Exclusive hadronic τ decays, within & beyond the Standard Model

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Abstract

Semileptonic exclusive tau decays in the Standard Model are reviewed. As it is well-known, they are a privileged arena to learn about low-energy hadronization. They also allow for a number of clean new physics tests, which have attracted much attention recently, probing scales as high as a few TeV. In light of forthcoming Belle-II data, perspectives on this area are bright.

Contents

1   Introduction

The τ is the only lepton massive enough to decay intro hadrons (mesons). As such, it has been (still is and will continue to be) a privileged tool to learn about the hadronization of QCD currents at low energies in a clean environment [1, 2].

In this workshop we have heard several talks devoted to this subject. These covered the interesting aspects that can be studied in semileptonic tau decays in the Standard Model (SM), aiming as well to uncover possible effects of new physics beyond it. Specifically, lepton universality [†] and CKM unitarity tests [†*], studies of second class currents [°], CP and T violation [□],

hadronic contributions to the muon g-2 [%], as well as multiparticle final-state interactions and determination of resonance pole parameters [△] were the focus of these talks [1].

Here, I will first review briefly hadronic tau decays in the SM and then discuss searches for non-standard interactions making use of them [2]. I will end with some concluding remarks.

2 Semileptonic τ decays in the Standard Model

At leading order, τ decays either leptonically, into $\nu_\tau \overline{\nu_{e/\mu}}(e/\mu)$, or hadronically, into $\nu_\tau \bar{u}D$ ($D = d, s$) with roughly one third into the former and the remaining two thirds into the latter, detected in a variety of meson final states [4]. Rich and large datasets accumulated and analyzed at CLEO, LEP, BaBar and Belle (and soon at Belle-II!) have fostered an increasingly precise knowledge of them over the last decades.

The matrix element for semileptonic tau decays, $\tau^- \to \nu_\tau H^-$, reads

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{uD} \bar{u}_{\nu_\tau} \gamma^\mu (1 - \gamma_5) u_\tau \mathcal{H}_\mu, \tag{1}$$

where G_F is the Fermi constant, V_{uD} is the CKM matrix element ($D = d, s$ for strangeness-conserving/changing transitions), and the lepton current multiplies the hadron vector

$$\mathcal{H}_\mu = \left\langle H \middle| (\mathcal{V}_\mu - \mathcal{A}_\mu) e^{i\mathcal{L}_{QCD}} \middle| 0 \right\rangle, \tag{2}$$

which encodes the hadronization process, creating the final-state mesons from the QCD vacuum, via the left-handed charged weak current, in presence of strong interactions. Lorentz and discrete QCD symmetries allow to decompose $\mathcal{H}_\mu$ in terms of a number of allowed tensor structures times the corresponding form factors (FFs), which are scalar functions of the kinematical invariants [3]. Data is, obviously, of uttermost importance in understanding these FFs. Theoretically, we are assisted by chiral symmetry [6–8], axiomatic quantum field theory properties, parton dynamics [9], ... Dispersive FF representations are extremely convenient, as they are best suited to fulfill analyticity, unitarity and crossing symmetry.

In the one-meson case, the relevant $\mathcal{H}^\mu$ is ($P = \pi/K$)

$$\left\langle P^-(p) \middle| \overline{D} \gamma^\mu \gamma_5 u \middle| 0 \right\rangle = -i\sqrt{2} f_P p^\mu, \tag{3}$$

where the P decay constant, f_P, is known from $P^- \to \mu^- \overline{\nu}_\mu$ ($f_\pi \sim 92$ MeV with this, chiral, normalization). If there is new physics, however, such experimentally determined f_P would differ from f_P^{QCD}, which is obtained from lattice data [10]. Radiative corrections (including structure-dependent effects) are essential in elucidating this [11–13]. LEP measurements of this one-meson tau decays branching ratios [14, 15] and by BaBar of their normalization with respect to the lepton channels [16] are fundamental for these tests. Belle observed, for the first time, $\tau^- \to \pi^- e^+ e^- \nu_\tau$ recently [17]. Measurements of these and other radiative processes are essential for the corresponding new physics tests and will hopefully be made at Belle-II.

For two-mesons, a convenient decomposition of $\mathcal{H}^\mu$ is

$$\left\langle P^-(p) P'^0(p_0) \middle| \overline{D} \gamma^\mu u \middle| 0 \right\rangle = C_{PP'-} \left\{ \left(p_- - p_0 - \frac{\Delta_{PP'}}{s} q \right)^\mu F_V^{PP'}(s) + \frac{\Delta_{PP'}}{s} q^\mu F_S^{PP'}(s) \right\}, \tag{4}$$

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[1] See, respectively, Gabriel López Castro (†), Alberto Lusiani (*), Bachir Moussallam (°), Zhi-Hui Guo and Frederic Noël (□), Álex Miranda (%), Fabian Krinner and Mikhail Mikhasenko (△) talks and corresponding contributions to these proceedings. These decays were also touched upon in the more general talks given by Bill Marciano, Zbigniew Was, Xiaorong Zhou, Denis Epifanov, Ami Rostomyan, Mogens Dam and Mike Roney and of course in the introductory historical talk given by Toni Pich and in the closing outlook talk by Hasaka-san (see these proceedings).

[2] A recent discussion of these topics can be found in ref. [3].

[3] An alternative, though equivalent, splitting is given using the so-called structure functions [5].

where $q^\mu = (p_- + p_0)^\mu$, $s = q^2$ and $\Delta_{PP'} = m_{P^-}^2 - m_{P'^0}^2$. The FFs above, $F_{S/V}^{PP'}(s)$ carry spin$^{\text{parity}}$ $0^+/1^-$ degrees of freedom, the scalar one being suppressed by the approximate $SU(3)$ flavor symmetry with respect to the usually dominant vector FF. $C_{PP'}$ are defined so that $F_V^{PP'} = 1 + \mathcal{O}(s)$ at low s ($C_{\pi\pi} = \sqrt{2}$, for instance).

$\tau^- \to \pi^- \pi^0 \nu_\tau$ and $\tau^- \to K^- K_S \nu_\tau$ decays admit a joint description in the isospin symmetry limit [18–20]. In this case, one can apply the standard (see e.g. refs. [20–22]) thrice-subtracted dispersive description of $F_V^{\pi\pi}(s)$ (and neglect $F_S^{\pi\pi}(s)$)

$$F_V^{\pi\pi}(s) = \exp\left[\alpha_1 s + \frac{\alpha_2}{2} s^2 + \frac{s^3}{\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\delta_1^1(s')}{(s')^3 (s' - s - i0)}\right],\qquad(5)$$

where the subtraction constants $F_V(0) = 1$ and $\{\alpha_i\}_{i=1,2}$ determine the polynomial expansion in s of this FF close to the origin. According to Watson's final-state interactions theorem [23], $\delta_1^1(s)$ equals the $\pi\pi$ scattering phaseshift in the elastic region, which is extremely well-known [24–26] and includes the dominant effect of the $\rho(770)$ resonance exchange as well as the leading (resummed) chiral logs. Asymptotically, this phase goes to π (and the FF modulus vanishes as $1/s$). Appropriate (smooth) interpolants for $\delta_1^1(s)$ between these two extreme regions will capture the dynamics of the $\rho(1450)$ and $\rho(1700)$ states, enabling the determination of their mass and width pole parameters [20–22] through fits to the data [15,27–30]. Di-pion radiative tau decays have been studied in Refs. [31–35] (see also Z. H. Guo and Á. Miranda, these proceedings) and provide an independent evaluation of the leading hadronic vacuum polarization contribution to the muon g-2 [36], $a_\mu^{HVP,LO}$.

In the $K\pi$ tau decay modes, the scalar FF is no longer negligible. It can be obtained by means of a coupled-channel ($K(\pi/\eta/\eta')$) strangeness-changing meson-meson scattering analysis [37,38]. The dispersive vector FF is constructed in analogy to the $\pi\pi$ case [39,40], and can benefit from $K_{\ell3}$ decays data [41–43]. In the $SU(3)$ flavor symmetry limit, the $K\eta^{(\prime)}$ vector FFs can be related to the $K\pi$ one [44] and a joint fit to $\tau^- \to K_S \pi^- \nu_\tau$ [45] and $\tau^- \to K^- \eta \nu_\tau$ [46] data improves particularly the extraction of the $K^*(1410)$ pole parameters [47]. $\tau^- \to K^- \pi^0 \nu_\tau$ branching fraction was best measured by BaBar [48] but the corresponding spectrum remains unpublished and the $\tau^- \to K^- \eta' \nu_\tau$ decays have not been discovered yet [49].

The $\pi^- \eta^{(\prime)}$ decay modes are much more difficult to understand. On the one hand, only upper limits exist on these [49,50]. On the other, G-parity suppresses them [4] and complicates their analysis [53]. As a result, two dispersive analyses of these decays [54,55] differ by an order of magnitude (due to the uncertain scalar contribution, see also Refs. [56–58]) in the predicted branching ratio for the η channel (see B. Moussallam, these proceedings).

No fully-dispersive treatment of three-meson tau decays [5] exist, in which three-body final-state interactions are accounted for completely. Chiral Lagrangian studies have been done for the 3π [59–61], $KK\pi$ [62] and $\eta\pi^-\pi^0$ [63] decay modes (which were incorporated into the corresponding version [64,65] of the TAUOLA library [66,67], together with the two-meson tau decays, featuring dispersive form factors, discussed before) and for $\tau^- \to (VP)^- \nu_\tau$ decays [68], where two/three of the pseudoscalar mesons (π, K) are close to the on-shell condition for a vector resonance (ρ, ω, ϕ). Dispersive analyses based on two-body final-state interactions were completed for the 3π [69] and $K\pi\pi$ [39] decay modes (see also M. Mikhasenko, these proceedings). Best measurements of the spectra of these decays still come from ALEPH [70–72] and CLEO [73]. We hope Belle-II will finally improve upon them. Higher multiplicity modes are omitted in this overview.

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[4]Other competing isospin-violating effects have been addressed in refs. [51,52].

[5]Four form factors appear in these processes: two of them carrying 1^+ degrees of freedom, one corresponding to 1^- (which is linked to the chiral anomaly) and another one to 0^+ (that is suppressed in the chiral limit).

3 Semileptonic τ decays beyond the Standard Model

For new physics searches, the main advantage of the effective field theory (EFT) formalism is that it allows to consistently use data at different energies, increasing the reach of the individual contributions by exploiting their synergy. For what concerns us here, semileptonic tau decays nicely complement both the traditional low-energy precision probes (Kaon and pion as well as nuclear beta decays) and the high-energy measurements (electroweak precision observables and LHC data).

The most general effective Lagrangian describing hadronic tau decays at dimension six is [74] [6]

$$
\begin{aligned}
\mathcal{L} = -\frac{G_F V_{uD}}{\sqrt{2}} \Big[& (1+\epsilon_L^\tau)\overline{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \overline{u}\gamma^\mu(1-\gamma_5)D + \epsilon_R^\tau\overline{\tau}\gamma_\mu(1-\gamma_5)\nu_\tau \cdot \overline{u}\gamma^\mu(1+\gamma_5)D \\
& +\overline{\tau}(1-\gamma_5)\nu_\tau \cdot \overline{u}(\epsilon_S^\tau-\epsilon_P^\tau\gamma_5)D + \epsilon_T^\tau\overline{\tau}\sigma_{\mu\nu}(1-\gamma_5)\nu_\tau \cdot \overline{u}\sigma^{\mu\nu}(1-\gamma_5)D \Big] + \text{h.c.},
\end{aligned}
\tag{6}
$$

where $\sigma^{\mu\nu} = i[\gamma^\mu, \gamma^\nu]/2$ and ϵ_i $(i = L, R, S, P, T)$ are effective couplings characterizing heavy new physics. The SM case is recovered for all $\epsilon_i = 0$. Although we kept the lepton flavor (τ) index in the ϵ_i, we omitted the corresponding quark flavor index (D). The $D = d$ and $D = s$ couplings would be the same according to minimal flavor violation (MFV) [76]. We note that the product $G_F V_{ud}$ denotes its determination from superallowed nuclear Fermi β decays (that includes, in fact, $\tilde{V}_{ud}^e$). As such [77], $G_F \tilde{V}_{uD}^e = G_F(1 + \epsilon_L^e + \epsilon_R^e)V_{uD}$, which will introduce a dependence of our results on $\epsilon_L^e + \epsilon_R^e$. The new physics scale suppressing these interactions, can be related to the ϵ_i^τ by $\Lambda \sim v/\sqrt{V_{uD}\epsilon_i}$, $v \sim 246$ GeV.

This framework has been applied to study tau decays into the following meson systems: $\eta^{(\prime)}\pi^-$ [78], $\pi^-\pi^0$ [79], $\pi^-(\pi^0/\eta)$ in combination with inclusive $\overline{u} \to d$ transitions [80], $(K\pi)^-$ [81] (with great focus on the CP asymmetry in $K_S\pi^-$ [81–85]), $K\eta^{(\prime)}$ [86] and a joint analysis of all one- and two-meson tau decays [87], recently updated using improved radiative corrections for the $(\pi/K)^-$ modes [13].

The $\eta^{(\prime)}\pi^-$ modes are very sensitive to non-SM scalar interactions, as the ϵ_S^τ dependence is enhanced by a factor $\sqrt{s}/(m_d-m_u)$. Unfortunately, the big uncertainty in the corresponding FF of these yet undiscovered modes implies a large error on the obtained bounds. Still, the limits on ϵ_S^τ [78] from the non-observation of the $\eta\pi^-$ mode bind $\Lambda > 3.5$ TeV at 90% C. L. (other limits given below correspond also to this confidence interval). The Belle-II discovery of this channel will allow to check these limits, and the measurement of its spectrum to increase the reach on new physics. Bounds are compatible, although slightly worse (and more uncertain) for the $\eta^{\prime}\pi^-$ case.

The $\pi^-\pi^0$ modes are quite sensitive to tensor interactions, ϵ_T^τ. Despite the parametric enhancement of ϵ_S^τ contributions is the same as for $\eta^{(\prime)}\pi^-$, the dynamical suppression of the corresponding scalar FF [54] spoils the prospects for binding it competitively. As in all other measured channels, a simultaneous fit of the (SM) dispersive FF parameters and of the ϵ_i to data is currently impossible, unfortunately. Notwithstanding, this could be achieved if Dalitz plot and angular distribution measurements (these and other observables are covered extensively in the refs. quoted in this section) were done. We hope this (and other similar observations) motivates Belle-II analyses in this direction. The most stringent limit is obtained [79] from a fit to Belle data [29] and binds $\Lambda > 3.5$ TeV. More precise Belle-II measurements will probe higher energy scales.

CP violation has attracted a lot of attention towards the $\tau \to K_S\pi\nu_\tau$ decays. This was

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[6]It can be obtained as the low-energy limit of the SM effective field theory (SMEFT) [75]. Contributions from right-handed neutrinos vanish at the leading order in the ϵ_i, to which we will stick.

triggered by the BaBar measurement of the corresponding CP asymmetry

$$A_{CP}^{\tau} = \frac{\Gamma(\tau^+ \to \pi^+ K_S \overline{\nu}_{\tau}) - \Gamma(\tau^- \to \pi^- K_S \nu_{\tau})}{\Gamma(\tau^+ \to \pi^+ K_S \overline{\nu}_{\tau}) + \Gamma(\tau^- \to \pi^- K_S \nu_{\tau})}, \tag{7}$$

yielding $A_{CP}^{\tau}|_{\exp} = -3.6(2.3)(1.1) \times 10^{-3}$ [88], which is 2.8σ away from the SM prediction, $A_{CP}^{\tau}|_{th} = 3.6(1) \times 10^{-3}$ [89], that is determined by the minutely known neutral Kaon mixing [4]. It must be noted that the corresponding Belle measurement [90] of a binned CP asymmetry was compatible with zero at the permille level in the four measured bins. Within the SMEFT, it is impossible [82] that the anomaly found by BaBar can be explained as a result of heavy new physics. The corresponding contribution would be proportional to both the strong and the weak phase difference between the vector and tensor FFs. The former vanishes in the elastic region [23, 82, 91] and grows slowly, according to unitarity constraints, due to inelastic effects [81, 82]. The latter must be, at most, $\sim \mathcal{O}(10^{-5})$ due to the experimental constraints on $D^0 - \overline{D}^0$ mixing and the neutron electric dipole moment [82]. This restricts the heavy new physics contribution to A_{CP}^{τ} to be $\leq 10^{-6}$ [81, 82], three orders of magnitude smaller than needed to explain the BaBar measurement. Turning to CP-conserving observables, the $\tau^- \to (K\pi)^- \nu_{\tau}$ decays have good sensitivity to both $\epsilon_{S,T}$. The corresponding bounds [81] on them imply $\Lambda > 3$ TeV. This limit agrees with those previously discussed for $\overline{u} \to d$ transitions and support the possible universality of the ϵ^{τ} for $D = d, s$, *i.e.* MFV. Again, we hope that Belle-II data on strangeness-changing tau decays will allow to be sensitive to higher new physics' scales.

The best limits set in exclusive hadronic tau decays are collected in Table 1.

Coefficient	Limit	Source
$\hat{\epsilon}_S^d$	$-(2.4 \pm 5.3) \times 10^{-3}$	$\tau^- \to \eta \pi^- \nu_{\tau}$ [78]
$\hat{\epsilon}_T^d$	$-\left(1.3_{-2.2}^{+1.5}\right) \times 10^{-3}$	$\tau^- \to \pi^0 \pi^- \nu_{\tau}$ [79]
$\hat{\epsilon}_S^s$	$(1.3 \pm 0.9) \times 10^{-2}$	$\tau^- \to (K\pi)^- \nu_{\tau}$ [81]
$\hat{\epsilon}_T^s$	$(0.7 \pm 1.0) \times 10^{-2}$	$\tau^- \to (K\pi)^- \nu_{\tau}$ [81]

Table 1: Best limits on the $\hat{\epsilon}_i^D := \epsilon_i^{\tau}/(1 + \epsilon_L^e + \epsilon_R^e)$ coefficients, set from exclusive hadronic tau decays.

Finally we turn to global fits of the ϵ_i, which use different sets of data simultaneously. Combining $\tau^- \to \pi^- \nu_{\tau}$, $\tau^- \to \pi^- \pi^0 \nu_{\tau}$ [7], $\tau^- \to \eta \pi^- \nu_{\tau}$ (with hadron input from [78]) and the inclusive non-strange tau hadronic width, the following limits [80]

$$\begin{pmatrix} \epsilon_L^{\tau} - \epsilon_L^e + \epsilon_R^{\tau} - \epsilon_R^{\tau} \\ \epsilon_R^{\tau} \\ \epsilon_S^{\tau} \\ \epsilon_P^{\tau} \\ \epsilon_T^{\tau} \end{pmatrix} = \begin{pmatrix} 1.0 \pm 1.1 \\ 0.2 \pm 1.3 \\ -0.6 \pm 1.5 \\ 0.5 \pm 1.2 \\ -0.04 \pm 0.46 \end{pmatrix} \times 10^{-2}, \tag{8}$$

were set, corresponding to $\Lambda \gtrsim 2$ TeV. Fig. 1 of Ref. [80] nicely shows that the addition of hadronic tau data to the electroweak precision observables [95] and LHC constraints allows to shrink the allowed contour in the plane defined by right-handed tau couplings vs. difference of left-handed couplings to taus and electrons by more than a factor two.

The separate analysis of the $\Delta S = 0, 1$ exclusive (one and two mesons) hadronic tau decays [87] cannot disentangle $\epsilon_{R,P}^{\tau}$. Limits on the ϵ_i are, as in eq.(8), of $\mathcal{O}(10^{-2})$ for both $\Delta S = 0, 1$ and are given in the following:

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[7] This channel was used via its impact on $a_{\mu}^{HVP,LO}$ [31, 32], see [34] for an updated evaluation in the Resonance Chiral Lagrangian framework [91–94].

$$\begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^\tau \\ \epsilon_R^\tau + \frac{m_\pi^2}{2M_\tau(m_u+m_d)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 2.2 \\ 0.3 \pm 1.2 \\ -0.2 \pm 0.5 \\ -0.1 \pm 1.3 \end{pmatrix} \times 10^{-2}, \begin{pmatrix} \epsilon_L^\tau - \epsilon_L^e + \epsilon_R^\tau - \epsilon_R^\tau \\ \epsilon_R^\tau + \frac{m_K^2}{2M_\tau(m_u+m_s)}\epsilon_P^\tau \\ \epsilon_S^\tau \\ \epsilon_T^\tau \end{pmatrix} = \begin{pmatrix} 0.5 \pm 1.5 \\ 0.4 \pm 0.9 \\ 0.8 \pm 0.9 \\ 0.9 \pm 0.8 \end{pmatrix} \times 10^{-2}$$
$$\tag{9}$$

with the left(right) constraints corresponding to the $\Delta S = 0(1)$ sectors. The compatibility of eqs. (8) and (9) confirms the robustness of these bounds. Assuming MFV, a joint fit of both $\Delta S = 0, 1$ sectors can be performed [87]. This untangles $\epsilon_{R,P}^\tau$ albeit at the price of a very big error on (and correlation between) them.

4   Conclusion

Exclusive semileptonic tau decays remain to be a clean laboratory for increasing our knowledge on hadronization at low energies, where resonance properties (like their pole positions) can be determined very accurately. At the current level of precision, QCD-driven descriptions are necessary and benefit from experimental and lattice data by using dispersion relations and the known chiral and asymptotic limits. Such thorough understanding of these decays in the Standard Model enables a number of searches for new physics. Other talks in the conference have discussed lepton universality and CKM unitarity tests, searches for second class currents, CP and T violation studies, and analyses of $a_\mu^{HVP,LO}$ using tau data. Here focus was on the effective field theory analyses of these decays beyond the standard W exchange, which bind the corresponding new physics scale at a few TeV, competitively and complementary to Kaon, pion and nuclear beta decays as well as to electroweak precision observables or LHC data. Altogether, prospects for semileptonic tau decays, in light of Belle-II data, are bright.

Acknowledgements

Note added   Just before sending this manuscript, Ref. [96] appeared. Although it is not covered in this contribution, it will be detailed in the proceedings of the next TAU conference.

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