Breaking Standard Model global symmetries with the tau lepton

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December 9, 2021

16th International Workshop on Tau Lepton Physics (TAU2021),
September 27 – October 1, 2021
doi:10.21468/SciPostPhysProc.?

Abstract

Properties of the dynamics of the Standard Model processes such as lepton flavour preservation and lepton, or baryon, number conservation, are usually taken for granted. The only reason is that it seems that these non-gauge accidental symmetries of the Standard Model seem to be conserved in Nature with great accuracy. In addition, the coupling of lepton doublets to the electroweak gauge bosons is universal and does not depend on the flavour. The discovery of the non-zero mass of the neutrinos and, maybe, the hints coming from the B-anomalies at LHCb, point out to a different scenario. I will briefly summarize some aspects of these contingent symmetries as seen from tau lepton processes.

Contents

1 Introduction

If one tried to summarize in a single idea the main achievement of the XX century in particle physics, the concept of gauge symmetry would arguably be very high in the list. The forces depicted by the gauge symmetry of the Standard Model (SM) show a vast faithfulness when faced with the Nature that they describe. In addition, in the last years it has been confirmed the existence of a scalar sector, the Higgs, that for the first time generates a rather tiny non-gauge interaction. This setting establishes the *fundamental* symmetries of the SM. However, and although the SM is characterized by its gauge symmetry and its spontaneous breaking, its Lagrangian has other symmetries that have not been called for explicitly and that, simply, are there. They are usually called *accidental* and they are respected by the fundamental symmetry [1]. Indeed, in the SM colour is fundamental, but flavour is accidental.

Another feature of the SM lies in its renormalizability. This condition is so demanding that the Lagrangian automatically obeys additional symmetries not introduced specifically. This happens, for instance, with Quantum Electrodynamics where the most general renormalizable, gauge and Lorentz invariant Lagrangian generates an interaction that preserves all flavours [2]. In the Electroweak Theory case, only lepton flavour and baryon number are conserved (see Ref. [3]). Accordingly, non-renormalizable interactions arising from higher mass scales may violate those accidental symmetries.

The reason for the existence of more than one generation of fermions in the SM is still not asserted. Evidently, it has consequences on the dynamics such as GIM mechanism or CP violation, and the rich flavour physics, although the last one is very different in the lepton or the quark sectors. The non-interacting SM spectrum of particles has a global symmetry group that breaks under the introduction of the interaction. The SM electroweak gauge sector, with n_g generations (assuming the same number of generations for both quarks and leptons), has a large global symmetry [4], namely

$$U(n_g)^5 = U(n_g)_Q \times U(n_g)_{u_R} \times U(n_g)_{d_R} \times U(n_g)_\ell \times U(n_g)_{e_R} , \tag{1}$$

for the quark Q and lepton ℓ doublets, and the right-handed u_R, d_R and e_R. The $SU(n_g)$ components of this group are explicitly broken by the Yukawa sector. Hence, the largest group of global unitary field transformations commuting with the SM electroweak group, including gauge and non-gauge interaction (but massless neutrinos) and consisting now of three lepton generations ($n_g = 3$), is

$$U(1)_e \times U(1)_\mu \times U(1)_\tau \times U(1)_B \times U(1)_Y , \tag{2}$$

with e, μ, τ lepton flavours, B baryon number and Y hypercharge. As a consequence both lepton flavour and baryon number are conserved in the SM. In addition, lepton number, $L = N_e + N_\mu + N_\tau$, is also conserved. Notice that, when talking about leptons, the words generation, family or flavour, identify the same attribute.

Hence, the flavour physics in the SM shows the following features:

1/ If one takes a look to the gauge symmetry part only, we notice that the couplings of the gauge bosons to the electrically charged leptons are the same for the three generations or flavours. This is the property of *universality* of the lepton couplings in the SM. This is broken when neutrinos adquire non-degenerate masses, because the coupling of leptons to the charged current gets modified à la CKM.

2/ The flavour processes involving the quark sector are indeed complex and plenty. Apart of the conservation of the electric charge (at all perturbative orders) and the flavour conservation of the neutral current (at tree level), the only constraint from the symmetry

> is conservation of the baryon number: everything else is allowed. On the other side, the flavour physics granted to the lepton sector is rather poor: lepton flavour is conserved in all processes. The discovery of neutrino mixing of flavours, a feature beyond the SM, has come to ease this aspect, although only for non-electrically charged leptons.

If we intent to overcome the SM and look for new physics, the possible violation of accidental symmetries represents a good benchmark. After all, it seems that global symmetries seem to be more amenable to the experiment than the local ones [5]. We only need to expose the higher energy scale at which the higher-dimensional non-renormalizable operators violating those symmetries appear.

At present, the anomalies in decays of the B meson involving leptons, seemingly uncovered at the LHCb experiment [6], seem to point out to violations of universality in the lepton couplings. I will not dwell on this specific topic but will review the present status of the couplings involving the tau lepton in Section 2. After the discovery of massive neutrinos and, in consequence, that there is mixing between the three lepton flavours, it comes to mind immediately that there should also be processes involving lepton flavour violation for charged leptons, unless some unknown symmetry forbids it. There is a huge experimental effort on this line of research and we will comment on this topic, from the tau lepton point of view, in Section 3. Finally, in Section 4 I will recall how lepton and baryon number violations are intertwined.

2 Tau-lepton universality

Leptons interact with the gauge bosons of the electroweak SM both through electrically charged and neutral currents. For the first one the model predicts that the couplings will be the same only if neutrinos are massless, while the weak neutral couplings are the same for all leptons with equal electric charge. We will state both cases.

2.1 Charged-current interaction

The coupling of the lepton families to the charged gauge boson in the SM (with massless neutrinos) can be writen as

$$\mathcal{L}_{\text{SM}} \supset - \sum_{i=e,\mu,\tau} \frac{g_i}{2\sqrt{2}} \left[\overline{\nu}_i \gamma^\mu (1-\gamma_5) \ell_i W_\mu^\dagger \right] + h.c. , \tag{3}$$

with $g_e = g_\mu = g_\tau = g$, i.e. the coupling is universal for the three flavours. As a consequence the decay width of the W gauge boson into the charged lepton and its neutrino is the same for all flavours. The figures in the PDG [7] for the ratios of widths give, in a self-explanatory notation[1],

$$\mu/e = 0.996(8) , \qquad \tau/e = 1.043(24) , \qquad \tau/\mu = 1.070(26) , \tag{4}$$

showing a slight tension related with the tau coupling with respected to e and μ. In fact the figures in Eq. (4) come still from LEP data. The possibility of a breaking of universality with its origin in the tau lepton, i.e. imposing a global symmetry $[U(2)_{e,\mu} \times U(1)_\tau]^5$, coming from an energy scale much larger than the electroweak, and giving $D = 6$ operators, were analysed in Refs. [8] and [9]. These works concluded that they could not explain the seeming violation of universality coming from the tau lepton.

This tension seems to be fading away after the recent measurement by ATLAS [10] at LHC. They have found

$$\tau/\mu \, = \, 0.992(13) , \tag{5}$$

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[1] The figure in parentheses indicates the error in the last digit or digits.

in good agreement with the SM universality setting and showing that more precise experimental determinations may be required to close definitely this issue.

Charged-current universality is also examined through the dominant leptonic decays, namely $\tau \to \ell \overline{\nu}_\ell \nu_\tau$, for $\ell = e, \mu$. By assuming different couplings, like in Eq. (3), the phenomenological analyses of data [11, 12] give

$$|g_\mu / g_e| = 1.0017(16) \,, \qquad |g_\tau / g_e| = 1.0028(15) \,, \qquad |g_\tau / g_\mu| = 1.0011(14) \,, \tag{6}$$

that may show a slight discrepancy in the comparison of the tau and electron couplings.

2.2 Neutral-current interaction

The neutral gauge boson Z couples to the lepton families as given by

$$\mathcal{L}_{\text{SM}} \supset -\frac{g}{2\cos\theta_W} Z_\mu \sum_{f=e,\mu,\tau} \overline{f} \gamma^\mu (v_f - a_f \gamma_5) f \,, \tag{7}$$

where $v_f = T_3^f - 2Q_f \sin^2\theta_W$ and $a_f = T_3^f$, being T_3^f the weak isospin of lepton f and Q_f the charge of the lepton f in units of the positron electric charge. Hence, all leptons with equal electric charge have the same weak neutral couplings. Z physics was thoroughly studied at LEP and the ratios of flavour couplings are determined to be [12, 13]

$$\begin{aligned} v_\mu / v_e &= 0.961(61) \,, & a_\mu / a_e &= 1.0002(13) \,, \\ v_\tau / v_e &= 0.959(29) \,, & a_\tau / a_e &= 1.0019(15) \,, \\ v_\tau / v_\mu &= 0.997(68) \,, & a_\tau / a_\mu &= 1.0017(17) \,, \end{aligned} \tag{8}$$

that, but for a minor discrepancy in the comparison of the couplings of tau and electron, is in good agreement with the universality of those couplings.

It is fair to conclude that the present discrepancies at B decays in LHCb [6], both in neutral as in charged driven processes, if confirmed might have as origin a higher energy scale beyond the SM.

3 Tau-lepton flavour violation

The global symmetry of the SM, when the Yukawa couplings are switched on, but with massless neutrinos, is represented by the group of Eq. (2) that indicates that lepton flavour is conserved in all processes. However, this feature was spoiled by the discovery of neutrino mixing that breaks conspicuously that conservation of flavour in a non-minimal way. Hence, once the symmetry is broken one could also expect the existence of processes where the flavour of charged leptons might not be conserved. Still, in spite of the enormous experimental effort, there are no signals of those processes. The most stringent upper-bound has been reached by the MEG experiment: $B(\mu^+ \to e^+ \gamma) < 4.2 \times 10^{-13}$ at 90% CL [14].

After BaBar and Belle, the Belle II detector [15] at the SuperKEKB collider in KEK (Tsukuba, Japan) is foreseen to provide a large amount of data in tau decay processes. In particular it has a dedicated program to look for lepton flavour violation (LFV) in decays of the tau lepton, both hadronic, i.e. $\tau \to \ell \pi$, $\tau \to \ell \pi \pi$, $\tau \to \ell K$, etc, and leptonic, i.e. $\tau \to \ell \gamma$, $\tau \to \ell' \ell^+ \ell^-$, and so on, with $\ell, \ell' = e, \mu$. The present bounds on branching ratios for these processes lie between 10^{-7} and 10^{-8} and are collected in Ref. [16], mainly due to BaBar and Belle, but also with the effort of LHC experiments, like LHCb [17]: $B(\tau^- \to \mu^- \mu^+ \mu^-) < 8.0 \times 10^{-8}$ and ATLAS [18]: $B(\tau \to 3\mu) < 3.76 \times 10^{-7}$, both at 90% CL. Meanwhile the bounds expected for Belle II, with an estimated integrated luminosity of 50 ab^{-1}, can be read from Ref. [15], reaching an improvement of, at least, one order of magnitude, i.e. $B < 10^{-9} - 10^{-10}$. For instance, at present we have $B(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ at 90% CL [7] (to be compared with the present bound on $\mu^+ \to e^+\gamma$ above) and Belle-II could better that figure by two orders of magnitude [15].

From a theory point of view, a huge work program on LFV is been performed, mainly since the discovery of neutrino mixing. It immediately came out that the inclusion of massive neutrinos in the SM would break lepton flavour in processes involving charged leptons. For instance, he prediction provided from either Dirac or Majorana massive fermions in the decay $\mu \to e\gamma$, was shown to be fairly negligible, although combination of both types of mass terms could provide a more playful discussion [19]. SUSY models, little Higgs, left-right symmetric models, Z' models, and others have widely been employed in the analyses of LFV tau decay processes reaching branching ratios that fall in the region that span the B-factories, i.e. $\mathcal{O}(10^{-7}-10^{-10})$.

All these theory models rely in the existence of an spectrum of particles in an energy region way beyond the electroweak scale. As commented in the introduction, it seems natural that accidental symmetries may be broken by non-renormalizable terms, to be added to the SM Lagrangian, that have their origin in higher energy scales. Hence, I useful tool to study LFV lies in the framework of effective field theories driven by the mass scale of New Physics and the symmetries and flavour structure of the SM, the so-called Standard Model Effective Field Theory (SMEFT) [20] at the electroweak scale, Λ_{EW}, that reads

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{D>4} \left(\frac{1}{\Lambda_{\text{LFV}}^{D-4}} \sum_i C_i^{(D)} \mathcal{O}_i^{(D)} \right), \tag{9}$$

where the $\mathcal{O}_i^{(D)}$ operators contain the SM spectrum of particles and its fundamental symmetries, but breaking the accidental lepton flavour conservation, and Λ_{LFV} is the scale of the ultraviolet theory responsible for lepton flavour violation, with $\Lambda_{\text{EW}} \ll \Lambda_{\text{LFV}}$. The dimensionless Wilson coefficients $C_i^{(D)}$ in Eq. (9) are determined by the new physics theory at the Λ_{LFV} scale. If the later is unknown these couplings are also unknown. Notice that $[\mathcal{O}_i^{(D)}] = [E^D]$, and a proper effective theory would require $C_i^{(D)} = \mathcal{O}(1)$. The lowest dimension operators implementing LFV but conserving baryon-number have $D = 6$ [20, 21].

A thorough analysis of LFV tau decays, in the SMEFT framework, was carried out in Ref. [22], where both leptonic and semihadronic decays were considered. It was shown that the study of both decay rates and differential distributions, could identify which $D = 6$ operators are more relevant and, consequently, offering a tool to discriminate among different ultraviolet theories. Following this work, we have carried out, a complete analysis of both, semihadronic decays of the tau lepton and ℓ-τ conversion, with $\ell = e, \mu$, in the presence of nuclei [23]. A previous study of this later process was performed in Ref. [24].

The hadronization of quark final states represents an still unsolved problem for QCD because it happens in a fairly non-perturbative energy region. The tau lepton, as the only lepton decaying into hadrons, represents a tidy environment to work with. Hence, in the study of LFV processes involving the tau lepton we have to care about this feature, either through model-independent dispersive methods [25] or within phenomenological approaches like resonance chiral theory [26,27] as we have done in Ref. [23]. In this later work we studied a fairly amount of semihadron decays, namely $\tau \to \ell P$, $\tau \to \ell PP$ and $\tau \to \ell V$, with $\ell = e, \mu$, P pseudoscalar mesons and V light vector resonances. Moreover we studied the processes $\ell N(A, Z) \to \tau X$, with $N(A, Z) = Fe(56, 26), Pb(108, 82)$, maybe feasibles at NA64 (CERN) [24]. Indeed, including 14 specific observables in our analysis, we concluded that

- Data on tau decays constrain the dynamics stronger than ℓ-τ conversion, though the later could be used to disentangle the relative weights of different $D = 6$ operators.

- The Wilson coefficient of the dipole operator $\mathcal{O}_\gamma = \cos\theta_W \mathcal{O}_{eB} - \sin\theta_W \mathcal{O}_{eW}$,

$$\mathcal{O}_{eB} = (\overline{L}_r \sigma^{\mu\nu} e_s)\, \varphi\, B_{\mu\nu}\,, \qquad\qquad \mathcal{O}_{eW} = (\overline{L}_r \sigma^{\mu\nu} e_s)\, \sigma_I\, \varphi\, W^I_{\mu\nu}\,, \tag{10}$$

  with the notation of Ref. [21], is the most constrained one, providing a bound, based on Belle data, of $\Lambda_{\text{LFV}} > 120\,\text{TeV}$, or $\Lambda_{\text{LFV}} > 330\,\text{TeV}$ (foreseen for Belle II) at 99.8% CL, for $C_\gamma \sim 1$. This also shows the relevant role to be played by Belle II.

4 Lepton or baryon number violation

The remaining global symmetry after Yukawas are introduced in the SM (2) shows that, a part of the baryon number B and the individual lepton flavours, also the total lepton number L is conserved. It is interesting to notice that the divergences of their corresponding currents are non-zero and equal,

$$\partial^\mu J^L_\mu = \partial^\mu J^B_\mu = n_g\, \frac{g^2}{32\pi^2}\, \varepsilon^{\mu\nu\alpha\beta}\, W^i_{\mu\nu}\, W^i_{\alpha\beta}\,, \qquad\qquad \partial^\mu J^{B-L}_\mu = 0\,, \tag{11}$$

where $W^i_{\mu\nu}$, $i = 1,2,3$ is the SU(2) field strength. As a consequence, B and L symmetries are anomalous, but B-L is not. The morale is that both global symmetries may be intertwined. An important consequence of this fact is that extensions of the SM cannot include a gauge boson related with B or L violation, but it is possible to have one associated to the global symmetry B-L. This gauge boson would violate B and L but would preserve B-L. However, it that boson exists it should be very heavy ($M_{\not{B}-\not{L}} > 10^{14}\,\text{TeV}$) because the fair stability of the proton. For instance, the B-L allowed decay $p \to \pi^0 \mu^+$ has a lifetime $\tau > 16 \times 10^{33}$ years at 90% CL [7].

Let us take a look to these symmetries from the SMEFT point of view (9). Then, we have to consider a new ultraviolet scale $\Lambda_{\text{L,B}}$ where we can have violation of lepton and/or baryon numbers. As above, we only consider those operators with SM spectrum of particles only. There is one $D = 5$ operator that provides processes with $\Delta L = 2$, $\Delta B = 0$ [28]. Five $D = 6$ operators furnish dynamics with $\Delta L = \Delta B$ [21,29]; accordingly, the later conserve B-L, hence if they dominate proton decay the selection rule requires that there should be an antilepton in the decay products. There are 20 $D = 7$ operators [30], all of them with $\Delta L \neq 0$ and 7 of them with $\Delta B = 1$. Those that do not violate B satisfy $\Delta L = 2$. Baryon-number violating operators satisfy $\Delta L = -1$. As a conclusion all $D = 7$ operators produce $\Delta(B - L) = 2$ processes. Higher dimensional operators have a more complex structure [31,32]. Although, it has been proven [33] that the correspondence between the dimensions D of the operators and the B and L quantum number transitions have some systematic adjustment,

$$\begin{aligned} D &= \text{even} \qquad \longrightarrow \qquad |\Delta B - \Delta L| = 0, 4, 8, 12, \ldots 2(2n)\,, \\ D &= \text{odd} \qquad \longrightarrow \qquad |\Delta B - \Delta L| = 2, 6, 10, 14, \ldots 2(2n+1)\,, \end{aligned} \tag{12}$$

with $n \in \mathbb{N}_0$.

Tau decays open a new line to look for the violation of this global symmetry. Indeed, decays into proton (or antiproton) have been looked for recently by LHCb and Belle, as shown in Table 1. Unfortunately, we think that those branching ratios, if they exist, should be really tiny. The reason is that they should also provide channels of decay for the proton (with a virtual tau lepton) [34] and, as pointed out above, the lifetime of the proton is, at least, huge. Notwithstanding, it is important to pursue the experimental search of these processes, a task also for Belle-II, as a surprise could be around the corner.

BaBar and Belle also have looked for processes with $\Delta L = 2$ but not involving baryons [16] and these will also be a target for Belle-II. An analysis within SMEFT with $D = 5, 7$ operators

$B \times 10^8$	Belle [35]	LHCb [17]
$B(\tau^- \to \overline{p}\mu^+\mu^-)$	< 1.8	< 23
$B(\tau^- \to p\mu^-\mu^-)$	< 4.0	< 44
$B(\tau^- \to \overline{p}e^+e^-)$	< 3.0	
$B(\tau^- \to pe^-e^-)$	< 3.0	
$B(\tau^- \to \overline{p}e^+\mu^-)$	< 2.0	
$B(\tau^- \to \overline{p}e^-\mu^-)$	< 1.8	

Table 1: Upper bounds on $\Delta B = \Delta L = \pm 1$ processes at LHCb and Belle, at 90% CL.

of the processes $\tau^+ \to \ell^- P^+ P^+$, with $\ell = e, \mu$ and P pseudoscalar mesons has also been carried out [36]. Alas, their conclusion is that for Wilson coefficients equal unity and $\Lambda_L \sim 1\,\text{TeV}$ the branching ratios for those processes are around $\mathcal{O}(10^{-20})$, namely ten orders of magnitude lower than present experimental bounds.

5 Conclusion

Gauge symmetries of the SM are all fundamental while its global symmetries are accidental. Moreover, as emphasized by S. Weinberg, "most of the experimentally discovered symmetries of elementary particle physics are *accidental* symmetries" [37].

The properties of the tau lepton make it a relevant benchmark in the study of violations of the SM global symmetries. This is mainly due to its high mass, that enlarges the number of its decays. The attention has increased after the signals of seeming violation of universality in B decays at LHCb or the search of evidence of lepton flavour violation in charged leptons after the discovery of neutrino mixing. If any of these facets is detected, it would unveil a new energy regime $\Lambda > \Lambda_{\text{EW}}$ where new physics awaits for us. In this quest, Belle-II will provide, in the next years, a huge amount of information on tau decays, that will allow us to keep the hunting.

Although I have paid more attention to the use of effective field theories in the analyses of tau data with this goal, because I think that they provide a wide model-independent framework able to provide sound information, I would also like to comment that models of interaction are also welcome. In my opinion, an interplay between models and effective theory frameworks provides and ideal working scenario.

Acknowledgements

I would like to thank the organizers of the 16th International Workshop on Tau Lepton Physics (TAU2021) for their kind invitation to provide this talk. This work has been supported in part by Grant No. MCIN/AEI/FPA2017-84445-P and by MCIN/AEI/10.13039/501100011033 Grant No. PID2020-114473GB-I00, by PROMETEO/2017/053 and PROMETEO/2021/071 (GV).

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