

Dilaton in scalar QFT: a no-go theorem

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Abstract

Spontaneous scale invariance breaking and the associated Goldstone boson, the dilaton, is investigated in renormalizable, unitary, interacting non-supersymmetric scalar field theories. Even though at leading order it is possible to construct models which give rise to spontaneous scale invariance breaking and indeed a massless dilaton can be identified, it is shown that the particular form of the running coupling β -function spoil this property already at 1-loop. **As a result spontaneous scale symmetry breaking** can not occur in renormalizable, unitary 4 dimensional scalar QFT.

1 Introduction

A massless (or approximately massless) dilaton is thought to arise in a number of field theories where (approximate) scale invariance is spontaneously broken. If this happens an effective field theory should be able to describe the low energy dynamics of the dilaton along with other potentially light degrees of freedom. These other light degrees of freedom are often also Goldstone bosons originating from spontaneous breaking of other symmetries besides scale invariance. These other symmetries might be also approximate and in this case the corresponding Goldstones would also be only approximately massless.

Conformal invariance, or invariance only by a subgroup, scale transformations, may play a role in the Standard Model and its extensions in a number of ways. The only scale symmetry breaking parameter of the Standard Model is the Higgs mass, at least at leading order. Quantum effects lead to additional scale symmetry breaking (dimensionful parameters), most notably by anomalous breaking of scale invariance and by spontaneous breaking of global symmetries. One class of ideas explores the possibility that scale symmetry itself is spontaneously broken and to what extent the Higgs boson (or other light particles) may be identified with the corresponding light dilaton [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. In particular, the smallness of the ratio between the Higgs mass and the Planck scale, which appears to be an enormously fine tuned quantity, is then explained by scale symmetry and its spontaneous breaking [13]. The same could be said about another fine tuning problem of the Standard Model, the smallness of the cosmological constant as well [14]. For early appearances of spontaneous breaking of scale invariance and its implications in QFT see [15, 16, 17].

The Higgs boson may also be viewed as a generic composite particle in several extensions of the Standard Model inspired by strong dynamics [18, 19] with or without a dilatonic interpretation. In these scenarios the spectrum of a strongly interacting new sector is thought to include a composite light scalar and if so, may be identified with the Higgs. Furthermore, the lightness may be related to the dilatonic nature of the particle although this interpretation is far from clear [20, 21]. In any case, the strongly interacting nature of the underlying theory makes the study of detailed properties of the low energy excitations difficult, and served as the major motivation for a surge in non-perturbative studies recently [22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37] as well as descriptions in terms of effective theories [38, 39, 40].

Whether a particular light degree of freedom in a given field theory can be identified as a dilaton is often non-trivial. Even though the effective theory might be weakly coupled, the underlying theory is often strongly coupled, complicating the identification of the physical degrees of freedom between the two. The fact that the light particles in question, one of which would be the hypothetical dilaton, are often not exactly massless further complicates the precise identification, especially if there are no parameters which would control the masses separately. For example, non-abelian gauge theories are often thought to give rise to a light dilaton if the fermion content is not far from the conformal window, but the lack of an obvious control parameter for its mass makes this conclusion more conjectural than firmly established. In this example the underlying theory is strongly coupled so the bridge from it to the effective theory is beyond analytical tools. Hence it is not possible to simply derive the effective theory in a top-down approach. As is often the case with effective theories one needs to consider the most general model with the given degrees of freedom and symmetries and match the unknown coefficients to observables in the underlying theory. However, if it is not known whether the underlying theory does give rise to a dilaton or not, one will not know whether a dilaton field should be included in the effective theory to begin with. Even if a dilaton is included in the effective theory, its interactions are not sufficiently constrained

by scale symmetry. In contrast, the effective theory describing Goldstones corresponding to the chiral symmetry breaking in gauge theory is essentially unique and fixed by the pattern of symmetry breaking. If a dilaton is to be coupled to these Goldstones, the form of the coupling is not fixed by scale symmetry and there are different conjectures for the detailed form of the coupled system.

The main motivation for the present work was to construct a renormalizable, weakly coupled and unitary theory in which spontaneous scale symmetry breaking unambiguously takes place and is fully in the realm of perturbation theory. In this case it would be possible to follow the dynamics from the underlying theory down to the effective theory describing the massless or light dilaton and potentially the other Goldstones, in a fully controlled manner. In order to avoid special features specific only to supersymmetric theories, non-supersymmetric models are sought.

Ideally, one would start with a weakly coupled CFT with a vanishing β -function and break scale symmetry spontaneously only, generating massive particles as well as the massless dilaton. We know of no renormalizable, non-supersymmetric and unitary such example [9]. Hence we relax the requirement of an exactly conformal underlying QFT and investigate unitary and massless scalar field theories. The couplings of course run, which corresponds to anomalous scale symmetry breaking, but nonetheless all particles can be arranged to be massless. More precisely either by tuning the masses or by working with a regularization such as $\overline{\text{MS}}$ which forbids the generation of mass terms if the bare masses are zero, one indeed has an underlying theory with massless particles only. The anomalous breaking only enters as logarithmic scale dependence for the couplings and anomalous dimensions. **If spontaneous scale symmetry breaking could take place in addition to the anomalous breaking, a light dilaton could be unambiguously identified. Of course it would not be exactly massless, precisely because of the anomaly, but would nevertheless be light.**

The main result is that although at leading order there is no obstruction to construct models with the desired properties, these are spoiled already at 1-loop. Hence in renormalizable unitary scalar QFT **spontaneous scale symmetry breaking** can not take place.

It is well known that supersymmetric models with the desired properties do exist. Most notable are $\mathcal{N} = 2, 4$ SUSY Yang-Mills [41]. If the scalar vevs are all zero, scale symmetry (even the larger full conformal group) is intact and in QFT all particles are massless with a vanishing β -function. Giving non-zero vevs to the scalars leads to spontaneous scale symmetry breaking and a corresponding exactly massless dilaton, along with other massless and massive states. Supersymmetry ensures that the flat direction required for the dilaton is not lifted by quantum corrections at any order; see e.g. [42].

Recently, the first non-supersymmetric example was found [43]. In this example the double scaling limit of γ -deformed $\mathcal{N} = 4$ SUSY Yang-Mills [44, 45, 46, 47] is considered at strong deformation and weak coupling, in the large- N limit, leading to so-called fishnet CFT's [48, 43]. The resulting theory is purely bosonic and renormalizable, however it is non-unitary. Non-unitarity is in fact crucial to show that the flat direction giving rise to a massless dilaton is not lifted by quantum effects.

These observations about supersymmetric and non-unitary non-supersymmetric examples were our primary motivation for the present work. Our result shows that a well-defined dilaton in renormalizable, non-supersymmetric and unitary perturbative QFT is not easy to construct; in purely scalar QFT it is in fact impossible. For a recent review on the fate of scale symmetry in QFT along with phenomenological applications in cosmology, gravity and particle physics in general, see [14].

In section 2 a number of examples are provided at leading order demonstrating that a classical dilaton can easily arise, either as the sole massless mode or together with other Goldstone bosons. Radiative corrections are spelled out in section 3 for all the examples and

the general argument is given in section 4 why not only in the examples but in all scalar QFT 1-loop corrections lift the flat directions responsible for the dilaton. We end with a set of conclusions and future outlook in section 5.

2 Dilaton at leading order

We are seeking interacting renormalizable scalar field theories described by scale invariant actions with the property that this symmetry breaks spontaneously. The generic form at tree level is then,

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi_a \partial^\mu \phi_a - \mathcal{V}(\phi), \quad (1)$$

where the potential \mathcal{V} contains only dimensionless couplings. No particular global symmetry is imposed, only scale symmetry, leading to

$$\mathcal{V}(\phi) = \frac{\lambda_{abcd}}{4!} \phi_a \phi_b \phi_c \phi_d. \quad (2)$$

The real couplings λ_{abcd} and real fields ϕ_a are understood to be bare quantities. Scale invariance is clearly present, the action $S = \int d^4x \mathcal{L}$ is invariant under the space-time symmetry $x \rightarrow e^{-s}x$ once the scalar field ϕ_a is transformed according to its mass dimension, $\phi_a(x) \rightarrow e^s \phi_a(e^s x)$. In particular linear, quadratic or cubic terms in the fields are not allowed.

The potential \mathcal{V} is required to be non-negative for stability of the vacuum. Then clearly $\phi_a = 0$ always corresponds to a vacuum state, one which does not break scale invariance and all particles are massless. We would like to construct potentials which possess non-trivial minima $\phi_a = v_a \neq 0$. Once such a minimum exists the corresponding vacuum will break scale invariance and some states will acquire masses proportional to v_a .

There is no obstruction at leading order, the simplest example is given by a two component model,

$$\mathcal{V} = \frac{\lambda}{4} \phi_1^2 \phi_2^2. \quad (3)$$

Clearly the potential is non-negative and possesses infinitely many stable minima. The choice $(\phi_1, \phi_2) = (0, 0)$ corresponds to a scale symmetric vacuum, while $(\phi_1, \phi_2) = (v, 0)$ and $(\phi_1, \phi_2) = (0, v)$, with arbitrary v , correspond to vacua which do break scale invariance spontaneously.

Expanding around the scale invariant vacuum leads to two massless bosons interacting through a quartic interaction. More interesting is the expansion around a scale symmetry breaking vacuum, for definiteness let us choose $(\phi_1, \phi_2) = (0, v)$, and the corresponding fluctuating fields will be denoted by η and χ ,

$$\begin{aligned} \phi_1 &= \eta \\ \phi_2 &= v + \chi. \end{aligned} \quad (4)$$

The potential becomes,

$$\mathcal{V} = \frac{\lambda}{4} v^2 \eta^2 + \frac{\lambda}{2} v \eta^2 \chi + \frac{\lambda}{4} \eta^2 \chi^2, \quad (5)$$

which clearly describes a massive particle η with $M^2 = \frac{1}{2}\lambda v^2$ and a massless particle χ , the dilaton ¹. The two types of particles are interacting through a cubic and quartic interaction.

Goldstone's theorem applies, the direction given by χ is a flat direction of the potential and hence corresponds to a massless particle. It is well-known that spontaneous breaking of space-time symmetries behave differently from spontaneous breaking of global symmetries in terms of counting Goldstone bosons, both in non-Lorentz invariant [49] and in Lorentz invariant theories [50]. In the case of scale symmetry the naive counting however does apply, one spontaneously broken symmetry corresponds to one Goldstone boson.

It is possible to generalize the model (3) to describe an interacting n -component and a 1-component field, ϕ_a and Φ . The potential

$$\mathcal{V} = \frac{h_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d + \frac{1}{2}g_{ab}\phi_a\phi_b\Phi^2 \quad (6)$$

with dimensionless couplings h_{abcd} and g_{ab} fulfilling suitable stability conditions gives rise to a scale symmetry respecting vacuum $(\phi_a, \Phi) = (0, 0)$ as well as scale symmetry breaking ones $(\phi_a, \Phi) = (0, v)$ with arbitrary v . Expanding around one of these latter vacua we have fluctuating fields η_a and χ ,

$$\begin{aligned} \phi_a &= \eta_a \\ \Phi &= v + \chi, \end{aligned} \quad (7)$$

leading to the potential,

$$\mathcal{V} = \frac{1}{2}g_{ab}\eta_a\eta_b v^2 + g_{ab}v\eta_a\eta_b\chi + \frac{1}{2}g_{ab}\eta_a\eta_b\chi^2 + \frac{h_{abcd}}{4!}\eta_a\eta_b\eta_c\eta_d. \quad (8)$$

Before symmetry breaking we had $n+1$ massless particles. After scale symmetry breaking we have n massive particles with mass matrix $M_{ab}^2 = g_{ab}v^2$ and a massless dilaton described by χ and again the two types of particles are interacting through cubic and quartic interactions. Goldstone's theorem applies again, one spontaneously broken symmetry corresponds to one massless Goldstone boson.

Something curious is nonetheless going on relative to generic field theories with Goldstone bosons. Generically, without symmetry breaking particles are massive and Goldstone's theorem provides an explanation why some particles *become* massless once the symmetry is spontaneously broken. In our case, since we start with a scale invariant action, all particles are massless from the start if scale symmetry is intact. Hence massless particles do not require a special explanation, their vanishing mass is simply a consequence of intact scale symmetry. After scale symmetry breaking a mass scale is generated and **Goldstone's** theorem ensures that one particle *remains* massless. At the same time all other particles acquire a mass. The non-trivial content of Goldstone's theorem in this case seems to be the precise number of particles which become *massive*, as opposed to becoming *massless*, after symmetry breaking.

As a third and final example let us incorporate spontaneous global symmetry breaking along with scale symmetry breaking, in which case we expect a dilaton as well as other Goldstone bosons. A potential with these properties is,

$$\mathcal{V} = \frac{\lambda}{4!}(\phi_a\phi_a - \Phi^2)^2. \quad (9)$$

The model is clearly scale invariant and also has an $O(n)$ symmetry. The trivial vacuum $(\Phi, \phi_a) = (0, 0)$ again breaks neither scale invariance nor the global symmetry, while the

¹In this particular example a global $\mathbb{Z}_2 \times \mathbb{Z}_2$ given by flipping the sign of the two fields is also broken to \mathbb{Z}_2 but this is of no significance for our discussion. Same applies to the models (6) and (9).

non-trivial vacua,

$$(\Phi, \phi_1, \dots, \phi_{n-1}, \phi_n) = (v, 0, \dots, 0, v), \quad (10)$$

with arbitrary v does break both. In particular $O(n)$ breaks to $O(n-1)$ hence we expect n Goldstones, $n-1$ from the breaking of the global symmetry and an additional one as the dilaton. Indeed, if fields $\eta_0, \eta_1, \dots, \eta_n$ are introduced,

$$\begin{aligned} \Phi &= v + \eta_0 \\ \phi_A &= \eta_A, \quad 1 \leq A \leq n-1 \\ \phi_n &= v + \eta_n, \end{aligned} \quad (11)$$

the potential becomes

$$\mathcal{V} = \frac{\lambda}{6} v^2 (\eta_n - \eta_0)^2 + \frac{\lambda}{6} v (\eta_n - \eta_0) (\eta_A^2 + \eta_n^2 - \eta_0^2) + \frac{\lambda}{4!} (\eta_A^2 + \eta_n^2 - \eta_0^2)^2. \quad (12)$$

A change of basis to $\xi = (\eta_n - \eta_0)/\sqrt{2}$ and $\chi = (\eta_n + \eta_0)/\sqrt{2}$ then leads to

$$\mathcal{V} = \frac{\lambda}{3} v^2 \xi^2 + \frac{\lambda\sqrt{2}}{6} v \xi (\eta_A^2 + 2\xi\chi) + \frac{\lambda}{4!} (\eta_A^2 + 2\xi\chi)^2, \quad (13)$$

which shows that ξ is massive with $M^2 = \frac{2}{3}\lambda v^2$ and the remaining n particles are massless, χ is the dilaton and η_A are the $n-1$ Goldstones corresponding to the breaking $O(n) \rightarrow O(n-1)$.

Goldstone's theorem applies again, out of the $n+1$ massless particles, exactly one becomes massive after symmetry breaking, n remain massless.

Model (9) may be generalized further to the so-called biconical models. The field Φ in this case has m components enlarging the original symmetry to $O(n) \times O(m)$. We will not discuss this setup further, rather just note that it has recently attracted interest; see [51] for a detailed discussion.

For completeness we note that the dilaton χ is often parametrized as $\chi = f e^{\sigma/f}$ with a new field σ and dimensionful parameter f . Scale transformations $x \rightarrow e^{-s} x$ are realized non-linearly on σ ,

$$\sigma(x) \rightarrow \sigma(e^s x) + f s. \quad (14)$$

The discussion so far was completely classical and we turn to loop corrections in the next section.

3 Radiative corrections

As shown in the previous section there is no obstruction to unambiguously define an exactly massless dilaton classically using a suitably chosen potential. One might wonder if a consistent renormalizable QFT can be built using the corresponding tree-level potentials. Were such a construction successful, it would provide an example of a renormalizable interacting QFT describing some massive particles and an approximately massless dilaton which is not just an effective theory². The main conclusion from this section will be that this is actually not possible in the examples given in section 2. In the next section it will be shown that the same conclusion applies to any scalar QFT.

²Here we ignore the fact that scalar theories are trivial and focus on perturbative renormalizability only.

One note is in order about our starting point: scalar QFT is never conformal, scale symmetry is anomalous, the trace of the energy-momentum tensor is proportional to the non-zero β -function. Even though the anomaly is present, it is possible to either tune all masses to zero or use a regularization procedure (for example dimensional regularization) in which renormalized masses are not generated if the bare masses are set to zero. This means that the anomaly itself does not generate masses, the resulting scale dependence only enters the coupling constants and anomalous dimensions in a logarithmic fashion. Hence it is meaningful to ask whether spontaneous breaking (on top of the already present anomalous breaking) leads to **an approximately** massless dilaton and some massive particles or not.

Clearly, the ideal setup would be a CFT with a zero β -function and only spontaneous scale symmetry breaking by some vevs. We do not know of any non-supersymmetric, unitary, interacting QFT of this type, hence relax the requirements to allow for anomalous breaking to begin with. **This will result in a setup where a massless dilaton is not expected, but a light one would arise if spontaneous breaking take place along with the anomalous breaking.**

Let us work within dimensional regularization and $\overline{\text{MS}}$ scheme and all couplings and fields will be assumed to be renormalized in this section.

First, let us see the effect of loop corrections on our simplest model (3). If we are to have the property that a non-trivial scale symmetry breaking vacuum exists in QFT, $\langle\phi_2\rangle \neq 0$, the form of the potential should either remain the same as in (3) or only a term of the type ϕ_1^4 should be generated. In other words the term ϕ_2^4 is forbidden. Unfortunately there is no symmetry which would prohibit this term at all loop order, hence it is expected that it will be generated perturbatively. Indeed, if all terms are included at tree-level which are generated at 1-loop, so that the theory is renormalizable, we must start with the potential,

$$\mathcal{V} = \frac{\lambda_1}{4!}\phi_1^4 + \frac{\lambda_2}{4!}\phi_2^4 + \frac{\lambda_{12}}{4}\phi_1^2\phi_2^2. \quad (15)$$

The set of renormalized couplings $\lambda_1, \lambda_2, \lambda_{12}$ do close under the RG flow, at 1-loop we have,

$$\begin{aligned} \mu \frac{d\lambda_1}{d\mu} &= \frac{3}{16\pi^2} (\lambda_1^2 + \lambda_{12}^2) \\ \mu \frac{d\lambda_2}{d\mu} &= \frac{3}{16\pi^2} (\lambda_2^2 + \lambda_{12}^2) \\ \mu \frac{d\lambda_{12}}{d\mu} &= \frac{1}{16\pi^2} \lambda_{12} (\lambda_1 + \lambda_2 + 4\lambda_{12}) . \end{aligned} \quad (16)$$

It is clear what the problem is: the subspace $\lambda_2 = 0$ is not invariant under the RG flow. The tree-level potential corresponding to $\lambda_2 = 0$ and $\lambda_1 \neq 0, \lambda_{12} \neq 0$ has the desired property in terms of giving rise to **spontaneous breaking**, but already at 1-loop $\lambda_2 \neq 0$. This means that the flat direction which was essential for **spontaneous breaking** is lifted by quantum effects and a dilaton **can not be identified**.

It is worth emphasizing that a non-zero β -function in itself is not necessarily a problem a priori. For instance if λ_2 would renormalize multiplicatively, perhaps because of some symmetry argument, a non-trivial RG flow in the $(\lambda_1, \lambda_2 = 0, \lambda_{12})$ plane would not lead to a lifting of the flat direction even at 1-loop. If the multiplicative renormalization of λ_2 would hold to all loops, then in full perturbative scalar QFT a non-zero β -function would be compatible with **spontaneous scale symmetry breaking**. The fact that this does not happen follows from the particular form of the β -function and not directly from its non-vanishing nature.

A similar analysis holds for the model (9). In order to include all terms at leading order

which are generated perturbatively, we must consider,

$$\mathcal{V} = \frac{\lambda_1}{4!}(\phi_a\phi_a)^2 + \frac{\lambda_2}{4!}\Phi^4 + \frac{\lambda_{12}}{4}\phi_a\phi_a\Phi^2. \quad (17)$$

The β -functions for the three couplings,

$$\begin{aligned} \mu \frac{d\lambda_1}{d\mu} &= \frac{3}{16\pi^2} \left(\frac{n+8}{9} \lambda_1^2 + \lambda_{12}^2 \right) \\ \mu \frac{d\lambda_2}{d\mu} &= \frac{3}{16\pi^2} (\lambda_2^2 + n\lambda_{12}^2) \\ \mu \frac{d\lambda_{12}}{d\mu} &= \frac{1}{16\pi^2} \lambda_{12} \left(\frac{n+2}{3} \lambda_1 + \lambda_2 + 4\lambda_{12} \right). \end{aligned} \quad (18)$$

again lead to the conclusion that radiative corrections destroy the property of having a non-trivial vacuum. In order to preserve the form (9) the combinations λ_1/λ_2 and λ_1/λ_{12} should be RG-invariant but they are not, already at 1-loop.

Similarly, the term Φ^4 will be generated at 1-loop in the model (6) which is missing at the classical level. Once it is included the flat direction is lifted and spontaneous scale symmetry breaking does not take place anymore.

4 Generic scalar QFT

The conclusions about the three examples hold also generally and is our main result. Let us start with a generic tree-level potential (2) for an N -component scalar field and assume the three ingredients necessary for our desired construction, a non-zero stable perturbative vacuum exists $\langle\phi_a\rangle = v_a$ at tree-level, the model is perturbatively renormalizable, and the theory is unitary. In this section all quantities, ϕ_a , v_a , \mathcal{V} , λ_{abcd} , etc., are assumed to be renormalized.

Multiplicative renormalization of the fields ϕ_a brings in μ -dependence as usual. The corresponding anomalous dimensions are γ_{ab} and at 1-loop we have $\gamma_{ab} = 0$. Having a non-zero vacuum $\langle\phi_a\rangle = v_a(\mu) \neq 0$ at tree-level means,

$$\mathcal{V} = \frac{1}{4!} \lambda_{abcd} v_a v_b v_c v_d = 0. \quad (19)$$

Note that we are minimizing the (renormalized) tree-level potential, hence we are not studying the well-known question of generating symmetry breaking from radiative corrections a la Coleman-Weinberg [52].

If condition (19) is to hold for all μ in perturbation theory, we must have,

$$(\beta_{abcd} + 4\lambda_{abce}\gamma_{de}) v_a v_b v_c v_d = 0, \quad (20)$$

by taking the μ -derivative of (19), where the general scalar β -functions at 1-loop are,

$$\mu \frac{d\lambda_{abcd}}{d\mu} = \beta_{abcd} = \frac{1}{16\pi^2} (\lambda_{abef}\lambda_{efcd} + \lambda_{acef}\lambda_{efbd} + \lambda_{adef}\lambda_{efbc}). \quad (21)$$

We are working at 1-loop and can thus conclude from (20),

$$(\lambda_{abef}\lambda_{efcd} + \lambda_{acef}\lambda_{efbd} + \lambda_{adef}\lambda_{efbc}) v_a v_b v_c v_d = 0, \quad (22)$$

which is equivalent to,

$$\lambda_{abcd}v_a v_b = 0, \quad (23)$$

and again this condition must hold for all μ . Taking the derivative and using (21) again leads to,

$$\lambda_{abcd}v_d = 0. \quad (24)$$

If the field ϕ_a is decomposed into parallel and orthogonal components to v_a then (24) means that $\mathcal{V}(\phi)$ can not depend on the parallel components at all. Which means that the N -component model decouples into a free massless 1-component model and $N - 1$ -components with a quartic potential. Continuing the argument down to $N = 1$ we conclude that the only possibility is $\mathcal{V}(\phi) = 0$ i.e. no interaction, only N independent free massless scalars.

This is the main result of our paper: spontaneous scale symmetry breaking and hence a **light dilaton, which only acquires a non-zero mass from anomalous scale symmetry breaking**, is not possible in interacting, perturbatively renormalizable and unitary scalar QFT.

Note again that a non-zero β -function, i.e. anomalous scale symmetry breaking, does not directly lead to the above conclusion. The particular form of the β -function was important and if, hypothetically, the null-direction of the potential would have been preserved by the RG flow, then a **light** dilaton would have been compatible with a non-vanishing β -function.

5 Conclusion and outlook

The dynamical appearance of a dilaton and its detailed properties are a non-trivial problem in QFT. An appealing playground would be a concrete top-down QFT construction which is perturbative, non-supersymmetric, renormalizable and unitary, beyond of course the prerequisite spontaneous breaking of scale invariance itself. All 4 ingredients are important: the perturbative nature of the construction would ensure that all properties can reliably be calculated, the non-supersymmetric requirement would guarantee that the construction is generic enough, renormalizability would ensure that the low energy effective theory describing the dilaton and potentially other Goldstones is UV complete, and unitarity would make sure that the construction has a well-defined Hamiltonian version.

It might seem at first that fulfilling all 4 requirements would not be difficult, but actually there is no known example, even if scale symmetry is allowed to be broken anomalously, i.e. before spontaneous breaking couplings and anomalous dimensions are allowed to be scale dependent while all particles are massless. In this work we have shown that in purely scalar QFT such a construction is in fact impossible.

Interestingly, the difficulty lies in fulfilling all 4 requirements simultaneously. If models are allowed to be supersymmetric, examples do exist, most notably $\mathcal{N} = 2$ or $\mathcal{N} = 4$ SUSY Yang-Mills theories [41]. If renormalizability is dropped, one may choose specific models from a large class of effective theories; see [53, 54, 55] for recent developments. If unitarity is dropped, again examples are known in the form of fishnet CFT's [48, 43]. If the interaction is allowed to be strong, beyond the realm of perturbation theory, much less is known rigorously but one may argue that gauge theories with sufficiently large fermion content could serve as examples.

This latter situation was part of the motivation for our study. Even if it is accepted that an approximately massless dilaton is present in the spectrum, it is not at all clear what the effective theory is describing the coupled system of Goldstone bosons from chiral symmetry breaking and the dilaton. Consequently it is not at all clear what the various couplings

are and what the detailed properties of the dilaton itself, e.g. its potential, is. A generic top-down model with calculable properties would probably have shed some light on some of these details.

It is still possible that non-supersymmetric, renormalizable, unitary and perturbative theories do exist with spontaneous scale symmetry breaking in 4 dimensions. One certainly needs to look beyond purely scalar QFT's and we hope to address larger classes of models in the future. It may also be the case that such theories do not exist, in which case a general proof would be desirable.

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