{p_j^{\star}}{p_j^{+}} - \frac{p_i^{\star}}{p_i^{+}}\right), \qquad \tilde{v}^{*}_{ij} = p_i^{+}\left(\frac{p_j^{\bullet}}{p_j^{+}} - \frac{p_i^{\bullet}}{p_i^{+}}\right). \tag{6}$$

The $\tilde{v}_{ij}$, $\tilde{v}^{*}_{ij}$ symbols are directly proportional to the spinor products $\langle ij\rangle$ and $[ij]$.

3 Wilson lines in MHV Lagrangian

In the original work [3], the MHV action was obtained using only analytic properties of the transformations and the S-matrix equivalence theorem. The momentum space solutions for $\hat{A}^{\bullet}$ and $\hat{A}^{\star}$ fields were explicitly found in [4]. The Wilson line interpretation of the new fields in the MHV action was first discussed in [1] where the plus helicity field, $B^{\bullet}_{a}[\hat{A}^{\bullet}](x)$, was shown to be the straight infinite Wilson line $B^{\bullet}_{a}[A^{\bullet}](x) = \mathcal{W}^{a}_{(+)}[A](x)$, where for a generic vector field K^{μ} we defined

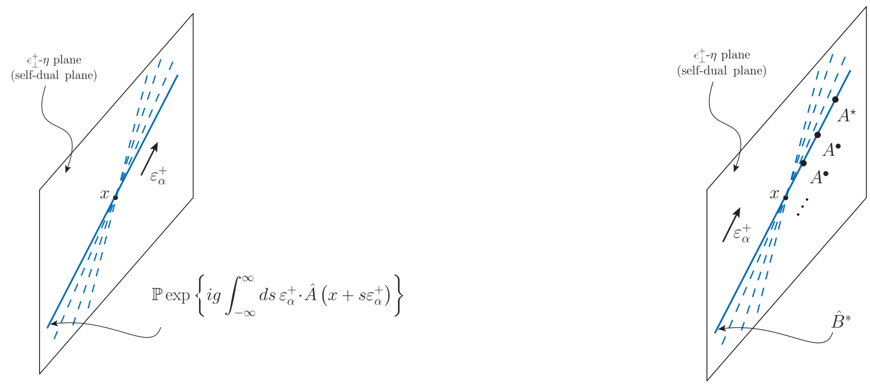
$$\mathcal{W}^{a}_{(\pm)}[K](x) = \int_{-\infty}^{\infty} d\alpha\,\mathrm{Tr}\left\{\frac{1}{2\pi g}t^{a}\partial_{-}\mathbb{P}\exp\left[ig\int_{-\infty}^{\infty} ds\,\varepsilon^{\pm}_{\alpha}\cdot\hat{K}\left(x+s\varepsilon^{\pm}_{\alpha}\right)\right]\right\}, \tag{7}$$

with $\varepsilon^{\pm\mu}_{\alpha} = \varepsilon^{\pm\mu}_{\perp} - \alpha\eta^{\mu}$. Notice, that the latter four vector resembles the gluon polarization vector. Considering $\alpha = p\cdot\varepsilon^{\pm}_{\perp}/p^{+}$, it in fact is the transverse polarization of a gluon carrying momentum p. Thus, in momentum space the Wilson line $B^{\bullet}_{a}[\hat{A}^{\bullet}](x)$ lies along the plus helicity polarization vector. Interestingly, the two vectors defining the direction of the Wilson line, $\varepsilon^{+}_{\perp}$ and η, span the so-called self-dual plane (the plane on which the tensors are self-dual). Note, however, that the Wilson line is integrated over all possible directions α on the self-dual plane gaining thus a projective character (see Fig. 1a).

The minus helicity field $B^{\star}_{a}[\hat{A}^{\bullet},\hat{A}^{\star}](x)$, on the other hand, was shown in [7] to be a similar Wilson line, but with an insertion of the minus helicty gluon field at certain point on the line (see Fig. 1b), more precisely

$$B^{\star}_{a}[A^{\bullet},A^{\star}](x) = \int d^{3}\mathbf{y}\left[\frac{\partial_{-}^{2}(y)}{\partial_{-}^{2}(x)}\frac{\delta\mathcal{W}^{a}_{(+)}[A](x^{+};\mathbf{x})}{\delta A^{\bullet}_{c}(x^{+};\mathbf{y})}\right]A^{\star}_{c}(x^{+};\mathbf{y}), \tag{8}$$

where $\partial_{-}(x) = \partial/\partial x^{-}$. Because it is natural to think of the $A^{\star}$ fields as associated to Wilson lines that live in the anti-self-dual plane spanned by ε^{-}_{α} and η (while the $B^{\bullet}$ is on the self-dual plane), we conjectured that the solution (8) should just be a cut through a larger structure that spans both the planes.



(a) (b)

Figure 1: Left: The straight infinite Wilson line $B^\bullet$ living on the self-dual plane spanned by $\varepsilon_\alpha^+ = \varepsilon_\perp^+ - \alpha\eta$ and integrated over all α (the dashed, tilted Wilson lines represent the change of α). Right: The $B^\star$ field can bee seen as the straight infinite Wilson line similar to the one on the left, but here one $A^\bullet$ field has been substituted by the $A^\star$ field in the expansion (with a suitable symmetry factor). (Source of the figure: Ref. [7].)

4 New classical action

The canonical transformation, Eq. (2), eliminates one of the triple gloun vertex $(++-)$ while the other triple gloun vertex $(+--)$ still exists in the MHV action. Triple point vertices are not very effective building blocks for calculating amplitudes, and, actually they are not physical amplitudes themselves – in the on-shell limit they are zero (for real momenta). Motivated by the geometric considerations mentioned before and the above arguments we proposed in [8] another set of field transformations that lead to a new action.

4.1 Field Transformation

The new canonical field transformations are based on path ordered exponentials of the gauge fields, spreading over the self-dual and anti-self-dual planes [8]:

$$\left\{\hat{A}^\bullet,\hat{A}^\star\right\} \to \left\{\hat{Z}^\bullet\left[A^\bullet,A^\star\right],\hat{Z}^\star\left[A^\bullet,A^\star\right]\right\}, \tag{9}$$

It maps the kinetic term and both the triple-gluon vertices of the Yang-Mills action to a free term in the new action. In order to preserve the functional measure in the partition function, up to a field independent factor, it is necessary that the transformation is canonical. Despite the complexity of the transformation (9), we found that the generating functional $\mathcal{G}[A^\bullet,Z^\star]$ for the transformation has the following simple form:

$$\mathcal{G}[A^\bullet,Z^\star](x^+) = -\int d^3\mathbf{x}\;\mathrm{Tr}\,\hat{\mathcal{W}}^{-1}_{(-)}[Z](x)\,\partial_-\hat{\mathcal{W}}_{(+)}[A](x)\,, \tag{10}$$

The Yang-Mills and the new fields are related as:

$$\partial_- A^\star_a(x^+,\mathbf{y}) = \frac{\delta\,\mathcal{G}[A^\bullet,Z^\star](x^+)}{\delta A^\bullet_a(x^+,\mathbf{y})}\,, \qquad \partial_- Z^\bullet_a(x^+,\mathbf{y}) = -\frac{\delta\,\mathcal{G}[A^\bullet,Z^\star](x^+)}{\delta Z^\star_a(x^+,\mathbf{y})}\,, \tag{11}$$

In [8] we demonstrated that the transformation (10) is identical to two consecutive canonical transformations: first, mapping the self-dual component in the Yang-Mills action to the kinetic term in the MHV action, and then mapping the anti-self-dual part of the latter to the kinetic term in the new action

$$\mathcal{L}_{-+}[B^\bullet,B^\star] + \mathcal{L}_{--+}[B^\bullet,B^\star] \longrightarrow \mathcal{L}_{-+}[Z^\bullet,Z^\star]\,, \tag{12}$$

Following this, the solution for Z fields reads (see Fig. 2)

$$
\begin{aligned}
& Z^{\star}_{a}[B^{\star}](x) = \mathcal{W}^{a}_{(-)}[B](x)\,, \\
& Z^{\bullet}_{a}[B^{\bullet},B^{\star}](x) = \int d^3\mathbf{y} \left[\frac{\partial_-^2(y)}{\partial_-^2(x)} \frac{\delta \mathcal{W}^{a}_{(-)}[B](x^+;\mathbf{x})}{\delta B^{\star}_{c}(x^+;\mathbf{y})} \right] B^{\bullet}_{c}(x^+;\mathbf{y})\,.
\end{aligned} \tag{13}
$$

4.2 Structure of the action

The new action can be most easily derived [8] by substituting the inverse of Z fields (13) in the MHV action. For the $B^{\star}$ field we find

$$
\widetilde{B}^{\star}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \dots d^3\mathbf{p}_n \, \overline{\widetilde{\Psi}}{}_{n}^{a\{b_1 \dots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\dots,\mathbf{p}_n\}) \prod_{i=1}^{n} \widetilde{Z}^{\star}_{b_i}(x^+;\mathbf{p}_i)\,, \tag{14}
$$

with

$$
\overline{\widetilde{\Psi}}{}_{n}^{a\{b_1\cdots b_n\}}(\mathbf{P};\{\mathbf{p}_1,\dots,\mathbf{p}_n\}) = -(-g)^{n-1} \frac{\widetilde{v}_{(1\cdots n)1}}{\widetilde{v}_{1(1\cdots n)}} \frac{\delta^3(\mathbf{p}_1+\dots+\mathbf{p}_n-\mathbf{P})\,\mathrm{Tr}(t^a t^{b_1}\cdots t^{b_n})}{\widetilde{v}_{21}\widetilde{v}_{32}\cdots\widetilde{v}_{n(n-1)}}\,. \tag{15}
$$

The expansion for the $B^{\bullet}$ field reads

$$
\widetilde{B}^{\bullet}_{a}(x^+;\mathbf{P}) = \sum_{n=1}^{\infty} \int d^3\mathbf{p}_1 \dots d^3\mathbf{p}_n \, \overline{\widetilde{\Omega}}{}_{n}^{a b_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\dots,\mathbf{p}_n\}) \widetilde{Z}^{\bullet}_{b_1}(x^+;\mathbf{p}_1) \prod_{i=2}^{n} \widetilde{Z}^{\star}_{b_i}(x^+;\mathbf{p}_i)\,, \tag{16}
$$

where

$$
\overline{\widetilde{\Omega}}{}_{n}^{a b_1\{b_2\cdots b_n\}}(\mathbf{P};\mathbf{p}_1,\{\mathbf{p}_2,\dots,\mathbf{p}_n\}) = n\left(\frac{p_1^+}{p_{1\cdots n}^+}\right)^2 \overline{\widetilde{\Psi}}{}_{n}^{a b_1\cdots b_n}(\mathbf{P};\mathbf{p}_1,\dots,\mathbf{p}_n)\,. \tag{17}
$$

Upon substitution of the expansions (14)-(16) in the MHV action we obtain the following generic structure of the new action:

$$
\begin{aligned}
S^{(\mathrm{LC})}_{\mathrm{Y-M}}[Z^{\bullet},Z^{\star}] = \int dx^+ \Big\{ & -\int d^3\mathbf{x}\, \mathrm{Tr}\, \hat{Z}^{\bullet}\Box\hat{Z}^{\star} \\
& + \mathcal{L}^{(\mathrm{LC})}_{--++} + \mathcal{L}^{(\mathrm{LC})}_{--+++} + \mathcal{L}^{(\mathrm{LC})}_{--++++} + \dots \\
& + \mathcal{L}^{(\mathrm{LC})}_{---++} + \mathcal{L}^{(\mathrm{LC})}_{---+++} + \mathcal{L}^{(\mathrm{LC})}_{---++++} + \dots \\
& \vdots \\
& + \mathcal{L}^{(\mathrm{LC})}_{---\dots-++} + \mathcal{L}^{(\mathrm{LC})}_{---\dots-+++} + \mathcal{L}^{(\mathrm{LC})}_{---\dots-++++} + \dots \Big\}\,,
\end{aligned} \tag{18}
$$

For convenience, we shall call the new action as *Z-field action* hereafter. It has the following properties:

i) There are no three point interaction vertices. This is because they have been effectively resummed inside the Wilson lines.

ii) No all-plus, all-minus, single-minus $(-+\cdots+)$ and single-plus $(-\cdots-+)$ vertices.

iii) It includes MHV vertices, $(--+\cdots+)$, that in the on-shell limit give alone the corresponding amplitudes.

iv) It includes $\overline{\mathrm{MHV}}$ vertices, $(-\cdots-++)$, that in the on-shell limit give alone the corresponding amplitudes.

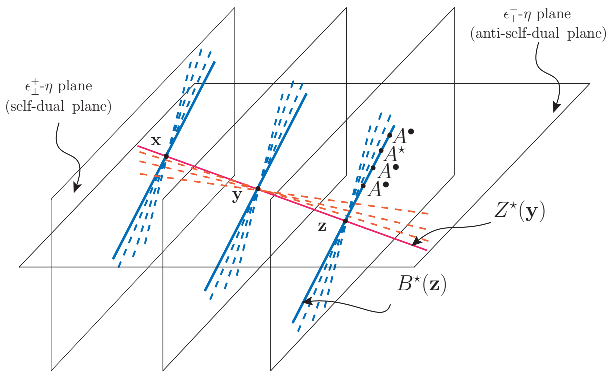


Figure 2: The geometric depiction of the $Z^\star$ field (the structure of $Z^\bullet$ is fairly similar). $Z^\star$ field is a Wilson line (with exactly the same analytic form as $B^\bullet$) of only $B^\star$ fields on anti-self-dual plane. Notice, each vertical plane is self-dual plane with $B^\star$ embedded in it as was shown in Fig. 1b. (Source of the figure: Ref. [8].)

v) All vertices have an easy-to-calculate form.

Let us now discuss a general form of the vertex. Without loss of generality, we consider all the negative helicity fields adjacent and moreover concentrate on the color ordered vertex, defined by

$$\mathcal{U}^{b_1\dots b_n}_{-\dots-+\dots+}(\mathbf{p}_1,\dots,\mathbf{p}_n) = \sum \mathrm{Tr}\left(t^{b_1}\dots t^{b_n}\right)\mathcal{U}\left(1^-,\dots,m^-,(m+1)^+,\dots,n^+\right), \tag{19}$$

where m is the number of minus helicity legs. The sum is over noncyclic permutations. In a similar fashion we define color ordered kernels for the B fields (14)-(16). Furthermore, for compact expressions, we use $[i,i+1,\dots,j]$ to denote the momentum $\mathbf{p}_{i(i+1)\dots j} = \mathbf{p}_i + \mathbf{p}_{i+1} + \dots + \mathbf{p}_j$. In this notation, the generic form of the color ordered vertex reads [8]:

$$\begin{aligned}\mathcal{U}\left(1^-,\dots,m^-,(m+1)^+,\dots,n^+\right) = &\sum_{p=0}^{m-2}\sum_{q=p+1}^{m-1}\sum_{r=q+1}^{m}\\ &\mathcal{V}\left([p+1,\dots,q]^-,[q+1,\dots,r]^-,[r+1,\dots,m+1]^+,(m+2)^+,\dots,(n-1)^+,[n,1,\dots,p]^+\right)\\ &\overline{\Omega}\left(n^+,1^-,\dots,p^-\right)\,\overline{\Psi}\left((p+1)^-,\dots,q^-\right)\,\overline{\Psi}\left((q+1)^-,\dots,r^-\right)\,\overline{\Omega}\left((r+1)^-,\dots,m^-,(m+1)^+\right)\end{aligned} \tag{20}$$

This can be easily understood as follows. The substitution of B fields in terms of Z fields can only multiplicate negative helicity legs. Thus we start with MHV vertex with $n-m$ positive helicity legs. Also, since we have considered all negative helicity legs adjacent, the only possible contributions are the ones where the B fields is substituted to at least one of the four adjacent $(--++)$ legs in the MHV vertex. Summing over all such contribution gives (20). This generic formula doesn't seem to simplify any further. However, it is operational and can be used for calculating amplitudes. We discuss this in the following.

4.3 Amplitudes

Using the Z-field action we computed several tree-level amplitudes. The MHV and $\overline{\text{MHV}}$ on-shell amplitudes can be directly obtained from the corresponding vertices. Consider the 5-point $\overline{\text{MHV}}$ vertex. All the contributions to the vertex (20) are shown in Fig. 3. In the on-shell limit, the sum of these contributions collapses to the known formula:

$$\mathcal{A}(1^-,2^-,3^-,4^+,5^+) = g^3\left(\frac{p_4^+}{p_5^+}\right)^2 \frac{\widetilde{v}_{54}^4}{\widetilde{v}_{15}\widetilde{v}_{54}\widetilde{v}_{43}\widetilde{v}_{32}\widetilde{v}_{21}}. \tag{21}$$

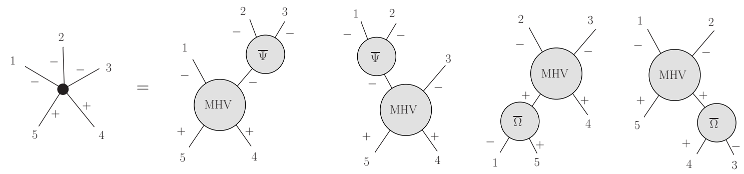


Figure 3: Contributions to the color-ordered $(---++)$ $\overline{\text{MHV}}$ vertex. (Source of the figure: Ref. [8].)

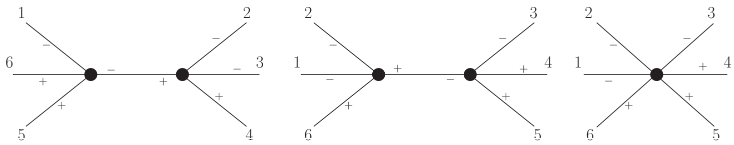


Figure 4: Contributions to the 6-point $(---+++)$ NMHV amplitude. (Source of the figure: Ref. [8].)

For the 6-point NMHV amplitude $(---+++)$ we get just three contributions shown in Fig. 4. In the on-shell limit, the sum of these contributions reduces to the known result [9]. For 7-point NNMHV amplitude $(----+++)$ we get just five contributions shown in Fig. 5. We also

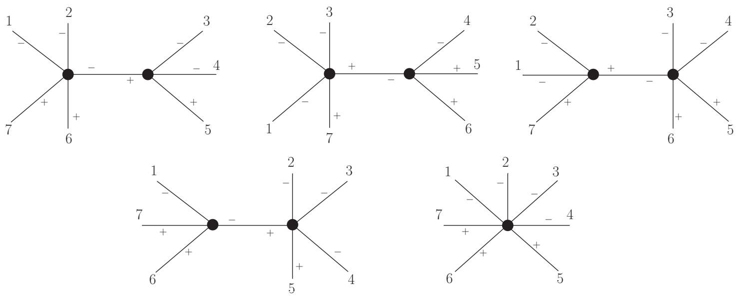


Figure 5: Contributions to the 7-point $(----+++)$ NNMHV amplitude. (Source of the figure: Ref. [8].)

calculated other amplitudes, up to 8-point NNMHV, and showed they agree with the standard methods [10]. The number of diagrams we encountered in the latter case was 13.

5 Conclusions

We developed a new action for gluodynamics by canonically transforming (Eq. (11)) the light-cone Yang-Mills action. The most striking property of the new action is that it has no triple-gluon vertex. Consequently, the number of diagrams needed to calculate the amplitudes is greatly reduced. Also, the geometric structure of the field transformations leading to the new action is incredibly rich and requires further investigation. Finally, a formulation at loop level seems feasible [5, 11–15] and is under development.

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