Flavor violating ℓ_i decay into ℓ_j and a light gauge boson

Alejandro Ibarra[1], Marcela Marín[2⋆] and Pablo Roig[2]

1 Physik-Department, Technische Universität München, James-Franck-Straße, 85748 Garching, Germany.
2 Centro de Investigación y de Estudios Avanzados del Instituto Politécnico Nacional, Ciudad de México, México.
* bmarin@fis.cinvestav.mx

December 3, 2021

16th International Workshop on Tau Lepton Physics (TAU2021),
September 27 – October 1, 2021

Abstract

The $\ell_i \to \ell_j \chi$ decays, with χ a boson associated to the $U(1)_\chi$ symmetry, have not been described satisfactorily so far for light spin-one χ. In particular, observables exhibited an unphysical divergence in the limit of massless χ, associated to its longitudinal polarizations. Based on gauge symmetry, we show how to correct this issue. To this end, we consider two general models realizing the effective field theory description. Being the LFV generated either at tree level or at one loop, these processes are well behaved for light m_χ. We discuss the most salient phenomenological consequences and its relevance in the searches for this kind of decays.

1 Introduction

Lepton flavor is conserved in the original version of the Standard Model [1–3], where only left-handed neutrinos are present. Neutrino oscillations [4–6] constitute undeniable evidence for lepton flavor violation, although, for the charged sector, no lepton flavor violating process has been observed so far.

The simplest Lagrangian describing the lepton flavor interaction is $\mathcal{L}_{\rm LFV} = g\bar{\ell}_i\gamma^\rho\chi_\rho\ell_j$+h.c., with χ_ρ the 4-potential associated to the $U(1)_\chi$ symmetry. Due to the emission of the longitudinal component of the gauge boson, we have terms proportional to g^2/m_χ^2 into the rate for $\ell_i \to \ell_j\chi$. At first sight, observables diverge as $m_\chi \to 0$, preventing the matching of the effective theory to the well studied $\ell_i \to \ell_j\gamma$ decay. Moreover, the decays into several gauge bosons $\ell_i \to \ell_j\chi\cdots\chi$, could also contribute overwhelmingly to the total decay width, reminding the "hyperphoton catastrophe" for the electron decay into a neutrino and an ultralight photon [7–9].

We aim to present a detailed analysis of the decay rate $\ell_i \to \ell_j\chi$, emphasizing the $m_\chi \to 0$ regime (see also [10–12]). We have focused on the light χ case, associated with the spontaneous breaking of an Abelian gauge symmetry, $U(1)_\chi$ (for the case where χ is a light spinless particle, see *e.g.* [13–27]).

This contribution is based on [28], where we consider the two-flavor case ($\mu \to e$ transitions). Here we extend it to three flavors. In Section 2, we present the most general effective interaction leading to the decay $\ell_i \to \ell_j\chi$ in terms of form factors. In Sections 3 and 4, we present two gauge invariant and renormalizable models where the lepton flavor violation is generated either at tree or at the one-loop level. We calculate the rate for $\ell_i \to \ell_j\chi$ and we explicitly show it remains finite as $m_\chi \to 0$. Finally, in Section 5, we conclude.

2 Effective theory

We start with an effective theory description of the $\ell_i \to \ell_j \chi$ decay, where i, j are lepton flavor indices and χ is a light gauge boson with $m_\chi \ll m_i$. The transition amplitude is given by $M = \bar{u}(p_j)\Gamma^\alpha(p_i,p_j)u(p_i)\epsilon^*_\alpha(p_\chi)$, where $\Gamma^\alpha(p_i,p_j)$ can be written in terms of six dimensionless scalar form factors $F_k(p_\chi^2)$, $G_k(p_\chi^2)$, $k=1,2,3$, as:

$$\begin{aligned}\Gamma^\alpha = &\left(\gamma^\alpha - \frac{\not{p}_\chi p_\chi^\alpha}{p_\chi^2}\right)F_1(p_\chi^2) + i\frac{\sigma^{\alpha\beta}p_{\chi\beta}}{m_i+m_j}F_2(p_\chi^2) + \frac{2p_\chi^\alpha}{m_i+m_j}F_3(p_\chi^2) + \\ &\left(\gamma^\alpha - \frac{\not{p}_\chi p_\chi^\alpha}{p_\chi^2}\right)\gamma^5 G_1(p_\chi^2) + i\frac{\sigma^{\alpha\beta}\gamma^5 p_{\chi\beta}}{m_i+m_j}G_2(p_\chi^2) + \frac{2p_\chi^\alpha}{m_i+m_j}\gamma^5 G_3(p_\chi^2)\,. \end{aligned} \tag{1}$$

The conservation of the $U(1)_\chi$ charge requires the form factor $F_3(p_\chi^2)$ to vanish and the Ward identities imply that $p_\chi^\alpha \cdot \epsilon^*_\alpha(p_\chi) = 0$, so the decay rate can then be expressed in terms of four form factors and reads:

$$\begin{aligned}\Gamma(\ell_i \to \ell_j \chi) = &\frac{\lambda^{1/2}[m_i^2, m_j^2, m_\chi^2]}{16\pi m_i}\left[\left(1-\frac{m_j}{m_i}\right)^2\left(1-\frac{m_\chi^2}{(m_i-m_j)^2}\right)\left(2\left|F_1(m_\chi^2)-F_2(m_\chi^2)\right|^2\right.\right.\\ &\left. + \left|F_1(m_\chi^2)\frac{(m_i+m_j)}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{(m_i+m_j)}\right|^2\right) + \left(1+\frac{m_i}{m_j}\right)^2\left(1-\frac{m_\chi^2}{(m_i+m_j)^2}\right)\\ &\left.\left(2\left|G_1(m_\chi^2)-G_2(m_\chi^2)\frac{(m_i-m_j)}{(m_i+m_j}\right|^2 + \left|G_1(m_\chi^2)\frac{(m_i-m_j)}{m_\chi} + G_2(m_\chi^2)\frac{m_\chi}{(m_i+m_j)}\right|^2\right)\right]\,.\end{aligned} \tag{2}$$

To all appearances the rate has an unphysical divergence in the limit $m_\chi \to 0$ on account of the term coming from the emission of the longitudinal component of the vector boson. Therefore, in an effective field theory approach, great care should be taken when considering decays into ultralight gauge bosons, since in a gauge invariant and renormalizable theory, the $\ell_i \to \ell_j \chi$ rate must be finite and continuously matched to the result from $\ell_i \to \ell_j \gamma$ [29–31].

We present below two specific models in which, due to gauge invariance, the rate for $\ell_i \to \ell_j \chi$ is finite in the limit $m_\chi \to 0$.

3 $\ell_i \to \ell_j \chi$ at tree level

The particle content of the model, and the corresponding spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$, are summarized in Table 1 [1].

The kinetic terms of the particles of the model read:

$$\mathcal{L}_{\rm kin} = \sum_{j=1}^{3} i\left(\bar{L}_j \not{D} L_j + \bar{e}_{R_j}\not{D} e_{R_j}\right) + \sum_{j,k=1}^{3}(D_\mu \phi_{jk})^\dagger (D^\mu \phi_{jk})\,, \tag{3}$$

where D_μ denotes the covariant derivative, given by

$$\begin{aligned} D_\mu &= \partial_\mu + igW_\mu^a T_a + ig'YB_\mu + ig_\chi q\chi_\mu \quad \text{for the } SU(2)_L \text{ doublets}\,,\\ D_\mu &= \partial_\mu + ig'YB_\mu + ig_\chi q \chi_\mu \quad \text{for the } SU(2)_L \text{ singlets}\,, \end{aligned} \tag{4}$$

[1] The model can be made anomaly-free adding heavy particles with suitable charges, without modifying the discussion that follows.

	L_i	e_{R_i}	ϕ_{ij}
spin	1/2	1/2	0
$SU(2)_L$	2	1	2
$U(1)_Y$	$-1/2$	-1	Y_{ij}
$U(1)_\chi$	q_{L_i}	q_{e_i}	$q_{\phi_{ij}}$

Table 1: Spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ of the particles of the model described in Section 3. All fields are assumed to be singlets under $SU(3)_C$ and the subscripts i, $j = 1, 2, 3$.

with g, g' and g_χ the coupling constants of $SU(2)_L$, $U(1)_Y$ and $U(1)_\chi$ respectively.

Assuming that $Y_{jk} = 1/2$, then, for j,k such that $q_{\phi_{jk}} = q_{L_j} - q_{e_k}$ the following Yukawa couplings arise in the Lagrangian:

$$-\mathcal{L}_{\rm Yuk} = \sum_{j,k=1}^{3} y_{jk} \overline{L}_j \phi_{jk} e_{R_k} + \text{h.c.} \tag{5}$$

In full generality, we consider that the charges of the particles allow all Yukawa couplings and that all ϕ_{jk} acquire a non-zero vacuum expectation value, $\langle \phi_{jk} \rangle = v_{jk}$; so that ϕ_{jk} generate a mass for the χ boson:

$$m_\chi^2 = g_\chi^2 \sum_{i,j} q_{\phi_{ij}}^2 v_{ij}^2 \,. \tag{6}$$

After rotating Eq. (3) to the mass eigenstate basis (assuming CP-violating phase $\delta = 0$), the charged lepton masses satisfy ($m_\tau \gg m_\mu \gg m_e$ is used):

$$\begin{aligned}
m_\tau^2 \simeq\; & yv_{11}^2 + yv_{12}^2 + yv_{13}^2 + yv_{21}^2 + yv_{22}^2 + yv_{23}^2 + yv_{31}^2 + yv_{32}^2 + yv_{33}^2 \,, \\
m_\mu^2 \simeq\; & \frac{1}{m_\tau^2}\Big(yv_{21}^2\big(yv_{12}^2 + yv_{13}^2 + yv_{32}^2 + yv_{33}^2\big) + yv_{31}^2\big(yv_{12}^2 + yv_{13}^2 + yv_{22}^2 + yv_{23}^2\big) + \\
& yv_{11}^2\big(yv_{22}^2 + yv_{23}^2 + yv_{32}^2 + yv_{33}^2\big) + yv_{23}^2\big(yv_{12}^2 + yv_{32}^2\big) + yv_{13}^2\big(yv_{22}^2 + yv_{32}^2\big) + \\
& yv_{33}^2\big(yv_{12}^2 + yv_{22}^2\big) - 2yv_{21}yv_{22}yv_{31}yv_{32} - 2yv_{21}yv_{23}yv_{31}yv_{33} - 2yv_{22}yv_{23}yv_{32}yv_{33} - \\
& 2yv_{12}yv_{13}\big(yv_{22}yv_{23} + yv_{32}yv_{33}\big) - 2yv_{11}\big(yv_{12}yv_{21}yv_{22} + yv_{12}yv_{31}yv_{32} + \\
& yv_{13}yv_{21}yv_{23} + yv_{13}yv_{31}yv_{33}\big)\Big) \,, \\
m_e^2 \simeq\; & \frac{1}{m_\tau^2 m_\mu^2}\big(yv_{31}(yv_{13}yv_{22} - yv_{12}yv_{23}) + yv_{32}(yv_{11}yv_{23} - yv_{13}yv_{21}) + \\
& yv_{33}(yv_{12}yv_{21} - yv_{11}yv_{22})\big)^2 \,,
\end{aligned} \tag{7}$$

where $yv_{jk} \equiv y_{jk}v_{jk}$, with $j,k = 1,2,3$ and the flavor violating terms have the form

$$-\mathcal{L} \supset \overline{\ell}_{i_R} i g_{ij}^{RR} \gamma^\rho \chi_\rho \ell_{j_R} + \overline{\ell}_{i_L} i g_{ij}^{LL} \gamma^\rho \chi_\rho \ell_{j_L} + \text{h.c.} \,, \tag{8}$$

with $\ell_i, \ell_j = e, \mu, \tau$ and

$$\begin{aligned}
g_{e\mu}^{RR} =\; & g_\chi \left(c_{12_R} s_{12_R} \left(c_{13_R}^2 q_{e_1} + c_{23_R}^2 (q_{e_3} s_{13_R}^2 - q_{e_2}) + s_{23_R}^2 (q_{e_2} s_{13_R}^2 - q_{e_3}) \right) + \right. \\
& \left. c_{23_R} s_{23_R} (q_{e_3} - q_{e_2}) s_{13_R} \cos(2\theta_{12_R}) \right) \,, \\
g_{e\tau}^{RR} =\; & g_\chi c_{13_R} \left(c_{23_R} s_{23_R} (q_{e_3} - q_{e_2}) s_{12_R} + c_{12_R} s_{13_R} \left(q_{e_1} - c_{23_R}^2 q_{e_3} - s_{23_R}^2 q_{e_2} \right) \right) \,, \\
g_{\mu\tau}^{RR} =\; & g_\chi \left(c_{23_R} s_{23_R} (q_{e_2} - q_{e_3}) c_{12_R} + s_{12_R} s_{13_R} \left(q_{e_1} - c_{23_R}^2 q_{e_3} - s_{23_R}^2 q_{e_2} \right) \right) \,,
\end{aligned} \tag{9}$$

where we have defined $s_{jk_R} \equiv \sin\theta_{jk_R}$ and $c_{jk_R} \equiv \cos\theta_{jk_R}$. The effective couplings $g^{LL}_{e\tau}$, $g^{LL}_{e\mu}$, and $g^{LL}_{\mu\tau}$, are defined analogously to the right couplings in eq. (9), substituting $\theta_{jk_R} \to \theta_{jk_L}$ and $q_{e_j} \to q_{L_j}$. Here θ_{jk_R} and θ_{jk_L} are mixing angles which can be written as a function of vacuum expectation values, v_{jk}, and Yukawa couplings, y_{jk}, as well as, the tau, muon, and electron masses in eq. (7). We note that all $g^{LL/RR}_{ij}$ vanish for intergenerational universality of the $U(1)_\chi$ charges, thus forbiding the $\ell_i \to \ell_j\chi$ decays in this tree-level model.

The rate for $\ell_i \to \ell_j\chi$ then reads:

$$\Gamma(\ell_i \to \ell_j\chi)\Big|_{m_j\to 0} = \frac{m_i}{32\pi}\left(\left|g^{LL}_{ij}\right|^2 + \left|g^{RR}_{ij}\right|^2\right)\left(2 + \frac{m_i^2}{m_\chi^2}\right)\left(1 - \frac{m_\chi^2}{m_i^2}\right)^2 . \tag{10}$$

Apparently, the term m_i^2/m_χ^2 would enhance the rate as $m_\chi \to 0$. However, if the gauge and fermion masses arise as a consequence of the spontaneous breaking of the $U(1)_\chi$ symmetry, the limit $m_\chi \to 0$ requires $v_{jk} \to 0$ for all j, k, which in turn implies $m_i \to 0$. One can explicitly check from Eqs. (6) and (7) that indeed when $m_\chi \to 0$ the term m_i^2/m_χ^2 is finite (as expected from the Goldstone boson equivalence theorem [32–35]), and depends on a function of the Yukawa couplings, the gauge coupling, and the charges and vacuum expectation values of the fields ϕ_{jk}. [2]

A complementary probe of the ℓ_i-ℓ_j flavor violation is the three-body decay $\ell_i^- \to \ell_j^-\ell_j^+\ell_j^-$ [3], which is generated in this model at tree-level via the exchange of a virtual χ. This process is generated through a flavor violating interaction vertex described in eq. (8) and a flavor conserving interaction vertex a form:

$$-\mathcal{L} \supset \bar{\ell}_{i_R} i g^{RR}_{ii}\gamma^\rho\chi_\rho\ell_{i_R} + \bar{\ell}_{i_L} i g^{LL}_{ii}\gamma^\rho\chi_\rho\ell_{i_L} , \tag{11}$$

where

$$\begin{aligned}
g^{RR}_{ee} &= g_\chi\left(c^2_{12_R}c^2_{13_R}q_{e_1} + \left(c_{23_R}s_{12_R} + c_{12_R}s_{13_R}s_{23_R}\right)^2 q_{e_2} + \left(c_{12_R}c_{23_R}s_{13_R} - s_{12_R}s_{23_R}\right)^2 q_{e_3}\right) ,\\
g^{RR}_{\mu\mu} &= g_\chi\left(c^2_{13_R}s^2_{12_R}q_{e_1} + \left(c_{12_R}c_{23_R} - s_{12_R}s_{13_R}s_{23_R}\right)^2 q_{e_2} + \left(c_{23_R}s_{12_R}s_{13_R} + c_{12_R}s_{23_R}\right)^2 q_{e_3}\right) ,\\
g^{RR}_{\tau\tau} &= g_\chi\left(s^2_{13_R}q_{e_1} + c^2_{13_R}\left(s^2_{23_R}q_{e_2} + c^2_{23_R}q_{e_3}\right)\right) ,
\end{aligned} \tag{12}$$

where the left couplings, g^{LL}_{ee}, $g^{LL}_{\mu\mu}$, and $g^{LL}_{\tau\tau}$, are obtained with obvious substitutions in eq. (12).

We focus in what follows in a scenario where 1 MeV $\lesssim m_\chi \lesssim m_i$. For $m_i = m_\mu$, the dominant decay channels are $\chi \to e^-e^+, \bar{\nu}_{L_1}\nu_{L_1}, \bar{\nu}_{L_2}\nu_{L_2}, \bar{\nu}_{L_3}\nu_{L_3}$. Using the electron interaction vertex from Eq. (11) and the neutrino interaction vertex from Eq. (3), we find that the total decay width of the χ-boson is:

$$\Gamma_\chi = \frac{m_\chi}{24\pi}\left(|g^{LL}_{ee}|^2 + |g^{RR}_{ee}|^2 + |g_\chi q_{L_1}|^2 + |g_\chi q_{L_2}|^2 + |g_\chi q_{L_3}|^2\right) , \tag{13}$$

and for $m_i = m_\tau$ the decays $\chi \to \mu^-\mu^+, \mu^-e^+ + \text{h.c.}$ can also be generated, with widths: $\Gamma(\chi \to \mu^-\mu^+) = m_\chi\left(|g^{LL}_{\mu\mu}|^2 + |g^{RR}_{\mu\mu}|^2\right)/(24\pi)$ and $\Gamma(\chi \to \mu e) = m_\chi\left(|g^{LL}_{e\mu}|^2 + |g^{RR}_{e\mu}|^2\right)/(24\pi)$.

We show in Fig. 1 the ratio between $\Gamma(\ell_i \to \ell_j\chi)$ and $\Gamma(\ell_i \to 3\ell_j)$ as a function of m_χ, for a representative case where χ couples only to the right-handed leptons or when χ couples only to the left-handed leptons . This result can be understood using the narrow width approximation

[2] An analogous behaviour occurs in the top decay $t \to bW^+$. The decay rate is $\Gamma(t \to bW^+) \sim m_t^3/m_W^2$ and naively diverges when $m_W \to 0$. However, since both masses arise as a consequence of the spontaneous breaking of the electroweak symmetry, $\Gamma(t \to bW^+) \sim m_t y_t^2/g^2$ and is finite.

[3] As well as processes of type $\ell_i^- \to \ell_j^-\ell_k^+\ell_j^-$ and $\ell_i^- \to \ell_j^-\ell_j^+\ell_k^-$. For simplicity and for the purposes of this work we will only analyze the decay $\ell_i^- \to \ell_j^-\ell_j^+\ell_j^-$.

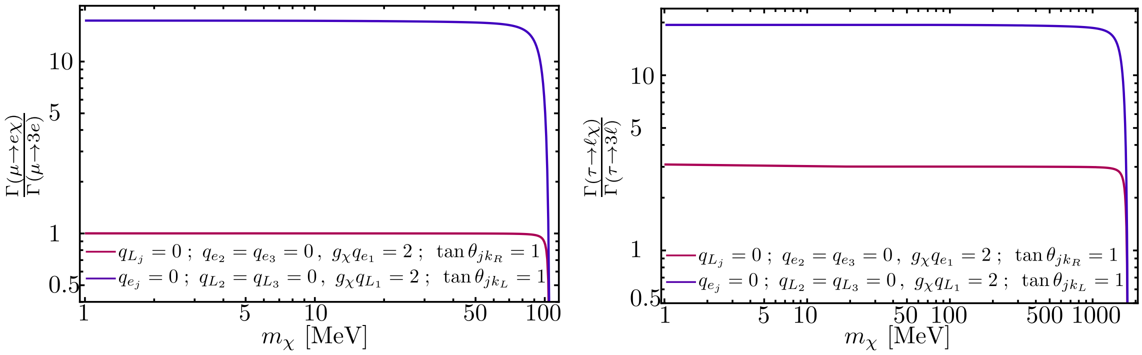


Figure 1: $\Gamma(\mu\to e\chi)/\Gamma(\mu\to 3e)$ (left plot) and $\Gamma(\tau\to\ell\chi)/\Gamma(\tau\to 3\ell)$ with $\ell = e\,,\mu$ (right plot) as a function of m_χ for the tree-level model, for the cases described in the text $q_{L_j}=0$, $q_{e_2}=q_{e_3}=0$, $g_\chi q_{e_1}=2$, and $\tan\theta_{jk_R}=1$ (magenta line); and $q_{e_j}=0$, $q_{L_2}=q_{L_3}=0$, $g_\chi q_{L_1}=2$, and $\tan\theta_{jk_L}=1$ (blue line).

(NWA), which holds when χ is produced close to the mass shell. Under this approximation, the decay rate for $\ell_i\to 3\ell_j$ reads:

$$\Gamma(\ell_i\to 3\ell_j)\Big|_{m_j\to 0} = \frac{m_i m_\chi}{768\pi^2\Gamma_\chi}\left(|g_{ji}^{LL}|^2+|g_{ji}^{RR}|^2\right)\left(|g_{jj}^{LL}|^2+|g_{jj}^{RR}|^2\right)\left(2+\frac{m_i^2}{m_\chi^2}\right)\left(1-\frac{m_\chi^2}{m_i^2}\right)^2 + \frac{m_\chi}{64\pi}\left(|g_{jj}^{LL}|^2|g_{ji}^{LL}|^2+|g_{jj}^{RR}|^2|g_{ji}^{RR}|^2\right)\frac{m_\chi}{m_i}\left(1-2\frac{m_\chi^2}{m_i^2}\right). \tag{14}$$

As for the decay $\ell_i\to\ell_j\chi$ the rate apparently diverges as $m_\chi\to 0$, but is in fact finite since m_i and m_χ are both generated after the breaking of the $U(1)_\chi$ symmetry. Further, and using Eq. (10), one reproduces the result $\Gamma(\mu\to e\chi)/\Gamma(\mu\to 3e)\simeq 1$ or $\simeq 17$, and $\Gamma(\tau\to\ell\chi)/\Gamma(\tau\to 3\ell)\simeq 3$ or $\simeq 19$, that we obtained numerically for our two representative scenarios. As m_χ becomes larger, the ratio becomes sensitive to the underlying model parameters, although this sensitivity is suppressed by a factor m_χ^2/m_i^2, and is hence typically weak, in agreement with the numerical results of Fig. 1.

4 $\ell_i\to\ell_j\chi$ at the one loop level

Now we turn to a renormalizable model with generation-independent $U(1)_\chi$ charges. Lepton flavor is now violated through a new Dirac fermion ψ and a new complex scalar η, which generate the process $\ell_i\to\ell_j\chi$ at one loop. Charged lepton masses are generated via a doublet scalar, with $U(1)_\chi$-charge $q_\phi = q_L - q_e$, so that the Yukawa coupling $y_{jk}\overline{L}_j e_{R_k}\phi$+h.c. is allowed. The spins and charges of the particles of the model are listed in Table 2.

As required by $U(1)_\chi$ charge conservation, $q_e = q_\psi + q_\eta$. We assume $Y_e = Y_\psi + Y_\eta$, for the Yukawa couplings $y_i\overline{e}_{R_i}\psi\eta$ to be allowed. We also assume that ϕ acquires a vacuum expectation value, but η does not, thereby χ acquires a mass: $m_\chi = g_\chi q_\phi\langle\phi\rangle$.

Rotating the Lagrangian to the mass eigenstates basis, we find that the interaction terms with the χ boson are:

$$\mathcal{L}\supset -ig_\chi q_L\left(\overline{e}_L\gamma^\nu e_L+\overline{\mu}_L\gamma^\nu\mu_L+\overline{\tau}_L\gamma^\nu\tau_L+\overline{\nu}_{L_1}\gamma^\nu\nu_{L_1}+\overline{\nu}_{L_2}\gamma^\nu\nu_{L_2}+\overline{\nu}_{L_3}\gamma^\nu\nu_{L_3}\right)\chi_\nu - ig_\chi q_e\left(\overline{e}_R\gamma^\nu e_R+\overline{\mu}_R\gamma^\nu\mu_R+\overline{\tau}_R\gamma^\nu\tau_R\right)\chi_\nu - ig_\chi q_\Psi\overline{\Psi}\gamma^\nu\Psi\chi_\nu - iq_\eta g_\chi\left[\eta^*(\partial^\nu\eta)-(\partial^\nu\eta^*)\eta\right]\chi_\nu\,, \tag{15}$$

	L_i	e_{R_i}	ϕ	ψ	η
spin	1/2	1/2	0	1/2	0
$SU(2)_L$	2	1	2	1	1
$U(1)_Y$	$-1/2$	-1	$+1/2$	Y_ψ	Y_η
$U(1)_\chi$	q_L	q_e	q_ϕ	q_ψ	q_η

Table 2: Spins and charges under $SU(2)_L \times U(1)_Y \times U(1)_\chi$ of the particles of the model described in Section 4. All fields are assumed to be singlets under $SU(3)_C$ and the subscript $i = 1, 2, 3$.

as well as a Yukawa coupling to the right-handed leptons $\mathcal{L} \supset h_k \overline{\ell}_{k_R} \eta \psi + \text{h.c.}$ with $\ell_k = e, \mu, \tau$.

The χ interactions and the Yukawa terms generate the process $\ell_i \to \ell_j \chi$ in this model at the one loop-level, through four diagrams, with χ emitted from either charged lepton, or from the heavy particles in the loop, Ψ and η (see fig. 2 in [28]). The form factors are finite and read:

$$\begin{aligned} F_1(m_\chi^2) = \quad G_1(m_\chi^2) &= \frac{g_\chi h_j h_i}{384\pi^2} \frac{m_\chi^2}{M_\eta^2} \left[q_\eta \mathcal{F}_{1\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{1\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right) \right], \\ F_2(m_\chi^2) = -G_2(m_\chi^2) &= \frac{g_\chi h_j h_i}{384\pi^2} \frac{m_i^2}{M_\eta^2} \left[q_\eta \mathcal{F}_{2\eta}\left(\frac{M_\psi^2}{M_\eta^2}\right) + q_\psi \mathcal{F}_{2\psi}\left(\frac{M_\psi^2}{M_\eta^2}\right) \right], \end{aligned} \tag{16}$$

where

$$\begin{aligned} \mathcal{F}_{1\eta}(x) &= \frac{-2 + 9x - 18x^2 + x^3 (11 - 6 \ln x)}{3 (1-x)^4}, \\ \mathcal{F}_{1\psi}(x) &= \frac{16 - 45x + 36x^2 - 7x^3 + 6(2 - 3x) \ln x}{3 (1-x)^4}, \\ \mathcal{F}_{2\eta}(x) &= \frac{1 - 6x + 3x^2 (1 - 2 \ln x) + 2x^3}{(1-x)^4}, \\ \mathcal{F}_{2\psi}(x) &= \frac{-2 - 3x(1 + 2 \ln x) + 6x^2 - x^3}{(1-x)^4}, \end{aligned} \tag{17}$$

which are regular at $x = 1$.

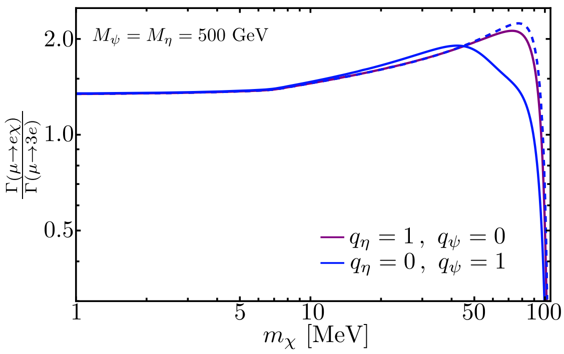
Using Eq. (2), and that $F_1 = G_1$, $F_2 = -G_2$, the decay rate can be recast as

$$\Gamma(\ell_i \to \ell_j \chi)\Big|_{m_j \to 0} = \frac{m_i}{8\pi} \left(1 - \frac{m_\chi^2}{m_i^2}\right)^2 \left[\left| F_1(m_\chi^2) \frac{m_i}{m_\chi} - F_2(m_\chi^2) \frac{m_\chi}{m_i} \right|^2 + 2 \left| F_1(m_\chi^2) - F_2(m_\chi^2) \right|^2 \right]. \tag{18}$$

In this rate, the factors from the emission of the longitudinal polarization cancel with the factors m_χ^2 implicit in the form factors F_1 and G_1, thus the rate for $\ell_i \to \ell_j \chi$ is finite in the limit $m_\chi \to 0$. Further, $F_1 m_i / m_\chi \propto m_\chi m_i / M_\eta^2$, and $F_2 m_\chi / m_i \propto m_\chi m_i / M_\eta^2$. As $M_\eta, M_\psi \gg m_i$, it follows that the rate in the limit $m_\chi \to 0$ will depend mostly on the form factors F_2 and G_2.

In this toy model, $\ell_i^- \to \ell_j^- \ell_j^+ \ell_j^-$ arises at one loop, through χ-penguin and box diagrams [4]. The χ-penguin are proportional to $h_j^2 h_i^2 g_\chi^4$ while the box diagram to $h_j^6 h_i^2$, so assuming that

[4] Similarly, the wrong-sign decays $\ell_i^- \to \ell_j^- \ell_k^+ \ell_j^-$ are generated. For simplicity, we will only focus on the decays $\ell_i^- \to \ell_j^- \ell_j^+ \ell_j^-$ here.



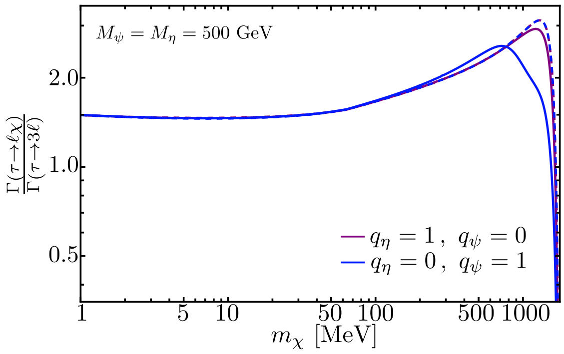


Figure 2: Ratio of rates $\Gamma(\ell_i \to \ell_j \chi)/\Gamma(\ell_i \to 3\ell_j)$ as a function of m_χ for the one-loop model presented in Section 4, assuming $q_L = 1 + q_e$, $h_i h_j = 1$ and $g_\chi = 1$. The solid lines show the full result obtained from Eq. (18), while the dashed ones neglect the contribution from F_1.

$h_j \ll g_\chi$, the decay will be dominated by the penguin diagrams (this happens unless there is a big mass hierarchy among the loop particles).

Focusing on the region 1 MeV $\lesssim m_\chi \lesssim m_i$, in the case when $m_i = m_\mu$, the dominant decay modes are $\chi \to e^- e^+\,, \overline{\nu}_{L_1} \nu_{L_1}\,, \overline{\nu}_{L_2} \nu_{L_2}\,, \overline{\nu}_{L_3} \nu_{L_3}$, with width:

$$\Gamma_\chi = \frac{g_\chi^2\, m_\chi}{24\pi}\left(|q_e|^2 + 4|q_L|^2\right), \tag{19}$$

and for the $m_i = m_\tau$ case, in addition to the above modes, we must consider the decay mode $\chi \to \mu^- \mu^+$ [5], with width: $\Gamma(\chi \to \mu^-\mu^+) = g_\chi^2\, m_\chi \left(|q_e|^2 + |q_L|^2\right)/(24\pi)$.

We show in Fig. 2 the ratio of rates $\Gamma(\ell_i \to \ell_j\chi)/\Gamma(\ell_i \to 3\ell_j)$ as a function of m_χ for two representative choices of charges, $q_\eta = 1$ while $q_\psi = 0$, and $q_\eta = 0$ while $q_\psi = 1$, and a mass choice of 500 GeV for the particles in the loop, M_η and M_ψ. These values are compatible with the current searches for exotic charged particles [36, 37]. For cases where $M_\eta > M_\psi$ or $M_\psi > M_\eta$ (as long as the values of M_η and M_ψ are compatible with current searches) the results are analogous. We find that the ratio is ~ 2, for both muon and tau decays. As in Section 3, this result can be understood analytically employing the narrow width approximation. Under it, the decay rate for $\ell_i^- \to \ell_j^- \ell_j^+ \ell_j^-$ reads

$$\begin{aligned}\Gamma(\ell_i \to 3\ell_j)\Big|_{m_j\to 0} = {} & \frac{g_\chi^2 |q_e|^2}{16\pi}\frac{m_\chi^2}{m_i}\left(1 - 2\frac{m_\chi^2}{m_i^2}\right)\left(2\left(|F_1(m_\chi^2)|^2 - F_1(m_\chi^2)F_2(m_\chi^2)\right) + |F_2(m_\chi^2)|^2\frac{m_\chi^2}{m_i^2}\right)\\ & + \frac{m_i m_\chi g_\chi^2}{96\pi^2\Gamma_\chi}(|q_e|^2 + |q_L|^2)\left(1 - \frac{m_\chi^2}{m_i^2}\right)^2\left[\left|F_1(m_\chi^2)\frac{m_i}{m_\chi} - F_2(m_\chi^2)\frac{m_\chi}{m_i}\right|^2 + 2|F_1(m_\chi^2) - F_2(m_\chi^2)|^2\right].\end{aligned} \tag{20}$$

Again the seeming divergence when $m_\chi \to 0$ is cancelled by the factor m_χ implicit in the form factor F_1, ergo, the rate for $\ell_i \to 3\ell_j$ is finite in the limit $m_\chi \to 0$ and comparable to the rate for $\ell_i \to \ell_j \chi$, quite independently of the masses and charges of the particles in the loop.

[5] The one-loop decay, $\chi \to \mu^- e^+ + \text{h.c.}$, could also be generated in this mass range, however, it is negligible with respect to tree level decays.

5 Conclusions

We have studied in detail the lepton flavor violating process $\ell_i \to \ell_j \chi$, with χ a massive gauge boson arising from the spontaneous breaking of a local $U(1)$ symmetry. We have given the most general effective parametrization of the interaction between two charged leptons and a massive gauge boson, yielding the decay rate in terms of the corresponding form factors. This decay width presents terms inversely proportional to the χ-boson mass, corresponding to the decay into the longitudinal component of the χ-boson, which naively lead to an unphysical enhancement of the rate when χ is very light.

We have provided two gauge-invariant and renormalizable models where the decay $\ell_i \to \ell_j \chi$ is generated either at tree level or at one loop. We focused on the scenario where the χ boson is light, and we have explicitly checked that the rate remains finite. We have also calculated the expected rate for the process $\ell_i \to 3\ell_j$, mediated by an off-shell χ. For these two models, the ratio of rates of $\ell_i \to \ell_j \chi$ and $\ell_i \to 3\ell_j$ is $\mathcal{O}(1-20)$ in the range of χ-masses considered. Taking into account that the limits of the latter decays are much more stringent than those on the former, it is evident that the upper limits on $\ell_i \to \ell_j \chi$ decays need to improve by 5-6 orders of magnitude to potentially observe a signal.

Acknowledgements

The speaker thanks the Local organizing committee of TAU2021 for the pleasurable conference.

References

[1] S. L. Glashow, *Partial Symmetries of Weak Interactions*, Nucl. Phys. **22**, 579 (1961), doi:10.1016/0029-5582(61)90469-2.

[2] S. Weinberg, *A Model of Leptons*, Phys. Rev. Lett. **19**, 1264 (1967), doi:10.1103/PhysRevLett.19.1264.

[3] A. Salam, *Weak and Electromagnetic Interactions*, Conf. Proc. C **680519**, 367 (1968), doi:10.1142/9789812795915_0034.

[4] Y. Fukuda *et al.*, *Evidence for oscillation of atmospheric neutrinos*, Phys. Rev. Lett. **81**, 1562 (1998), doi:10.1103/PhysRevLett.81.1562, hep-ex/9807003.

[5] Q. R. Ahmad *et al.*, *Measurement of the rate of $\nu_e + d \to p + p + e^-$ interactions produced by ^8B solar neutrinos at the Sudbury Neutrino Observatory*, Phys. Rev. Lett. **87**, 071301 (2001), doi:10.1103/PhysRevLett.87.071301, nucl-ex/0106015.

[6] Q. R. Ahmad *et al.*, *Direct evidence for neutrino flavor transformation from neutral current interactions in the Sudbury Neutrino Observatory*, Phys. Rev. Lett. **89**, 011301 (2002), doi:10.1103/PhysRevLett.89.011301, nucl-ex/0204008.

[7] L. B. Okun and Y. B. Zeldovich, *Paradoxes of unstable electron*, Phys. Lett. B **78**, 597 (1978), doi:10.1016/0370-2693(78)90648-2.

[8] L. B. Okun and M. B. Voloshin, *On the electric charge conservation*, JETP Lett. **28**, 145 (1978).

[9] M. Suzuki, *Slightly massive photon*, Phys. Rev. D **38**, 1544 (1988), doi:10.1103/PhysRevD.38.1544.

[10] Y. Farzan and I. M. Shoemaker, *Lepton Flavor Violating Non-Standard Interactions via Light Mediators*, JHEP **07**, 033 (2016), doi:10.1007/JHEP07(2016)033, 1512.09147.

[11] J. Heeck, *Lepton flavor violation with light vector bosons*, Phys. Lett. B **758**, 101 (2016), doi:10.1016/j.physletb.2016.05.007, 1602.03810.

[12] Y. Farzan and J. Heeck, *Neutrinophilic nonstandard interactions*, Phys. Rev. D **94**(5), 053010 (2016), doi:10.1103/PhysRevD.94.053010, 1607.07616.

[13] F. Wilczek, *Axions and Family Symmetry Breaking*, Phys. Rev. Lett. **49**, 1549 (1982), doi:10.1103/PhysRevLett.49.1549.

[14] B. Grinstein, J. Preskill and M. B. Wise, *Neutrino Masses and Family Symmetry*, Phys. Lett. B **159**, 57 (1985), doi:10.1016/0370-2693(85)90119-4.

[15] Z. G. Berezhiani and M. Y. Khlopov, *Cosmology of Spontaneously Broken Gauge Family Symmetry*, Z. Phys. C **49**, 73 (1991), doi:10.1007/BF01570798.

[16] J. L. Feng, T. Moroi, H. Murayama and E. Schnapka, *Third generation familons, b factories, and neutrino cosmology*, Phys. Rev. D **57**, 5875 (1998), doi:10.1103/PhysRevD.57.5875, hep-ph/9709411.

[17] M. Hirsch, A. Vicente, J. Meyer and W. Porod, *Majoron emission in muon and tau decays revisited*, Phys. Rev. D **79**, 055023 (2009), doi:10.1103/PhysRevD.79.055023, [Erratum: Phys.Rev.D 79, 079901 (2009)], 0902.0525.

[18] J. Jaeckel, *A Family of WISPy Dark Matter Candidates*, Phys. Lett. B **732**, 1 (2014), doi:10.1016/j.physletb.2014.03.005, 1311.0880.

[19] A. Celis, J. Fuentes-Martin and H. Serodio, *An invisible axion model with controlled FCNCs at tree level*, Phys. Lett. B **741**, 117 (2015), doi:10.1016/j.physletb.2014.12.028, 1410.6217.

[20] A. Celis, J. Fuentes-Martín and H. Serôdio, *A class of invisible axion models with FCNCs at tree level*, JHEP **12**, 167 (2014), doi:10.1007/JHEP12(2014)167, 1410.6218.

[21] I. Galon, A. Kwa and P. Tanedo, *Lepton-Flavor Violating Mediators*, JHEP **03**, 064 (2017), doi:10.1007/JHEP03(2017)064, 1610.08060.

[22] L. Calibbi, F. Goertz, D. Redigolo, R. Ziegler and J. Zupan, *Minimal axion model from flavor*, Phys. Rev. D **95**(9), 095009 (2017), doi:10.1103/PhysRevD.95.095009, 1612.08040.

[23] Y. Ema, K. Hamaguchi, T. Moroi and K. Nakayama, *Flaxion: a minimal extension to solve puzzles in the standard model*, JHEP **01**, 096 (2017), doi:10.1007/JHEP01(2017)096, 1612.05492.

[24] F. Björkeroth, E. J. Chun and S. F. King, *Flavourful Axion Phenomenology*, JHEP **08**, 117 (2018), doi:10.1007/JHEP08(2018)117, 1806.00660.

[25] M. Bauer, M. Neubert, S. Renner, M. Schnubel and A. Thamm, *Axionlike Particles, Lepton-Flavor Violation, and a New Explanation of a_μ and a_e*, Phys. Rev. Lett. **124**(21), 211803 (2020), doi:10.1103/PhysRevLett.124.211803, 1908.00008.

[26] J. Heeck and H. H. Patel, *Majoron at two loops*, Phys. Rev. D **100**(9), 095015 (2019), doi:10.1103/PhysRevD.100.095015, 1909.02029.

[27] C. Cornella, P. Paradisi and O. Sumensari, *Hunting for ALPs with Lepton Flavor Violation*, JHEP **01**, 158 (2020), doi:10.1007/JHEP01(2020)158, 1911.06279.

[28] A. Ibarra, M. Marín and P. Roig, *Flavor violating muon decay into an electron and a light gauge boson* (2021), 2110.03737.

[29] S. T. Petcov, *The Processes $\mu \to e + \gamma, \mu \to e + \bar{e}, \nu' \to \nu + \gamma$ in the Weinberg-Salam Model with Neutrino Mixing*, Sov. J. Nucl. Phys. **25**, 340 (1977), [Erratum: Sov.J.Nucl.Phys. 25, 698 (1977), Erratum: Yad.Fiz. 25, 1336 (1977)].

[30] W. J. Marciano and A. I. Sanda, *Exotic Decays of the Muon and Heavy Leptons in Gauge Theories*, Phys. Lett. B **67**, 303 (1977), doi:10.1016/0370-2693(77)90377-X.

[31] B. W. Lee and R. E. Shrock, *Natural Suppression of Symmetry Violation in Gauge Theories: Muon - Lepton and Electron Lepton Number Nonconservation*, Phys. Rev. D **16**, 1444 (1977), doi:10.1103/PhysRevD.16.1444.

[32] J. M. Cornwall, D. N. Levin and G. Tiktopoulos, *Derivation of Gauge Invariance from High-Energy Unitarity Bounds on the s Matrix*, Phys. Rev. D **10**, 1145 (1974), doi:10.1103/PhysRevD.10.1145, [Erratum: Phys.Rev.D 11, 972 (1975)].

[33] C. E. Vayonakis, *Born Helicity Amplitudes and Cross-Sections in Nonabelian Gauge Theories*, Lett. Nuovo Cim. **17**, 383 (1976), doi:10.1007/BF02746538.

[34] B. W. Lee, C. Quigg and H. B. Thacker, *Weak Interactions at Very High-Energies: The Role of the Higgs Boson Mass*, Phys. Rev. D **16**, 1519 (1977), doi:10.1103/PhysRevD.16.1519.

[35] M. S. Chanowitz and M. K. Gaillard, *Multiple Production of W and Z as a Signal of New Strong Interactions*, Phys. Lett. B **142**, 85 (1984), doi:10.1016/0370-2693(84)91141-9.

[36] A. M. Sirunyan *et. al.*, *Search for supersymmetric partners of electrons and muons in proton-proton collisions at* $\sqrt{s}$ = 13 TeV, Phys. Lett. B **790**, 140 (2019), doi:10.1016/j.physletb.2019.01.005, 1806.05264.

[37] G. Aad *et al.*, *Search for electroweak production of charginos and sleptons decaying into final states with two leptons and missing transverse momentum in $\sqrt{s} = 13$ TeV pp collisions using the ATLAS detector*, Eur. Phys. J. C **80**(2), 123 (2020), doi:10.1140/epjc/s10052-019-7594-6, 1908.08215.