

Matchmakereft: automated tree-level and one-loop matching

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Abstract

We introduce matchmakereft, a fully automated tool to compute the tree-level and oneloop matching of arbitrary models onto arbitrary effective theories. Matchmakereft performs an off-shell matching, using diagrammatic methods and the BFM when gauge theories are involved. The large redundancy inherent to the off-shell matching together with explicit gauge invariance offers a significant number of non-trivial checks of the results provided. These results are given in the physical basis but several intermediate results, including the matching in the Green basis before and after canonical normalization, are given for flexibility and the possibility of further cross-checks. As a non-trivial example we provide the complete matching in the Warsaw basis up to one loop of an extension of the Standard Model with a charge -1 vector-like lepton singlet. Matchmakereft has been built with generality, flexibility and efficiency in mind. These ingredients allow matchmakereft to have many applications beyond the matching between models and effective theories. Some of these applications include the one-loop renormalization of arbitrary theories (including the calculation of the one-loop renormalization group equations for arbitrary theories); the translation between different Green bases for a fixed effective theory or the check of (off-shell) linear independence of the operators in an effective theory. All these applications are performed in a fully automated way by matchmakereft.

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1 Introduction

Effective field theories (EFTs) are the most appropriate tool to perform calculations in multiscale problems when using mass-independent renormalization schemes. The process of matching and running turns an often complicated, and sometimes not even perturbatively convergent, multi-scale problem into a succession of simpler single-scale calculations with the possibility of resummation of large logarithms for a better perturbative convergence [1]. The absence of direct experimental indications of new physics beyond the Standard Model (SM) seems to imply a hierarchy between the scale of new physics and the energies at which experiment is performed. In these circumstances, EFTs are not only applicable, but they become a powerful tool that allows us to very efficiently solve the problem of comparing experimental data with theoretical predictions. This is a highly non-trivial problem that involves complicated, usually loop-level, calculations of many experimental observables. Calculations that have to be repeated for each new physics model and for each observable. The way EFTs simplify this process is by splitting in two, mostly independent, steps. The first one, the bottom-up approach to EFTs, provides a model-independent parametrization of experimental observables that can be systematically improved but has to be computed only once (for the given precision) for each experimental observable. This efficient parametrization can be provided in the form of global fits to experimental data and a very important effort from the community has been devoted to this task in the last few years (see [2-13] and references there in for some recent global fits). The second step, the top-down approach to EFTs, sacrifices model-independence in favour of model-discrimination. It consists of the process of matching, in which the Wilson coefficients (WCs) of the EFT are computed in terms of the parameters of the ultraviolet (UV) model. This process of matching has to be repeated for each UV model but it can be automated.

When dealing with searches for new physics beyond the SM, the relevant EFT seems to be the SMEFT (see [14] for a recent review). The SMEFT WCs can be run down to the electroweak scale thanks to the renormalization group equations (RGEs) computed in [15–18] at dimension 6 ignoring baryon and lepton number violating operators (see [19,20] for calculations of a subset at dimension 6, [21,22] for the inclusion of lepton-number violating operators, [18] for baryon-number violating ones, [23] for dimension-7 operators and [24,25] for recent efforts towards the calculation of the dimension 8 RGEs). The SMEFT has been matched to the low-energy EFT (LEFT), the relevant EFT below the electroweak scale, both at tree level [26] and at one-loop [27] and then run again using the LEFT RGEs [28] to the relevant experimental energy scale to compute the corresponding experimental observables. This process of running in the SMEFT, matching to the LEFT and running in the LEFT has been implemented in automated tools like DsixTools [29, 30] or Wilson [31]. This leaves the calculation of the matching of arbitrary UV models onto the SMEFT as the only missing step towards a fully automated calculation of the phenomenological implications of new physics models.

We present in this article matchmakereft, https://ftae.ugr.es/matchmakereft/, a fully automated tool to perform tree-level and one-loop matching of arbitrary UV models onto arbitrary EFTs. The tree-level matching of the most general extension of the SM onto the SMEFT at dimension 6 has been recently computed in [32], building on previous partial results [33–36]. Going up to the one-loop order in the matching is far more complex and some degree of automation is needed. Functional methods, pioneered in [37] extending ideas from the 80s [38, 39], have seen an impressive progress in the recent years [40–51] and they have resulted in computer tools that help with some of the most computational-intensive steps of the calculation [52, 53] or that are automated but apply only to specific sets of models [54]. However, to the best of our knowledge, there is currently no code that can provide the complete one-loop matching of arbitrary models onto arbitrary EFTs in a fully automated way. ¹ Matchmakereft comes to fill this gap.

Matchmakereft uses a diagrammatic approach to tree-level and one-loop matching, performed in the background field method (BFM) when gauge theories are involved. The matching is, as of version 1.0.0, done off-shell which, together with gauge invariance, provides a significant redundancy that results in a number of non-trivial cross-checks of the calculation. It has been designed with efficiency, generality and flexibility in mind, what allows a number of applications beyond the direct matching of UV models to EFTs. Current applications include the renormalization of arbitrary (effective) theories, the calculation of the RGEs of ar-

¹See [55, 56] for alternative partial efforts using a diagrammatic approach.

bitrary (effective) theories, EFT basis translation and checks of (off-shell) linear independence of operators. All these calculations are done in a fully automated way.

The rest of the article is organised as follows. We describe the way matchmakereft performs the tree-level and one-loop calculation in Section 2. Model creation in matchmakereft is explained in detail in Section 3. The different commands available in matchmakereft are defined in Section 4 and common pitfalls when using matchmakereft are described in Section 5. Some physical applications are given in Section 6 and we conclude and provide some outlook in Section 7. In the appendices we provide some more technical information, including comprehensive installation instructions in Appendix A, details about the handling of γ_5 in Appendix B, a minimal but complete example of the capabilities of matchmakereft in Appendix C, as well as the complete Green basis for the baryon-number preserving SMEFT, including our definition of evanescent operators, in matchmakereft in Appendix D.

2 Matchmakereft in a nutshell

2.1 Model classification

Models in matchmakereft are classified according to two criteria. Depending on their spectrum they can be **light models**, if only light (but not necessarily massless) particles are present in the spectrum; or **heavy models**, when there is at least one heavy particle in the spectrum. Depending on their role in the process of matching we have **UV models**, which can be light or heavy, that are to be matched onto **EFTs**, which are necessarily light models. Models in matchmakereft are created using FeynRules [57], as described in detail in Section **3**.

Matchmakereft performs an off-shell matching, in the BFM when gauge theories are involved, of a UV model onto an EFT. This is done by computing, in dimensional regularization in $D = 4 - 2\epsilon$ space-time dimensions, the hard-region contribution to the one-light-particle-irreducible relevant amplitudes at tree and one-loop level in the UV theory and equating it to the tree level contribution in the EFT for arbitrary kinematic configurations of the external particles (see [58,59] for two recent excellent reviews of the matching process). The amplitudes, with only physical external light particles, that have to be computed are fixed in an automated way by matchmakereft but the user has flexibility on changing this list as described in Section 4. All the relevant diagrams in the UV model and the EFT are then automatically computed by QGRAF [60] and the corresponding amplitudes are dressed by matchmakereft using the Feynman rules computed during the creation of the model.

2.2 Amplitude calculation

Matchmakereft runs in two different modes, depending on whether the UV model is light or heavy, called RGEmaker and Matching modes, respectively. In RGEmaker mode, which is used to compute the RGEs of an arbitrary theory, the UV model contains no heavy particles and matchmakereft computes the UV-divergent contribution proportional to $1/\epsilon$ of the corresponding one-particle-irreducible amplitudes. In Matching mode, there are heavy particles in the spectrum and both the finite and $1/\epsilon$ (both UV and IR) hard-region contributions to the corresponding one-light-particle-irreducible amplitudes are computed. In this case diagrams including only light particles are not included in the calculation, as they cancel in the matching.

The calculation of the hard region contribution to the amplitudes is performed with FORM [61] and proceeds as follows:

• Hard region expansion. This corresponds to the following expansion of the integrand

of the amplitude

$$k^2 \sim M^2 \gg p^2 \sim m^2,\tag{1}$$

where k represents the loop momentum, M a heavy mass, p any of the external momenta and m a light mass. This is done by iterating the following identities

$$\frac{1}{(k+p)^2 - M^2} = \frac{1}{k^2 - M^2} \left[1 - \frac{p^2 + 2k \cdot p}{(k+p)^2 - M^2} \right],$$
$$\frac{1}{(k+p)^2 - m^2} = \frac{1}{k^2} \left[1 - \frac{p^2 + 2k \cdot p - m^2}{(k+p)^2 - m^2} \right].$$
(2)

These identities are imposed iteratively until the power of infrared (IR) scales (external momenta or light masses) is the correct one to match the maximum dimension of the operators appearing in the EFT as automatically computed in matchmakereft.

• Tensor reduction. Tensor reduction is performed by means of the following identities

$$k^{\mu_1}k^{\mu_2} = g^{\mu_1\mu_2}\frac{k^2}{D},\tag{3}$$

$$k^{\mu_1}k^{\mu_2}k^{\mu_3}k^{\mu_4} = g^{\mu_1\mu_2\mu_3\mu_4}\frac{k^4}{D^2 + 2D},\tag{4}$$

$$k^{\mu_1} \dots k^{\mu_6} = g^{\mu_1 \dots \mu_6} \frac{k^{\circ}}{D^3 + 6D^2 + 8D},$$
(5)

$$k^{\mu_1} \dots k^{\mu_8} = g^{\mu_1 \dots \mu_8} \frac{k^{\circ}}{D^4 + 12D^3 + 44D^2 + 48D}, \dots$$
(6)

where $g^{\mu_1...\mu_n}$ is the totally symmetric combination of metric tensors.

- Dirac algebra. Once the integrals have been reduced to scalar integrals we proceed to perform the corresponding Dirac algebra, in *D* dimensions in Matching mode (in 4 dimensions in RGEmaker mode). Version 1.0.0 of matchmakereft uses an anticommuting γ^5 prescription as discussed in Section 2.3. In the case of fermion number violating particles and/or interactions, we follow the rules proposed in Ref. [62, 63].
- **Partial fractioning**. The following identity is used to separate propagators with different masses

$$\frac{1}{(k^2 - m_1^2)(k^2 - m_2^2)} = \frac{1}{m_1^2 - m_2^2} \left[\frac{1}{k^2 - m_1^2} - \frac{1}{k^2 - m_2^2} \right],\tag{7}$$

where masses can be light or heavy and one of them can be vanishing.

• Integration by parts. After partial fractioning scaleless integrals are set to zero, except in RGEmaker mode, in which case we keep the UV poles using,

$$\int \frac{d^D k}{(2\pi)^D} \frac{1}{k^4} = \frac{i}{(4\pi)^2} \frac{1}{\epsilon} + \dots,$$
(8)

before eliminating the remaining scaleless integrals. In Matching mode the following identity is used to reduce the massive integrals to tadpoles

$$\frac{1}{(k^2 - m^2)^{n+1}} = \frac{D - 2n}{2nm^2} \frac{1}{(k^2 - m^2)^n}.$$
(9)

At this point we are left with a tadpole integral

$$a_0(m) = \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2 - m^2} = i \frac{m^2}{16\pi^2} \left[\frac{1}{\bar{\epsilon}} + 1 - \log\left(\frac{m^2}{\mu^2}\right) \right] + \mathcal{O}(\epsilon), \tag{10}$$

where

$$\frac{1}{\bar{\epsilon}} \equiv \frac{1}{\epsilon} + \gamma_E - \log(4\pi), \tag{11}$$

with $\gamma_E \approx 0.5772$ the Euler-Mascheroni constant. In matchmakereft $1/\bar{e}$ is denoted by invepsilonbar.

2.3 About γ^5 and evanescent operators

In order to ensure maximum generality matchmakereft assumes no four-dimensional properties when running in Matching mode. In particular no Fierz relations or reduction of products of three or more gamma matrices is perform during the matching procedure. This means that all evanescent structures, which are equivalent to operators in the Green basis only in D = 4have to be explicitly defined as part of the Green basis. As an example, we provide in Appendix D the complete Green basis for the SMEFT at dimension 6 as needed for the matching with matchmakereft of general theories with renormalizable couplings.² This extends the Green basis of [64] with the general set of evanescent structures including fermionic operators.

Regarding γ^5 it is well known that its implementation in dimensional regularization schemes is problematic (see [65] and references there in for an account of the current status). The current version of matchmakereft (1.0.0) implements an anticommuting γ^5 together with an implementation of the hermiticity properties of the WCs that is enough for the case of the SMEFT (and extensions with multiple scalar doublets) as we discuss more carefully in Appendix **B**.

2.4 Wilson coefficient matching

Once the amplitudes have been computed, the output is written in two files for each amplitude, one including the gauge structure and the other including the kinematic and flavour structures. Matchmakereft then uses Mathematica match the amplitudes for all kinematic configurations between the UV model and the EFT both at tree level and one loop. Once the off-shell matching has been performed, that is the Wilson coefficients of the Green basis have been computed in terms of the couplings and masses of the UV model, matchmakereft automatically performs a canonical normalization of the results and then reduces the matching to a physical basis as defined by the user (see Section 4 for details). Results at all three levels (Green basis with non-canonical kinetic terms, canonically normalised Green basis and Physical basis) are reported by matchmakereft together with the corresponding renormalisation of the gauge couplings as fixed by gauge boson renormalisation in the BFM. If RGEmaker mode has been invoked then matchmakereft can also automatically compute the beta functions for all the WCs of the EFT used.

The off-shell matching used introduces a large degree of kinematic redundancy. In gauge theories the BFM also introduces a large degree of explicit gauge redundancy. These redundancies provide a very powerful mechanism to cross-check the results obtained in matchmakereft and when any of them is not fulfilled matchmakereft will issue a warning and provide some extra information that can be useful to debug the problem.

²Non-renormalizable theories can be matched also with matchmakereft but an extension of the basis with a larger number of gamma matrices in four-fermion operators would be needed. Similarly, if bosonic evanescent operators appear in the process of matching a specific UV model they would have to be included in the EFT basis.

3 Model creation

Model creation is fully automated in matchmakereft but it relies on the explicit input from the user. Thus, it is the step that should be performed with the greatest care, as it is the most likely culprit in case of problems with the matching calculation. Model creation is greatly simplified by the use of FeynRules but a few important points should be taken into account.

3.1 Required files

Matchmakereft expects a number of files with all the relevant information to create the model. The detailed structure of each of these files will be defined below but we list them here first: 3

- Model files (compulsory): one or more files modfile1.fr, ..., modfilen.fr that define the model in FeynRules format. One of the files, the last one of the list during the creation of the model (see below) is special, as it will define the name of the matchmakereft model, which is modelfilen_MM, and the name that extra files with additional information need to have. Matchmakereft expects the Lagrangian of the to be defined as Ltot. If a different name is used the model will not be created.
- Gauge information file (compulsory only if gauge groups are present): a file called modfilen.gauge that has the definition of all the gauge functions, including structure constants, group generators in different representations and Clebsch-Gordan coefficients, appearing in the model (see below for more information). The user can choose any gauge basis of interest but they are responsible for the consistency of the chosen basis.
- Symmetry file (optional): a file called modfilen.symm that indicates possible symmetries in the parameters of the model. This is particularly important in the case of the EFT model and it is compulsory in this case if symmetries are present. The content of the file should be a Mathematica list in which the symmetries are given in the form of replacement rules. As an example we show the case of the symmetries of the Wilson coefficient of the Weinberg operator (denoted by alphaWeinberg[i,j]=alphaWeinberg[j,i]) and the four-lepton operator O_{ll} = l_iγ^μl_jl_kγ_μl_l (denoted by alphaOll[i,j,k,l]= alphaOll[k,l,i,j]):

```
1 listareplacesymmetry=
2 {
3 alphaWeinberg[i_, j_] -> alphaWeinberg[j, i],
4 alphaWeinbergbar[i_, j_] -> alphaWeinbergbar[j, i],
5 alphaOll[i_, j_, k_, l_] -> alphaOll[k, l, i, j]
6 }
```

where the underscore denoting dummy indices on the left hand side of the rules is compulsory and the bar at the end of a name denotes complex conjugation.

• Redundancy file (compulsory): a file called modfilen.red that provides the redundancies that express the Wilson coefficient of a physical basis in terms of the ones in the Green basis. It is compulsory but it can be empty if no redundancies are needed (for example if the model is a UV model, if the physical and Green bases coincide or if we just want the results in the Green basis).

³The installation of matchmakereft comes with a number of sample models that can be obtained with the command copy_models (see below). We encourage the user to check these examples for details on how to implement new models.

• Hermiticity properties file: complex conjugation is a very important process in matchmakereft, providing extra cross-checks of the correctness of the calculation. For that reason it is important to provide the information of which WCs have special (anti)hermiticity properties. A file called modfilen.herm can be provided by the user defining a Mathematica list called listahermiticity defining those WCs whose hermitian conjugate can be defined in terms of the original coefficient. As an example, for the case of the hermitian operator $(\mathcal{O}_{Hq}^{(1)})_{ij} = H^{\dagger} i \widetilde{D}_{\mu} H \overline{\ell}_i \gamma^{\mu} \ell_j$, its Wilson coefficient, denoted alphaOHq1[i,j] satisfies alphaOHq1bar[i,j]=alphaOHq1[j,i]. This is provided in the form of the hermiticity file as follows

```
1 listahermiticity = {
2 alpha0Hq1bar[i_,j_]->alpha0Hq1[j,i]
3 }
```

3.2 Gauge structure

Matchmakereft is especially efficient when the matching is performed in the unbroken phase of gauge theories as it keeps gauge indices as dummy indices during the calculation of the amplitudes, replacing them with their explicit values only at the end of the calculation. When creating a model, all gauge functions, including structure constants, generators in different representations and Clebsh-Gordan coefficients need to have a specific name that does not correspond to any function already present in Mathematica or FeynRules. Structure constants and generators do not have to be defined as FeynRules parameters but Clebsch-Gordan coefficients do. The numerical values of these gauge functions are provided in a file wigh a modfilen.gauge extension with the definition of a mathematica list called replacegaugedata that consist of a list of substitutions in the form of Mathematica sparse arrays. As a simple example, the $SU(2)_L$ weak gauge group can be defined as follows in one of the .fr files:

```
M$GaugeGroups = {
2
    SU2L == {
3
      Abelian
                             -> False,
      CouplingConstant -> g2,
GaugeBoson -> Wi,
4
5
      StructureConstant -> fsu2,
6
      Representations -> {{Ta,SU2D}}
7
    }
8
 },
9
```

where we have defined the structure constant symbol and one representation with generator symbol Ta and index definition SU2D. Note that the adjoint representation does not need to be explicitly defined as it is defined by the structure constants and the definition of the corresponding gauge bosons which, in this case reads (again, provided in one of the .fr files),

```
M$ClassesDescription = {
   V[2] == {
2
                      -> Wi,
     ClassName
3
     SelfConjugate -> True,
4
     Indices
Mass
                     -> {Index[SU2W]},
5
                      -> 0,
6
     FullName
                    -> "light"
7
8
   }
9
 };
```

A few things are worth emphasizing from the above example. First, the mass is set to zero because we are in the unbroken phase of the SM. Second, we define physical fields in entire

gauge multiplets, rather than components.⁴ Finally, we assign the FullName variable to "light" (FullName->"light"). This is compulsory in matchmakereft. Every particle has to be defined with FullName equal to either "light" or "heavy" to define the corresponding particle as light (and therefore to be kept in the EFT model) or heavy (to be integrated out in the UV model).

The corresponding indices (not only gauge, also flavour indices if present) have to be defined with a finite range within the .fr files. As an example the ones corresponding the the adjoint (SU2W) and fundamental (SU2D) representations of $SU(2)_L$, together with flavor indices for fermion generations (Generation), can be defined as follows

```
1 IndexRange[Index[SU2W]] = Range[3];
2 IndexRange[Index[SU2D]] = Range[2];
3 IndexRange[Index[Generation]] = Range[3];
4 IndexStyle[SU2W,n];
5 IndexStyle[SU2D,1];
6 IndexStyle[Generation, f1];
```

Only massless particles can have flavor indices in the current version of matchmakereft (1.0.0).

In order to show how new particles with non-trivial quantum numbers and Clebsch-Gordan coefficients are defined we show here the case of a heavy scalar triplet under $SU(2)_L$ and the SM Higgs

```
1 M$ClassesDescription = {
    S[105] == {
2
       ClassName -> tphi,
SelfConjugate -> True,
3
4
                           \rightarrow {Index[SU2W]},
       Indices
5
                            -> Mtphi,
       Mass
6
                  -> "heavy"
7
       FullName
       QuantumNumbers \rightarrow {Y \rightarrow 0}
8
    },
9
10
    S[11] == {
11
      ClassName -> Phi,
Indices -> {Index[SU2D]},
12
13
     SelfConjugate -> False,
14
     Mass -> muH,
FullName -> "light"
15
16
       QuantumNumbers \rightarrow {Y \rightarrow 1/2}
17
    }
18
19
20 };
```

where the new particle is defined as heavy and has a non-zero mass while the SM Higgs boson is defined as light (but also has a non-vanishing mass). We also see that U(1) quantum numbers have to be defined explicitly as is standard in FeynRules. A trilinear coupling between the heavy scalar and two Higgs bosons, which has a non-trivial gauge structure following the corresponding Clebsch-Gordan coefficients, has to be defined explicitly like in the following example

```
1 M$Parameters = {
2
3 C223 == {
4 ParameterType -> Internal,
5 Indices -> {Index[SU2D], Index[SU2D], Index[SU2W]},
6 ComplexParameter -> True
```

⁴It is actually possible to define also the field components separately as the physical fields. In fact, this can be advantageous when creating complicated models that take very long to generate in FeynRules.

7 }, 8 9 ... 10 }

and in the corresponding Lagrangian

```
1 Ltot := Block[{ii,jj,nn},
2 2 C223[ii,jj,nn] kappatphi tphi[nn] Phibar[ii] Phi[jj] + ...]
```

As mentioned above the explicit values of the gauge functions are given in a file called modfilen.gauge (assuming the last FeynRules model file is called modfilen.fr) which, in the example we are showing would contain the following information

```
replacegaugedata = {
1
     fsu2 -> SparseArray[Automatic, {3, 3, 3}, 0,
2
               \{1, \{\{0, 2, 4, 6\}, \{\{2, 3\}, \{3, 2\}, 
3
                     \{1, 3\}, \{3, 1\}, \{1, 2\}, \{2, 1\}\},\
4
                                 \{1, -1, -1, 1, 1, -1\}\}
5
          Ta -> SparseArray[Automatic, {3, 2, 2}, 0,
6
               \{1, \{\{0, 2, 4, 6\}, \{\{1, 2\}, \{2, 1\}, \}
                     \{1, 2\}, \{2, 1\}, \{1, 1\}, \{2, 2\}\},\
8
                     \{1/2, 1/2, -I/2, I/2, 1/2, -1/2\}\}
9
          C223 -> SparseArray[Automatic, {2, 2, 3}, 0,
10
               \{1, \{\{0, 3, 6\}, \{\{1, 3\}, \{2, 1\}, \{2, 2\}, 
                     \{1, 1\}, \{1, 2\}, \{2, 3\}\},\
12
                     \{1/2, 1/2, -I/2, 1/2, I/2, -1/2\}\}
13
```

where we have implemented the usual definitions, $fsu2[i, j, k] = e^{ijk}$, $Ta[a, i, j] = \sigma_{ij}^a/2$ and $C223[i, j, a] = \sigma_{ij}^a/2$.

3.2.1 Background field method

Matchmakereft assumes that the BFM is used when gauge theories are involved. Following the $SU(2)_L$ example, the associated quantum and ghost fields have to be defined on top of the definition of Wi given above

```
V[102] == {
1
      ClassName
                       -> WiQuantum,
2
                       -> True,
3
      SelfConjugate
                       \rightarrow \{ Index [SU2W] \},
      Indices
4
                        -> 0,
5
      Mass
                      -> "light"
6
      FullName
 },
7
    U[1] == {
8
      ClassName
                     -> ghWi,
9
      SelfConjugate -> False,
10
     Indices -> {Index[SU2W]},
11
      Ghost
                      -> Wi,
12
      QuantumNumbers -> {GhostNumber -> 1},
13
     Mass
                     -> 0,
14
                     -> "light"
     FullName
15
16
   },
17 . . .
```

and the Lagrangian involving the $SU(2)_L$ part is given by

```
1 gotoBFM={Wi[a__]->Wi[a]+WiQuantum[a]};
2
3 Ltot :=
4 Block[{lag,mu,nu,ii,aa},
5 lag=-1/4 FS[Wi,mu,nu,ii] FS[Wi,mu,nu,ii]
6 -ghWibar[aa].DC[(DC[ghWi[aa],mu]/.gotoBFM),mu]
```

```
-DC[WiQuantum[mu,a],mu] DC[WiQuantum[nu,a],nu]/2;
lag/.gotoBFM
```

Note the compulsory use of Ltot for the name of the total Lagrangian of the model.

3.3 Defining the EFT model

As we have mentioned above, the EFT model has to be a light model (no heavy particles) and has to include all the relevant operators in a Green basis. The WCs in the EFT need to have a special format. They have to be named alphaXXX, where XXX stands for an arbitrary number of alpha-numeric characters (see Section 3.4). The amplitudes that matchmakereft computes are defined by the operators in the EFT. In that sense, one does not need to include all the operators of a Green basis but at least all the operators in a certain class (same fields), including redundant and evanescent operators. ⁵ Matchmakereft will then generate only the relevant amplitudes to match these operators. Unless the user is absolutely sure that they do not need them, renormalizable operators, including kinetic and mass terms, have to be included in the EFT model.

When performing the matching, matchmakereft will check that all off-shell kinematic configurations and all gauge directions are correctly matched. If these checks are not satisfied, matchmakereft will issue a warning, and store the relevant information. This usually happens because the user made a mistake when defining the models, either because the model is not correctly defined or because there are missing operators in the Green basis. See Section 5 for common problems and possible solutions when running matchmakereft. When doing the external momentum expansion all operators of dimension equal or smaller to the highest dimension of the operators appearing in the EFT will be generated. Thus, one has to include all operators of smaller dimensions within the same class. Failing to do so will make the matching fail but the user can check that all problems appear in sectors that are of no interest for them. Also sometimes some amplitudes are not correctly matched due to the use of an anticommuting γ^5 . Our proposed solution should be enough to ensure the correct results in the SMEFT and similar EFTs but a warning will still be issued and the relevant information stored so that the user can check if the solution is correct or not.

If the user is interested in the matching result in a physical basis, they have to provide the corresponding redundancies to reduce the WCs of the Green basis into the ones of the physical basis. This is done in the file denoted modfilen.red. As an example, let's consider the following redundant operators in the SMEFT at dimension 6

$$\mathcal{O}_{HD} = (H^{\dagger} D_{\mu} H)^{\dagger} (H^{\dagger} D_{\mu} H), \tag{12}$$

$$\mathcal{R}_{BDH} = (H^{\dagger} \overrightarrow{D}_{\mu} H) \partial_{\nu} B^{\mu\nu} \to g_1 \mathcal{O}_{HD} + \dots,$$
(13)

$$\mathcal{R}_{2B} = -\frac{1}{2} (\partial_{\mu} B^{\mu\nu}) (\partial^{\rho} B_{\rho\nu}) \rightarrow -\frac{g_1^2}{2} \mathcal{O}_{HD} + \dots, \qquad (14)$$

where the \rightarrow indicates an on-shell equivalence. These redudancies imply the following on-shell condition for the corresponding Wilson coefficients (with obvious notation)

alphaOHD
$$\rightarrow$$
 alphaOHD + 2g₁alphaRBDH - $\frac{g_1^2}{2}$ alphaR2B, (15)

which we implement in the file modfilen.red as follows

⁵See section 2.3 for a discussion about evanescent operators.

```
1 finalruleordered={
2 alphaOHD ->alphaOHD + 2*alphaRBDH*g1 - (alphaR2B*g1^2)/2,
3 ...
4 }
```

3.4 Protected words

There is quite a bit of flexibility in the process of model definition in matchmakereft but there are a few words that are protected and should be used only for their specific purpose. In general, all variables in matchmakereft should be made of alphanumeric characters, without including any special characters. The list of protected variables are the following

- alpha. All WCs should be defined as alphaXXX where XXX is an arbitrary string of alphanumeric characters. Similarly no other variable in the model should contain the substring alpha. An exception to this rule is that when computing the RGEs of an EFT the Wilson coefficients in the UV model can be kept with their original alphaXXX name as they will be changed into WCXXX automatically by matchmakereft.
- Ltot. Ltot should be used to define the complete Lagrangian of the model (and it should not be used for anything else).
- Quantum. Gauge bosons are split into a classical background and a quantum excitation. If the classical gauge boson is defined by ClassName->Vname then the quantum excitation has to be defined by ClassName->VnameQuantum.
- invepsilonbar is used for the dimensional regularization variable $1/\bar{e}$ so it should not be use explicitly in the definition of a model. Similarly epsilonbar is used of \bar{e} .
- Eps[] denotes the FeynRules Levi-civita tensor. When used with four indices it is interpreted by matchmakereft as the Minkowskian (with + -- metric signature) totally antisymmetric tensor and should therefore not be used for the Euclidean one (with a number of indices different from four it can be used as the Euclidean one). In case one needs to use the totally antisymmetric rank-4 tensor both with Minkowskian and Euclidean signatures, the latter should be explicitly defined as a gauge function and its numerical value defined in the corresponding modfilen.gauge file.
- onelooporder is a dummy variable to identify the one-loop order contribution.
- sSS is a dummy variable to identify the order in external momenta of a specific contribution.
- iCPV= ϵ_{0123} is used to fix the sign convention for the Levi-Civita symbol.

4 Matchmakereft usage

An updated version of this manual can be found, once matchmakereft is installed, in the directory matchmakereft-location/matchmaker/docs/ where matchmakereft-location is the directory listed under Location when the command pip show matchmakereft is used or the analogous location in Anaconda (see Appendix A).

Matchmakereft can be run in two different ways. The same commands are available in all different running modes although the syntax is slightly different on each of them.

4.1 Matchmakereft command line interface

The most straight-forward way to run matchmakereft is via the command line interface (CLI). This is obtained by just typing on the terminal (after matchmakereft has been installed, see Appendix A for details)

```
> matchmakereft
```

The user has then access to the CLI which looks as follows

```
Welcome to matchmakereft Please refer to arXiv:2112.10787 when using this code
```

```
matchmakereft >
```

Inside the CLI tab-completion is available and all file paths can be absolute or relative. The command help gives information on all available commands. The core commands in matchmakereft CLI are:

matchmakereft> test_installation

This command runs a number of minimal tests to check that matchmakereft has been correctly installed. The process is verbose and provides information on what is being computed. It takes about 6 minutes to complete in a core-i7@3.00 GHz laptop.

matchmakereft> copy_models Location

This command copies a number of sample models, including the complete baryon-number conserving SMEFT at dimension 6, in the directory Location (which can be . for the current directory).

matchmakereft> create_model modfile1.fr ... modfilen.fr

This command creates a matchmakereft model called modfilen_MM from the FeynRules model defined in one or more files with names modfile1.fr ... modfilen.fr as described in detail in Section 3. The matchmakereft model is created in the same directory modfilen.fr is present. Both relative and absolute paths can be given as input.

matchmakereft> match_model_to_eft UVModelName EFTModelName

This command performs the complete tree-level and one-loop matching of matchmakereft UV model with name UVModelName onto a matchmakereft EFT model with name EFTModelName. The result of the matching is written in a file called MatchingResult.dat under file UVModelName. Any possible problems with the matching are reported and stored in a file called MatchingProblems.dat under the same directory. Matchmakereft automatically checks if the matching is run in RGEmaker or Matching mode.

The result of the matching stored in MatchingResult.dat is a mathematica list called MatchingResult with four entries and the following structure

```
1 MatchingResult={
2 {
3 {{GreenTree,GreenTreeProblems},{GreenLoop,GreenLoopProblems}},
4 {{NormGreenTree,NormGreenTreeProblems},{NormGreenLoop,
NormGreenLoopProblems}},
5 {PhysTreeLoop},
6 {GaugeCouplingMatching}
7 }
```

where GreenTree and GreenLoop stands for the tree level and one-loop matching in the Green basis, respectively and GreenTreeProblems, GreenLoopProblems are filled if problems were found in the process of impossing hermiticity as discussed in Section 2.3. The second level, with the Norm prefix stand for the matching in the Green basis (again separately for tree

level and one-loop) after canonical normalization. The third level, denoted PhysTreeLoop stands for the matching in the physical basis in which the tree level and one-loop contributions have been merged into a single expression (with the dummy variable onelooporder denoting the one-loop contribution). If no physical basis is defined by providing an empty modfilen.red file then the Green basis is used as a physical basis. Finally, the fourth level, denoted GaugeCouplingMatching provides the redefinition of the gauge couplings after matching as fixed by the corresponding gauge boson canonical normalization in the background gauge.

matchmakereft> match_model_to_eft_onlytree UVModelName EFTModelName

Identical to match_model_to_eft but only the tree level matching is computed. With this feature, matchmakereft can be used as an automated basis translator, as one can simply use the corresponding EFT in a different basis as UV model and the matching will provide the complete translation between the two bases (see Section 6 for an explicit example).

matchmakereft> compute_rge_model_to_eft UVModelName EFTModelName

This command runs match_model_to_eft UVModelName EFTModelName in RGEmaker mode (the UV model has to be a light model) and then computes the beta functions for the WCs of the EFT given the UV model. They are stored in a file called RGEResult.dat under directory UVModelName. We define the beta function of a WC *C* as

$$\beta(C) = \mu \frac{\mathrm{d}C}{\mathrm{d}\mu}.\tag{16}$$

matchmakereft> clean_model ModelName

Matchmakereft is designed for the maximal efficiency so that if a specific process has been already computed it is not computed again. If for any reason the user wants to recreate the calculation of all the amplitudes this command should be invoked to clean the previous calculations.

matchmakereft> check_linear_dependence EFTModelName

Given a set of operators, defined as an EFT model in directory EFTModelName, this command checks if they are off-shell linearly independent or not. This command is useful when finding a Green basis as sometimes the off-shell relations between different operators are difficult to obtain analytically. If the set is not linearly independent matchmakereft will provide the relations between the different WCs (see Section 6 for an explicit example).

matchmakereft> exit

This command exits the CLI.

For the sake of flexibility the following commands are also available to perform independently some of the steps of the calculations:

matchmakereft> match_model_to_eft_amplitudes UVModelName EFTModelName

This command is used to compute all the relevant amplitudes in the UV model and the EFT but no calculation of the WCs is attempted.

matchmakereft> match_model_to_eft_amplitudes_onlytree UVModelName EFTModelName

This command is identical to match_model_to_eft_amplitudes but performs only the tree level calculation.

matchmakereft> compute_wilson_coefficients UVModelName EFTModelName

This command should be run after the call to either match_model_to_eft_amplitudes or match_model_to_eft_amplitudes_onlytree and it computes the WCs to complete the matching.

4.2 Matchmakereft as a Python module

The python CLI described in the previous section provides an interactive experience to user. However, matchmakereft can be also run by importing matchmakereft into a python script, a iPython shell or a Jupyter notebook as a module and then running the same commands as in the CLI adding the parameters of the corresponding function as a string. As an example, the commands to create a model stored in model UVmodel.fr and to match it to the EFT stored in directory EFT_MM (which we assume has been already created) are given e.g. by

```
1 from matchmakereft.libs.mm_offline import *
2 create_model("UVmodel.fr")
3 match_model_to_eft("UVmodel_MM EFT_MM")
```

All other commands in the CLI are also available to use as functions in a script that imports matchmakereft.

5 Troubleshooting in matchmakereft

Matchmakereft provides a significant number of cross-checks that usually catch problems with the installation or with the definition of the models. When a problem is encountered, matchmakereft tries to provide a useful warning message that can be used to figure out the origin of the problem. If the user encounters a problem that cannot be solved from the information provided by matchmakereft we encourage them to check the troubleshooting section in the latest matchmakereft manual and the Gitlab matchmakereft issue tracker (https://gitlab.com/m4103/matchmaker-eft/-/issues) to see if the problem has been encountered by other users and a solution is available. If no solution can be found, the issue tracker should be use to pose questions to the matchmakereft developers or to file possible bugs.

Most of the times an unsuccessful matching is due to a badly defined model. Some common pitfalls are:

- The complete Lagrangian of the model has to be named Ltot. Using a different name results in matchmakereft not creating the model properly.
- Operators badly defined in FeynRules (a common example is indices not properly contracted). This results in wrong Feynman rules that lead to incorrect matching.
- Model generation takes too long. This can happen with complicated models, in particular with effective operators of high mass dimension. As an example, the generation of the SMEFT model can easily take more than 30 minutes in a core-i7@3.00 GHz laptop. In this case it is useful to compute the Feynman rules directly with FeynRules to check that the model does not have any obvious problems. Also sometimes expanding in the gauge components can significantly speed up model creation (at the expense of a reduced gauge degeneracy and therefore a smaller set of cross-checks).
- QGRAF not running correctly. This could happen when vertices with a larger number of particles than the limit set in QGRAF are present. The solution is to modify correspondingly the limit in the QGRAF source and compile it again.
- FORM not running correctly. This is normally due to variables not being correctly defined (again due to an incorrect implementation of the model). Running directly with FORM the offending file can give hints on what is happening.

- FORM taking too long to run. Amplitudes with many external legs usually involve a very large number of diagrams that can take a long time to compute. The simplest solution is to not include the corresponding operators in the EFT model if the user is not interested in their matching (in the SMEFT case at dimension 6 the operator $\mathcal{O}_H = (H^{\dagger}H)^3$ is usually the one that takes the longest to be matched). Other options could imply splitting the diagrams into smaller sets and combining the result after the calculation (this is how the beta function of the $(H^{\dagger}H)^4$ operator was computed in [24]) but this process is not straight-forward. We expect to automate this procedure in future versions of matchmakereft.
- All amplitudes are computed but the matching is unsuccessful. This can be due to a number of reasons, the most common ones being: the WCs of the EFT model or the couplings in the UV model have some symmetry properties that have not been implemented in the corresponding modfilen.symm file; the hermiticity properties of the couplings in the EFT or UV models have not been properly defined, either in the definition of the model itself or in the corresponding modfilen.herm file; there are some missing operators in the Green basis of the EFT model.

6 Physics applications

A preliminary version of matchmakereft has been already used in a number of physical applications [24, 66, 67] but we list here some of the new applications that matchmakereft has.

6.1 Cross-checks

As we have emphasized, the large redundancy inherent in the off-shell matching in the BFM gives us confidence on the correctness of the results computed with matchmakereft. Never-theless we have tested matchmakereft against some of the few available complete one-loop matching results in the literature. We have found complete agreement except when explicitly described. The list of models we have compared to include:

- RGEmaker mode:
 - Complete RGEs for the ALP-SMEFT up to mass dimension-5 as computed in [66].
 Exact agreement was found up to a typo in the original reference.
 - RGEs for the purely bosonic and two-fermion operators in the Warsaw basis [68] as computed in [15–17] and implemented in DSixTools [29, 30]. Complete agreement was found.
- Matching mode:
 - Scalar singlet. The complete matching up to one-loop order of an extension of the SM with a scalar singlet was recently completed in [69], after several partial attempts [37,70]. We have found complete agreement with the results in [69].
 - Type-I see-saw model, as computed in [71]. Complete agreement was found.
 - Scalar leptoquarks, as computed in [64]. We have found some minor differences that we are discussing with the authors.
 - Charged scalar electroweak singlet, as computed in [72]. We agree with the result except for a sign in Eqs. (4.14), the terms with Pauli matrices in (4.15), (B.4) and

(B.5) (the latter is the culprit of the opposite sign in terms with Pauli matrices) and a factor of 2 in Eq. (4.17) and of 4 in (B.7). We have contacted the authors about these differences.

 Some partial cross-checks have been performed against the results in [73] with full agreement.

6.2 Complete one-loop matching of a new charged vector-like lepton singlet

In this section we provide the complete tree-level and one-loop matching of an extension of the SM with a new hypercharge -1, electroweak singlet vector-like lepton *E*. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \bar{E}(i\not\!\!\!D - M_E)E - \left[\tilde{\lambda}_i\bar{\ell}_i\phi E_R + \text{h.c.}\right],\tag{17}$$

where ℓ_i and ϕ stand for the SM lepton doublets and the Higgs boson, respectively and *i* is a SM flavor index. See [74] for direct experimental limits on such an extension of the SM.

This model is included in the distribution of matchmakereft and can be obtained via the copy_models command. Once the model is downloaded, and inside the corresponding directory, the following commands will generate the complete one-loop matching, including the complete matching in the Greeen basis. We use the CLI as an example and replace the output given by matchmakereft with ...,

```
matchmakereft> create_model UnbrokenSM_BFM.fr VLL_Singlet_Y_m1_BFM.fr
...
matchmakereft> match_model_to_eft VLL_Singlet_Y_m1_BFM_MM
SMEFT_Green_Bpreserving_MM
...
```

The non vanishing WCs in the Warsaw basis, including one-loop accuracy are given in the next sections. In order to reduce clutter we only write explicitly flavor indices when necessary. Also we use the following notation

$$[\tilde{\lambda}\tilde{\lambda}^*] \equiv \tilde{\lambda}_i \tilde{\lambda}_i^*, \qquad [\tilde{\lambda}\mathcal{M}\tilde{\lambda}^*] \equiv \tilde{\lambda}_i \mathcal{M}_{ij} \tilde{\lambda}_j^*, \tag{18}$$

with \mathcal{M}_{ii} and arbitrary matrix with flavor indices. We also define

$$L_E \equiv \log(\mu^2/M_E^2). \tag{19}$$

The tree level result agrees with the calculation in [34] (when taking into account the different notation in the Yukawa coupling).

6.2.1 SM couplings

The SM couplings receive the following (one-loop) corrections:

$$\mu_{H}^{2} = \mu_{H}^{(0)2} + \frac{\lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor}{16\pi^{2}} \left[2M_{E}^{2} - \frac{1}{2}\mu_{H}^{(0)2} - \frac{1}{3}\frac{\mu_{H}^{(0)4}}{M_{E}^{2}} - (\mu_{H}^{(0)2} - 2M_{E}^{2})L_{E} \right],$$
(20)

$$\lambda = \lambda^{(0)} + \frac{1}{16\pi^2} \left[\frac{[\lambda^* \lambda] (5g_2^{(0)2} \mu_H^{(0)2} - 6\lambda^{(0)} (4\mu_H^{(0)2} + 3M_E^2) + 18\mu_H^{(0)2} [\lambda^* \lambda])}{18M_E^2} + \frac{(2M_E^2 - \mu_H^{(0)2}) [\tilde{\lambda}^* Y_e^{(0)} Y_e^{(0)\dagger} \tilde{\lambda}]}{M^2} \right]$$
(21)

$$-\frac{1}{16\pi^{2}} \left[\frac{\lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor [-g_{2}^{(0)2} \mu_{H}^{(0)2} + 6\lambda^{(0)} M_{E}^{2} - 3M_{E}^{2} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor]}{3M_{E}^{2}} + \frac{2(-M_{E}^{2} + \mu_{H}^{(0)2}) \lfloor \tilde{\lambda}^{*} Y_{e}^{(0)} Y_{e}^{(0)\dagger} \tilde{\lambda} \rfloor }{M_{E}^{2}} \right] L_{E},$$
(22)

$$Y_{u} = Y_{u}^{(0)} - \frac{1}{16\pi^{2}} \left[\left(\frac{1}{4} + \frac{\mu_{H}^{(0)2}}{3M_{E}^{2}} \right) Y_{u}^{(0)} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor + \frac{1}{2} Y_{u}^{(0)} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor L_{E} \right],$$
(23)

$$Y_{d} = Y_{d}^{(0)} - \frac{1}{16\pi^{2}} \left[\left(\frac{1}{4} + \frac{\mu_{H}^{(0)2}}{3M_{E}^{2}} \right) Y_{d}^{(0)} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor + \frac{1}{2} Y_{d}^{(0)} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor L_{E} \right],$$
(24)

$$(Y_{e})_{ij} = (Y_{e}^{(0)})_{ij} - \frac{1}{16\pi^{2}} \left[\left(\frac{1}{4} + \frac{\mu_{H}^{(0)2}}{3M_{E}^{2}} \right) (Y_{e}^{(0)})_{ij} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor + \left(\frac{3}{8} + \frac{3\mu_{H}^{(0)2}}{4M_{E}^{2}} \right) \tilde{\lambda}_{i} \tilde{\lambda}_{k}^{*} (Y_{e}^{(0)})_{kj} \right] \\ - \frac{1}{16\pi^{2}} \left[\frac{1}{2} (Y_{e}^{(0)})_{ij} \lfloor \tilde{\lambda}^{*} \tilde{\lambda} \rfloor + \left(\frac{1}{4} + \frac{\mu_{H}^{(0)2}}{2M_{E}^{2}} \right) \tilde{\lambda}_{i} \tilde{\lambda}_{k}^{*} (Y_{e}^{(0)})_{kj} \right] L_{E},$$
(25)

where the (0) superscript denotes the original parameters in the SM Lagrangian. All other SM couplings receive no corrections. In the following we express our results in terms of the physical SM couplings, the ones on the left hand side of Eqs. (20-25).

6.2.2 Bosonic operators

Turning now to dimension 6 bosonic operators, we obtain the following non-vanishing WCs.

$$\alpha_{HW} = \frac{1}{16\pi^2} \frac{g_2^2 \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{24M_E^2},\tag{26}$$

$$\alpha_{HB} = \frac{1}{16\pi^2} \frac{g_1^2 \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{8M_E^2},\tag{27}$$

$$\alpha_{HWB} = -\frac{1}{16\pi^2} \frac{g_1 g_2 \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{6M_E^2},\tag{28}$$

$$\begin{aligned} \alpha_{H\Box} &= \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_1^4}{30} + \left(\frac{13g_1^2}{72} - \frac{5g_2^2}{24} - \frac{\lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{3} \right) \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \frac{3}{2} \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor \right] \\ &+ \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\left(\frac{g_1^2}{12} - \frac{g_2^2}{4} \right) \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor \right] L_E, \end{aligned}$$
(29)

$$\alpha_{HD} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{\sigma_1}{15} + \left(\frac{\sigma_1}{18} - \frac{\sigma_2}{2} \right) \left[\lambda \lambda^* \right] + \frac{1}{2} \left[\lambda^* Y_e Y_e^* \lambda \right] \right] + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{g_1^2}{3} \left[\tilde{\lambda} \tilde{\lambda}^* \right] + \left[\tilde{\lambda}^* Y_e Y_e^\dagger \tilde{\lambda} \right] \right] L_E,$$
(30)

$$\begin{aligned} \alpha_{H} &= \frac{1}{16\pi^{2}} \frac{1}{M_{E}^{2}} \bigg[\bigg(\frac{4\lambda^{2}}{3} - \frac{5\lambda g_{2}^{2}}{9} - 2\lambda [\tilde{\lambda}\tilde{\lambda}^{*}] + \frac{[\tilde{\lambda}\tilde{\lambda}^{*}][\tilde{\lambda}\tilde{\lambda}^{*}]}{3} + 2[\tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}\tilde{\lambda}] \bigg) [\tilde{\lambda}\tilde{\lambda}^{*}] \\ &+ 2\lambda [\tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}\tilde{\lambda}] - 2[\tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}Y_{e}Y_{e}^{\dagger}\tilde{\lambda}] \bigg] \\ &+ \frac{1}{16\pi^{2}} \frac{1}{M_{E}^{2}} \bigg[-\frac{2\lambda g_{2}^{2}}{3} [\tilde{\lambda}\tilde{\lambda}^{*}] + 4\lambda [\tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}\tilde{\lambda}] - 2[\tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}Y_{e}Y_{e}^{\dagger}\tilde{\lambda}] \bigg] L_{E}. \end{aligned}$$
(31)

All other bosonic operators do not receive any corrections up to one loop.

6.2.3 Bi-fermion operators

Regarding operators in the Warsaw basis with two fermion fields, the non-vanishing contributions in our model are the following.

$$(\alpha_{eW})_{ij} = -\frac{1}{16\pi^2} \frac{g_2}{24M_E^2} \tilde{\lambda}_i \tilde{\lambda}_k^* (Y_e)_{kj},$$
(32)

$$(\alpha_{eB})_{ij} = -\frac{1}{16\pi^2} \frac{g_1}{12M_E^2} \tilde{\lambda}_i \tilde{\lambda}_k^* (Y_e)_{kj},$$
(33)

$$(\alpha_{Hq}^{(1)})_{ij} = \frac{g_1^2}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_1^2}{45} + \frac{13}{216} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \frac{1}{36} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor L_E \right] \delta_{ij},$$
(34)

$$(\alpha_{Hq}^{(3)})_{ij} = \frac{g_2^2}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{5}{72} [\tilde{\lambda} \tilde{\lambda}^*] - \frac{1}{12} [\tilde{\lambda} \tilde{\lambda}^*] L_E \right] \delta_{ij},$$
(35)

$$(\alpha_{Hu})_{ij} = \frac{g_1^2}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{4g_1^2}{45} + \frac{13}{54} [\tilde{\lambda} \tilde{\lambda}^*] + \frac{1}{9} [\tilde{\lambda} \tilde{\lambda}^*] L_E \right] \delta_{ij},$$
(36)

$$(\alpha_{Hd})_{ij} = \frac{g_1^2}{16\pi^2} \frac{1}{M_E^2} \left[\frac{2g_1^2}{45} - \frac{13}{108} [\tilde{\lambda}\tilde{\lambda}^*] - \frac{1}{18} [\tilde{\lambda}\tilde{\lambda}^*] L_E \right] \delta_{ij},$$
(37)

$$\begin{aligned} (\alpha_{H\ell}^{(1)})_{ij} &= -\frac{\tilde{\lambda}_{i}\tilde{\lambda}_{j}^{*}}{4M_{E}^{2}} + \frac{1}{16\pi^{2}}\frac{1}{M_{E}^{2}} \left[\left(\frac{g_{1}^{4}}{15} - \frac{13g_{1}^{2}\lfloor\tilde{\lambda}\tilde{\lambda}^{*}\rfloor}{72} \right) \delta_{ij} + \left(\frac{31g_{1}^{2}}{288} - \frac{33g_{2}^{2}}{32} + \frac{13}{16}\lfloor\tilde{\lambda}\tilde{\lambda}^{*}\rfloor \right) \tilde{\lambda}_{i}\tilde{\lambda}_{j}^{*} \\ &- \frac{1}{2}\tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*}(Y_{e})_{kl}(Y_{e}^{\dagger})_{lj} - \frac{1}{2}(Y_{e})_{ik}(Y_{e}^{\dagger})_{kl}\tilde{\lambda}_{l}\tilde{\lambda}_{j}^{*} \right] \\ &+ \frac{1}{16\pi^{2}}\frac{1}{M_{E}^{2}} \left[-\frac{g_{1}^{2}\lfloor\tilde{\lambda}\tilde{\lambda}^{*}\rfloor}{12} \delta_{ij} + \left(\frac{25g_{1}^{2}}{48} - \frac{9g_{2}^{2}}{16} + \frac{3\lfloor\tilde{\lambda}\tilde{\lambda}^{*}\rfloor}{8} \right) \tilde{\lambda}_{i}\tilde{\lambda}_{j}^{*} \\ &- \frac{1}{2}\tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*}(Y_{e})_{kl}(Y_{e}^{\dagger})_{lj} - \frac{1}{2}(Y_{e})_{ik}(Y_{e}^{\dagger})_{kl}\tilde{\lambda}_{l}\tilde{\lambda}_{j}^{*} \right] L_{E}, \end{aligned}$$
(38)

$$(\alpha_{H\ell}^{(3)})_{ij} = -\frac{\tilde{\lambda}_i \tilde{\lambda}_j^*}{4M_E^2} + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{5g_2^2 \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{72} \delta_{ij} + \left(\frac{9g_1^2}{32} + \frac{77g_2^2}{288} + \frac{5}{16} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor \right) \tilde{\lambda}_i \tilde{\lambda}_j^* \right] \\ + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_2^2 \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{12} \delta_{ij} + \left(\frac{9g_1^2}{16} + \frac{7g_2^2}{48} + \frac{3\lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{8} \right) \tilde{\lambda}_i \tilde{\lambda}_j^* \right] L_E, \quad (39)$$
$$(\alpha_{He})_{ij} = \frac{1}{16\pi^2} \frac{1}{M^2} \left[g_1^2 \left(\frac{2g_1^2}{15} - \frac{13\lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor}{26} \right) \delta_{ij} + \frac{1}{24} (Y_e^{\dagger})_{ik} \tilde{\lambda}_k \tilde{\lambda}_l^* (Y_e)_{lj} \right]$$

$$\begin{aligned} \alpha_{He} \rangle_{ij} &= \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[g_1^2 \left(\frac{2g_1^2}{15} - \frac{13[\lambda\lambda^+]}{36} \right) \delta_{ij} + \frac{1}{24} (Y_e^{\dagger})_{ik} \tilde{\lambda}_k \tilde{\lambda}_l^* (Y_e)_{lj} \right. \\ &+ \left(-\frac{g_1^2}{6} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor \delta_{ij} + \frac{1}{4} (Y_e^{\dagger})_{ik} \tilde{\lambda}_k \tilde{\lambda}_l^* (Y_e)_{lj} \right) L_E \right], \end{aligned}$$

$$(40)$$

$$(\alpha_{uH})_{ij} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\left(\frac{2}{3} \lambda - \frac{5g_2^2}{36} - \frac{1}{2} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor \right) \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \frac{1}{2} \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor \right] \\ + \left(-\frac{g_2^2}{6} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor \right) L_E \right] (Y_u)_{ij},$$

$$(41)$$

$$(\alpha_{dH})_{ij} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\left(\frac{2}{3} \lambda - \frac{5g_2^2}{36} - \frac{1}{2} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor \right) \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \frac{1}{2} \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor + \left(-\frac{g_2^2}{6} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor + \lfloor \tilde{\lambda}^* Y_e Y_e^{\dagger} \tilde{\lambda} \rfloor \right) L_E \right] (Y_d)_{ij},$$

$$(42)$$

$$\begin{aligned} (\alpha_{eH})_{ij} &= \frac{\tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*}(Y_{e})_{kj}}{2M_{E}^{2}} + \frac{1}{16\pi^{2}}\frac{1}{M_{E}^{2}} \left[\left(\frac{2}{3}\lambda - \frac{5g_{2}^{2}}{36} - \frac{1}{2} \lfloor \tilde{\lambda}\tilde{\lambda}^{*} \rfloor \right) \lfloor \tilde{\lambda}\tilde{\lambda}^{*} \rfloor (Y_{e})_{ij} + \frac{1}{2} \lfloor \tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}\tilde{\lambda} \rfloor (Y_{e})_{ij} \\ &+ \left(5\lambda - \frac{14}{16} \lfloor \tilde{\lambda}\tilde{\lambda}^{*} \rfloor \right) \tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*}(Y_{e})_{kj} + \frac{37}{24} (Y_{e})_{ik} (Y_{e}^{\dagger})_{kl} \tilde{\lambda}_{l}\tilde{\lambda}_{m}^{*}(Y_{e})_{mj} - \frac{1}{4}\tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*}(Y_{e})_{kl} (Y_{e}^{\dagger})_{lm} (Y_{e})_{mj} \right] \\ &+ \frac{1}{16\pi^{2}} \frac{1}{M_{E}^{2}} \left[-\frac{g_{2}^{2}}{6} \lfloor \tilde{\lambda}\tilde{\lambda}^{*} \rfloor (Y_{e})_{ij} + \lfloor \tilde{\lambda}^{*}Y_{e}Y_{e}^{\dagger}\tilde{\lambda} \rfloor (Y_{e})_{ij} + \left(4\lambda - \frac{3}{4} \lfloor \tilde{\lambda}\tilde{\lambda}^{*} \rfloor \right) \tilde{\lambda}_{i}\tilde{\lambda}_{k}^{*} (Y_{e})_{kj} \\ &+ \frac{3}{4} (Y_{e})_{ik} (Y_{e}^{\dagger})_{kl} \tilde{\lambda}_{l}\tilde{\lambda}_{m}^{*} (Y_{e})_{mj} \right] L_{E}. \end{aligned}$$

6.2.4 Four-fermion operators

Finally, the following four-fermion operators receive non-vanishing WCs.

.

$$(\alpha_{qq}^{(1)})_{ijkl} = -\frac{1}{16\pi^2} \frac{g_1^4}{270M_E^2} \delta_{ij} \delta_{kl}, \tag{44}$$

$$(\alpha_{uu})_{ijkl} = -\frac{1}{16\pi^2} \frac{8g_1^4}{135M_E^2} \delta_{ij} \delta_{kl},$$
(45)

$$(\alpha_{dd})_{ijkl} = -\frac{1}{16\pi^2} \frac{2g_1^4}{135M_E^2} \delta_{ij} \delta_{kl},$$
(46)

$$(\alpha_{ud}^{(1)})_{ijkl} = -\frac{1}{16\pi^2} \frac{8g_1^4}{135M_E^2} \delta_{ij}\delta_{kl},\tag{47}$$

$$(\alpha_{qu}^{(1)})_{ijkl} = -\frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{4g_1^4}{135} \delta_{ij} \delta_{kl} + \frac{1}{18} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor (Y_u)_{il} (Y_u^{\dagger})_{kj} \right], \tag{48}$$

$$(\alpha_{qu}^{(8)})_{ijkl} = -\frac{1}{16\pi^2} \frac{1}{3M_E^2} [\tilde{\lambda} \tilde{\lambda}^*] (Y_u)_{il} (Y_u^{\dagger})_{kj},$$
(49)

$$(\alpha_{qd}^{(1)})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{2g_1^4}{135} \delta_{ij} \delta_{kl} - \frac{1}{18} \lfloor \tilde{\lambda} \tilde{\lambda}^* \rfloor (Y_d)_{il} (Y_d^{\dagger})_{kj} \right],$$
(50)

$$(\alpha_{qd}^{(8)})_{ijkl} = -\frac{1}{16\pi^2} \frac{1}{3M_E^2} [\tilde{\lambda} \tilde{\lambda}^*] (Y_d)_{il} (Y_d^{\dagger})_{kj},$$
(51)

$$(\alpha_{quqd}^{(1)})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{3M_E^2} [\tilde{\lambda}\tilde{\lambda}^*] (Y_u)_{ij} (Y_d)_{kl},$$
(52)

$$\begin{aligned} (\alpha_{\ell\ell})_{ijkl} &= \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_1^4}{30} \delta_{ij} \delta_{kl} + \frac{25g_1^2 + 11g_2^2}{288} (\delta_{ij}\tilde{\lambda}_k \tilde{\lambda}_l^* + \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl}) \\ &- \frac{11g_2^2}{144} (\delta_{il}\tilde{\lambda}_k \tilde{\lambda}_j^* + \delta_{jk}\tilde{\lambda}_i \tilde{\lambda}_l^*) - \frac{1}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* \tilde{\lambda}_k \tilde{\lambda}_l^* + \frac{3}{16} (\tilde{\lambda}_i \tilde{\lambda}_l^* (Y_e)_{km} (Y_e^{\dagger})_{mj} + \tilde{\lambda}_k \tilde{\lambda}_j^* (Y_e)_{im} (Y_e^{\dagger})_{ml}) \right] \\ &+ \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{g_1^2 + g_2^2}{48} (\delta_{ij}\tilde{\lambda}_k \tilde{\lambda}_l^* + \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl}) - \frac{g_2^2}{24} (\delta_{il}\tilde{\lambda}_k \tilde{\lambda}_j^* + \delta_{jk}\tilde{\lambda}_i \tilde{\lambda}_l^*) \right] \\ &+ \frac{1}{8} (\tilde{\lambda}_i \tilde{\lambda}_l^* (Y_e)_{km} (Y_e^{\dagger})_{mj} + \tilde{\lambda}_k \tilde{\lambda}_j^* (Y_e)_{im} (Y_e^{\dagger})_{ml}) \right] L_E, \end{aligned}$$
(53)

$$(\alpha_{ee})_{ijkl} = -\frac{1}{16\pi^2} \frac{2g_1^4}{15M_E^2} \delta_{ij} \delta_{kl},$$
(54)

$$(\alpha_{\ell e})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{2g_1^4}{15} \delta_{ij} \delta_{kl} + \frac{25g_1^2}{72} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} - \frac{1}{6} (Y_e)_{il} (Y_e^{\dagger})_{kj} - \frac{3}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_e^{\dagger})_{km} (Y_e)_{ml} \right] \\ + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[+ \frac{g_1^2}{12} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} - \frac{1}{4} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_e^{\dagger})_{km} (Y_e)_{ml} \right] L_E,$$
(55)

$$(\alpha_{\ell q}^{(1)})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{g_1^4}{45} \delta_{ij} \delta_{kl} + \frac{25g_1^2}{432} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{3}{16} \tilde{\lambda}_i \tilde{\lambda}_j^* ((Y_d)_{km} (Y_d^{\dagger})_{ml} - (Y_u)_{km} (Y_u^{\dagger})_{ml}) \right] \\ + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_1^2}{72} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{1}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* ((Y_d)_{km} (Y_d^{\dagger})_{ml} - (Y_u)_{km} (Y_u^{\dagger})_{ml}) \right] L_E,$$
(56)

$$(\alpha_{\ell q}^{(3)})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{11g_2^2}{144} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{3}{16} \tilde{\lambda}_i \tilde{\lambda}_j^* \Big((Y_d)_{km} (Y_d^{\dagger})_{ml} + (Y_u)_{km} (Y_u^{\dagger})_{ml} \Big) \right] \\ + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_2^2}{24} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{1}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* \Big((Y_d)_{km} (Y_d^{\dagger})_{ml} + (Y_u)_{km} (Y_u^{\dagger})_{ml} \Big) \right] L_E,$$
(57)

$$(\alpha_{eu})_{ijkl} = \frac{1}{16\pi^2} \frac{8g_1^4}{45M_E^2} \delta_{ij} \delta_{kl},$$
(58)

$$(\alpha_{ed})_{ijkl} = -\frac{1}{16\pi^2} \frac{4g_1^4}{45M_E^2} \delta_{ij} \delta_{kl},$$
(59)

$$(\alpha_{qe})_{ijkl} = \frac{1}{16\pi^2} \frac{2g_1^4}{45M_E^2} \delta_{ij} \delta_{kl}, \tag{60}$$

$$(\alpha_{\ell u})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{4g_1^4}{45} \delta_{ij} \delta_{kl} - \frac{25g_1^2}{108} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{3}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_u^{\dagger})_{km} (Y_u)_{ml} \right] + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{g_1^2}{18} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} + \frac{1}{4} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_u^{\dagger})_{km} (Y_u)_{ml} \right] L_E,$$
(61)

$$(\alpha_{\ell d})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[-\frac{2g_1^4}{45} \delta_{ij} \delta_{kl} + \frac{25g_1^2}{216} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} - \frac{3}{8} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_d^{\dagger})_{km} (Y_d)_{ml} \right] + \frac{1}{16\pi^2} \frac{1}{M_E^2} \left[\frac{g_1^2}{36} \tilde{\lambda}_i \tilde{\lambda}_j^* \delta_{kl} - \frac{1}{4} \tilde{\lambda}_i \tilde{\lambda}_j^* (Y_d^{\dagger})_{km} (Y_d)_{ml} \right] L_E,$$
(62)

$$(\alpha_{\ell e d q})_{ijkl} = \frac{1}{16\pi^2} \frac{1}{3M_E^2} [\tilde{\lambda} \tilde{\lambda}^*] (Y_e)_{ij} (Y_d^{\dagger})_{kl},$$
(63)

$$(\alpha_{\ell equ}^{(1)})_{ijkl} = -\frac{1}{16\pi^2} \frac{1}{3M_E^2} [\tilde{\lambda}\tilde{\lambda}^*](Y_e)_{ij}(Y_u)_{kl}.$$
(64)

6.3 Basis translation

Given a specific EFT, defined by its field content and symmetries, there are different options to choose a basis of operators (either Green or physical). Different bases are useful for different purposes and it is very useful to have a systematic way to translate the results from one basis to another. The Rosetta code [75] can be used to translate between popular physical bases for the dimension 6 SMEFT. However a more general approach, applicable to different EFTs and any two bases (not necessarily physical) would be very welcome. Matchmakereft can do this in a straight-forward way by performing the tree-level matching of the new basis (as a UV model) onto the old one (as an EFT).

As a trivial example we consider the following two operators that appear in the one-loop integration of a scalar singlet as shown in [70]

$$O_{R} = (H^{\dagger}H)(D_{\mu}H^{\dagger}D^{\mu}H) \rightarrow 2\lambda \mathcal{O}_{H} + \frac{1}{2}\mathcal{O}_{H\Box} + \frac{1}{2}\Big((Y_{u})_{ij}(\mathcal{O}_{uH})_{ij} + (Y_{d})_{ij}(\mathcal{O}_{dH})_{ij} + (Y_{e})_{ij}(\mathcal{O}_{eH})_{ij} + \text{h.c.}\Big),$$
(65)

$$O_T = \frac{1}{2} (H^{\dagger} \overleftrightarrow{D}_{\mu} H)^2 \to -2\mathcal{O}_{HD} - \frac{1}{2} \mathcal{O}_{H\Box}, \tag{66}$$

where in the second equality we have written these operators in terms of the corresponding ones in the Warsaw basis (see Appendix D for the definition of the operators).

When using matchmakereft to match at tree level a UV model consisting of the SM plus the two operators in the new basis, O_R and O_T , onto the SMEFT in the basis described in Appendix D we obtain the following tree-level matching in the physical basis (we use β for the WCs of the operators in the new basis)

$$\alpha_H = 2\lambda\beta_R, \qquad \qquad \alpha_{HD} = -2\beta_T, \qquad \qquad \alpha_{H\Box} = \frac{1}{2}(\beta_R - \beta_T), \qquad (67)$$

$$\alpha_{uH} = \frac{1}{2} \beta_R(Y_u)_{ij}, \qquad \alpha_{dH} = \frac{1}{2} \beta_R(Y_d)_{ij}, \qquad \alpha_{eH} = \frac{1}{2} \beta_R(Y_e)_{ij}, \qquad (68)$$

which exactly reproduce the above equations. This is of course just a minimal example to show the application of matchmakereft to basis translation but complete bases (both Green and physical) can be translated in an automated way using this procedure.

6.4 Off-shell operator independence

Constructing a Green basis, that can match arbitrary off-shell amplitudes, is in principle straightforward. One has to simply write all possible operators and then eliminate those that are related to others by integration by parts (momentum conservation at the amplitude level) or gauge (Fierz) identities. However, this procedure is sometimes quite cumbersome in practice, in particular when there are many (space-time) indices involved. Matchmakereft can be used in this case to check if the operators defined in the EFT are linearly independent for arbitrary off-shell kinematics or not. This is done by checking the rank of the system of equations obtained by matching the EFT to all vanishing amplitudes. If several operators are linearly dependent, the rank will be smaller than the number of operators and matchmakereft will solve the system of equations to provide the relationship of the list of the dependent operators in terms of a particular set of independent ones. This is achieved by defining an EFT with all the relevant operators (including possibly linearly dependent ones) and running the command check_linear_dependece EFTModel (see Section 4 for details). As a word of caution it should be emphasized that matchmakereft makes no assumption about the space-time dimension. Thus, operators that are linearly dependent in D = 4 but not in arbitrary D are listed as linearly independent in matchmakereft.

7 Conclusions and outlook

EFTs have become a standard tool in quantum field theory to address multi-scale problems. Using a mass-independent renormalization scheme, the process of computing observables in models with disparate scales consists of integrating out particles at the highest mass threshold, running the WCs by means of the RGEs down to the next threshold, integrating out the particles at that threshold and so on until we reach the energies at which experimental measurements are performed. We then compute the corresponding experimental observable using the EFT relevant at that scale.

The current situation in particle physics seems to indicate that indeed, physics beyond the SM, if present, is likely to be at a scale much larger than the energies at which we are performing our experiments. In searches beyond the SM, the relevant EFT seems to be the SMEFT at energies above the electroweak scale and the LEFT below that scale. The process of running in the SMEFT, matching to the LEFT and running in the LEFT down to experimental energies has been recently completed, up to mass dimension 6 and one-loop order, and it is now available in computer tools. The matching of arbitrary models to the (dimension 6) SMEFT has been also recently solved at tree level. Thus, the only missing piece to perform a complete one-loop analysis of the implications of experimental data on new physics models is the one-loop matching of arbitrary models onto the SMEFT.

In this article we have introduced matchmakereft, a computer tool that performs treelevel and one-loop matching of arbitrary models onto arbitrary EFTs. Matchmakereft is robust, efficient, flexible and fully automated and can be trivially used to perform the step to one-loop matching of new physics models onto the SMEFT to complete the program outlined above. Due to its flexibility matchmakereft has many more applications than just the matching of new physics models onto the SMEFT. First it can match any model onto any EFT. Thus, it can be used to match new physics models to operators of dimension higher than 6 in the SMEFT (see [76] for the definition of the bosonic sector of a complete Green basis for the SMEFT at dimension 8), or to other EFTs beyond the SMEFT that can be also phenomenologically interesting, including for instance new light (or heavy particles) like an axion-like particle or a right-handed neutrino [77]. Matchmakereft is also able to match divergences in EFTs to compute the RGEs of arbitrary theories. Again this includes other EFTs beyond the dimension-6 SMEFT (or LEFT) but also renormalizable (or not) arbitrary theories. Other applications of matchmakereft include an automated basis translation between two (Green or physical) bases of an EFT or the reduction of an off-shell linearly dependent set of operators to a minimal Green basis. All these applications are performed in an automated way, with minimal interaction from the user, that only needs to provide the information of the UV and the EFT models.

Version 1.0.0 of matchmakereft performs an off-shell matching in the background-field gauge, thus providing a significant (kinematic and gauge) redundancy that is used to perform numerous cross-checks of the computed WCs. There are nevertheless some limitations in version 1.0.0 that we plan to overcome in future versions of matchmakereft. Among these some of the most significant are the following:

- Flavor indices are currently not supported for massive particles. There is an experimental version of this in matchmakereft and we expect to implement it in the near future.
- Amplitudes with many external particles (as required to match operators with many fields) can have a very large number of Feynman diagrams at one loop. When the number is very large the calculation can become very slow. We plan to introduce new procedures to deal with this issue, including the splitting of groups of Feynman diagrams that contribute to a single amplitude and the parallelization of the calculation to allow

an efficient computation in multi-core or multi-node computer systems.

Matchmakereft has already been used in several physical applications and we have performed an extensive number of non-trivial cross-checks to ensure the robustness of the calculations produced by matchmakereft. Its flexibility and efficiency will allow the particle physics community to analyze in a fully systematic and automated way the one-loop phenomenology or arbitrary new physics models.

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A Matchmakereft installation

A.1 Prerequisites

In order to be able to run matchmakereft, some prerequisites need to be met. First of all, the following programs need to be installed:

- Mathematica, version 10 or higher.
- FORM : Installation can be checked by typing form -v in a terminal. Binaries or the source code can be downloaded from http://www.nikhef.nl/~form/
- QGRAF : Installation can be checked by typing qgraf in a terminal. Binaries or the source code can be downloaded from http://cfif.ist.utl.pt/~paulo/qgraf.html. Please note that versions of QGRAF earlier than 3.5 had a maximun multiplicity of vertices equal to 6 particles. If vertices with more than 6 particles are needed (this can happen if dimension-8 operators are considered for instance) the source code has to be modified accordingly and compiled again. The maximum multiplicity of QGRAF v3.5 is 8.

The binaries of both FORM and QGRAF need to be located in some path that is included in the binary path of the system, in such a way that they can be executed from any possible location. It is also necessary to have installed

• Python (3.5 or higher): Installation can be checked by typing python --version in a terminal. In some systems python3 might have to be explicitly invoked.

 FeynRules the mathematica package, is needed for the creation of matchmakereft models. It can be downloaded from https://feynrules.irmp.ucl.ac.be/.

Matchmakereft is available both in the Python Package Index (PyPI) https://pypi.org/ project/matchmakereft/ as well as in the Anaconda Python distribution https://anaconda. org/matchmakers/matchmakereft. Moreover,

- pip. In order to access packages from PyPI, pip needs to be installed on the system. It is available in most Linux distributions and can be easily installed on MacOS and Windows. Installation can be checked by typing pip -V in a terminal. In some systems pip3 has to be explicitly invoked. General information about installing packages fcan be found on https://packaging.python.org/en/latest/tutorials/installing-packages/.
- conda. Instructions about Anaconda installation on different OS can be found on https: //docs.anaconda.com/anaconda/install/index.html. Explicit information about installing conda packages can be found on https://docs.anaconda.com/anacondaorg/user-guide/ howto/.

A.2 Installing matchmakereft

A.2.1 PyPI

Once pip is installed in the system, matchmakereft can be installed by just typing in a terminal (we denote the terminal prompt as >)

> python3 -m pip install matchmakereft --user

or by typing

```
> pip install matchmakereft --user
```

Alternatively, if the installation has been downloaded from the project web page, it can be installed via

> python3 -m pip install matchmakereft-x.x.x.tar.gz --user

where the $x \cdot x \cdot x$ correspond to the version being installed.

We can get information about matchmakereft by writing

```
> pip3 show matchmakereft
Name: matchmakereft
Version: 1.0.0
Summary: One loop matching
Home-page: https://ftae.ugr.es/matchmakereft/
Author: Adrian Carmona, Achilleas Lazopoulos, Pablo Olgoso, Jose
Santiago
Author-email: adrian@ugr.es, lazopoulos@itp.phys.ethz.ch,
pablolgoso@ugr.es, jsantiago@ugr.es
License: Creative Commons Attribution-Noncommercial-Share Alike
license
Location: /home/user/.local/lib/python3.9/site-packages
Requires: requests, yolk3k, setuptools
Required-by:
```

If matchmakereft is already installed in the system, it is possible to check for possible updates by writing

> pip install --upgrade matchmakereft --user

whereas one can remove it by typing

> pip uninstall matchmakereft

A.2.2 Anaconda

For users employing Anaconda python distribution, matchmakereft can be installed by just typing

```
conda install -c matchmakers matchmakereft
```

A.3 Updating the system path

Once matchmakereft is installed, we have to ensure that the corresponding executable is included in the user path. This can be easily achieved by copying the following two lines in the $^{-}$.bashrc file (or the equivalent one if a different shell is used)

```
1 export PY_USER_BIN=$(python -c 'import site; print(site.USER_BASE + "/
bin")')
2 export PATH=$PY_USER_BIN:$PATH
```

If the user prefers to install without the --user option (s)he should ensure that the directory where the executables are installed is included in the path.

B Dealing with γ_5

When computing the matching equations with matchmakereft, we might encounter loop integrals that involve fermionic traces with chiral projectors, $P_{L,R}$. Among such traces, those that contain an odd number of γ_5 and six or more γ -matrices are known to be ambiguous, due to the impossibility of finding a regulator that is Lorenz invariant and preserves the chiral structure of the theory. In matchmakereft we regulate integrals within Dimensional Regularization, so we have to face the question of how to compute these traces consistently.

Limiting ourselves, at first, to matching renormalizable UV models to the SMEFT, the cases we have to deal with, in practice, are few. In order to have a trace in the first place, there can be no external fermions in the related diagram. Moreover, the ambiguity is proportional to the fully antisymmetric $\epsilon_{\mu\nu\rho\sigma}$ tensor, and can therefore contribute to the Wilson Coefficient of one of the few CP violating bosonic operators of the SMEFT, with two or three field strength tensors. We therefore need two or three external gauge bosons. In order to have a trace with at least 6 γ -matrices we, therefore need four or three internal fermionic propagators respectively. In summary, we only need to worry about boxes contributing to $H^{\dagger}HX_{\mu\nu}\tilde{X}^{\mu\nu}_{\rho}\tilde{X}^{\mu}_{\rho}$, with fermions in the internal lines, however, are non-ambiguous because there are no γ_5 's involved in the vertices: all heavy fermions have to be vector-like⁶.

Next, let's look at the box diagrams contributing to $H^{\dagger}HX_{\mu\nu}\tilde{X}^{\mu\nu}$. There are four internal fermionic propagators, of which one or more can correspond to heavy particles in the UV model. As mentioned, the corresponding traces have ambiguous and non-ambiguous parts. The unambiguous part can be computed in any γ_5 -scheme. The ambiguous part is proportional to D-4 and is only of interest if it multiplies one of the singularities of the integral. Moreover, since the UV theory is renormalizable, the maximum number of γ -matrices is 6, which means that terms in the numerator of such integrals with mass insertions have a lower number of γ 's and are therefore unambiguous. There are two types of singularities: UV singularities corresponding to the UV structure of the full theory, and IR singularities that appear after the hard region expansion is performed. The latter correspond to the UV singularities of the SMEFT.

⁶Triangle diagrams with only light particles are ambiguous, but they do not concern us during matching. The corresponding ambiguities are fixed by the anomaly cancellation mechanism that we assume any EFT has built in.

The generic structure of the potentially ambiguous part of such box integrals is, up to couplings and group theory factors:

$$I^{\mu\nu} \sim \int_{k} T_{\mu_{1}...\mu_{4}\mu\nu} \prod_{i=1}^{4} \frac{(k+q_{i})^{\mu_{i}}}{(k+q_{i})^{2} - m_{i}^{2}},$$
(69)

where $\int_k \equiv \int \frac{d^D k}{(2\pi)^D}$, and $T_{\mu_1...\mu_4\mu_{\nu}}$ denotes a trace with an odd number of γ_5 insertions and six γ 's, e.g.

$$Tr[\gamma_{\mu_1}\gamma_5\gamma_{\mu_2}\gamma_{\mu}\gamma_{\mu_3}\gamma_5\gamma_{\nu}\gamma_{\mu_4}\gamma_5].$$
(70)

Performing the hard region expansion and the tensor reduction we get

$$\begin{split} I^{\mu\nu} &\sim \int_{k} T_{\mu_{1}...\mu_{4}\mu\nu} \frac{1}{\prod_{i=1}^{4} (k^{2} - m_{i}^{2})} \\ &\left(k^{4} \frac{g^{\mu_{1}\mu_{2}\mu_{3}\mu_{4}}}{D(D+2)} (1 + \sum_{i} \frac{q_{i}^{2}}{k^{2} - m_{i}^{2}}) + k^{6} \frac{g^{\mu_{1}\mu_{2}\mu_{3}\mu_{4}\rho\sigma}}{D(D+2)(D+4)} \sum_{i \leq j} \frac{4q_{i}^{\rho} q_{j}^{\sigma}}{(k^{2} - m_{i}^{2})(k^{2} - m_{j}^{2})} \\ &- 2k^{4} \sum_{i,m} q_{i}^{\mu_{i}} q_{m}^{\rho} \frac{g^{\{\hat{\mu}_{i}\}\rho}}{D(D+2)} \frac{1}{k^{2} - m_{m}^{2}} + \sum_{i < j} k^{2} \frac{g^{\{\hat{\mu}_{i},\hat{\mu}_{j}\}}}{D} q^{\mu_{i}} q^{\mu_{j}} \right) \\ &\sim \int_{k} T_{\mu_{1}...\mu_{4}\mu\nu} \frac{g^{\mu_{1}\mu_{2}\mu_{3}\mu_{4}}}{D(D+2)} \frac{k^{4}}{\prod_{i=1}^{4} (k^{2} - m_{i}^{2})} + \int_{k} T_{\mu_{1}...\mu_{4}\mu\nu} \frac{k^{2}}{\prod_{i=1}^{4} (k^{2} - m_{i}^{2})} \frac{1}{D(D+2)} \\ &\left(g^{\mu_{1}\mu_{2}\mu_{3}\mu_{4}} \sum_{i} q_{i}^{2} \left(1 + \frac{m_{i}^{2}}{k^{2} - m_{i}^{2}}\right) + \frac{g^{\mu_{1}\mu_{2}\mu_{3}\mu_{4}\rho\sigma}}{(D+4)} \sum_{i \leq j} 4q_{i}^{\rho} q_{j}^{\sigma} \prod_{l=i,j} \left(1 + \frac{m_{l}^{2}}{(k^{2} - m_{l}^{2})}\right) \\ &- 2\sum_{i,m} q_{i}^{\mu_{i}} q_{m}^{\rho} g^{\{\hat{\mu}_{i}\}\rho} (1 + \frac{m_{m}^{2}}{k^{2} - m_{m}^{2}}) + \sum_{i < j} (D+2)g^{\{\hat{\mu}_{i},\hat{\mu}_{j}\}} q^{\mu_{i}} q^{\mu_{j}}\right), \end{split}$$
(71)

where $\{\hat{\mu}_i\} = \mu_j \mu_k \mu_r$ with $j, k, r \neq i$, and $\{\hat{\mu}_i \hat{\mu}_j\} = \mu_k \mu_r$ with $k, r \neq i, j$.

The first term, of $\mathcal{O}(p^0)$, is UV singular. It does not lead to ambiguities, though, since $g^{\mu_1\mu_2\mu_3\mu_4}$ makes the γ_5 -dependent part of the trace vanish. The rest of the terms, of order $\mathcal{O}(p^2)$, are UV finite, but can be IR singular, depending on the number of heavy propagators: for two, three or four heavy propagators, there is no singularity. For one heavy and three light propagators, however, we do have an IR singularity.

We, therefore, conclude that ambiguous contributions from γ_5 -odd traces in box integrals are present only in diagrams with one heavy and three light propagators. They appear as a product of the ambiguous (D-4) coefficient of the trace multiplying a $1/\epsilon$ pole of IR type resulting from the hard region expansion. We would like to remain within the naive anticommuting γ_5 scheme when computing traces in matchmakereft. To this end, we wish to fix the ambiguous contributions of the γ_5 -odd traces in box integrals with one heavy massive fermion, a posteriori. We note that the Wilson coefficient we are trying to match must be real: all operators of the type $H^{\dagger}HX_{\mu\nu}\tilde{X}^{\mu\nu}$ are hermitian. Therefore any contribution from γ_5 -odd traces, being imaginary, must be multiplied by a purely imaginary product of couplings of the full theory. But, in the case that ambiguities are present, i.e. when the UV theory has Yukawa terms with the Higgs boson, one heavy and one light fermion, leading to box diagrams with a single fermionic propagators, the corresponding Yukawa couplings are complex conjugates of each other, by virtue of the hermiticity of the UV Lagrangian. As a result, the product of all couplings is real and, therefore, the γ_5 -odd contributions are purely imaginary. They can be set to zero by hand, at the end of the computation. Note that this does not imply that the ambiguous contributions are zero: it is the sum of ambiguous and non-ambiguous traces that are set to zero.

Note that this procedure also works if the effective theory is not the SMEFT. In such cases, more than one scalar fields might be present, allowing for non-hermitian operators of the type $\phi_1^{\dagger}\phi_2 X_{\mu\nu}\tilde{X}^{\mu\nu}$. The constraint that the corresponding wilson coefficient is real does not apply any more. It is, however still true that the wilson coefficient of this operator should equal the complex conjugate of the wilson coefficient of the hermitian conjugate operator $\phi_2^{\dagger}\phi_1 X_{\mu\nu}\tilde{X}^{\mu\nu}$. This is, again, enough to remove the ambiguities.

C A minimal complete example

In this section we demonstrate many of the features of matchmakereft with a concrete example involving two scalar fields, a light, but not massless, field ϕ and a heavy field Φ . Our model is described by the Lagrangian:

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^2 - \frac{1}{2} m_L^2 \phi^2 + \frac{1}{2} (\partial_{\mu} \Phi)^2 - \frac{1}{2} M_H^2 \Phi^2 - \frac{\lambda_0}{4!} \phi^4 - \frac{\lambda_2}{4} \phi^2 \Phi^2 - \frac{\kappa}{2} \phi^2 \Phi, \qquad (72)$$

which we want to match to the EFT Lagrangian without the heavy scalar,

$$\mathcal{L}_{\rm EFT} = \frac{\alpha_{4k}}{2} (\partial_{\mu}\phi)^2 - \frac{\alpha_2}{2}\phi^2 - \frac{\alpha_4}{4!}\phi^4 - \frac{\alpha_6}{6!}\phi^6 - \frac{\ddot{\alpha}_6}{4!}\phi^3\partial^2\phi - \frac{\dot{\alpha}_6}{2}(\partial^2\phi)^2.$$
(73)

We will use this Lagrangian during off-shell matching. Subsequently, the kinetic term can be canonically normalized, and the redundant operators can be eliminated. Two of the three operators of dimension 6 are redundant. We choose ϕ^6 as the independent operator. Using equations of motion we can readily find that:

$$\phi^3 \partial^2 \phi \to -\alpha_2 \phi^4 - \frac{1}{3!} \alpha_4 \phi^6, \tag{74}$$

$$(\partial^2 \phi)^2 \to \alpha_2^2 \phi^2 + \frac{\alpha_2 \alpha_4}{3} \phi^4 + \frac{\alpha_4}{36} \phi^6.$$
 (75)

Eliminating these operators from the Lagrangian would induce the shifts

$$\alpha_2 \rightarrow \alpha_2 + \alpha_2^2 \hat{\alpha}_6 \tag{76}$$

$$\alpha_4 \rightarrow \alpha_4 - \tilde{\alpha}_6 \alpha_2 + 4\alpha_2 \alpha_4 \hat{\alpha}_6 \tag{77}$$

$$\alpha_6 \rightarrow \alpha_6 - 5\tilde{\alpha}_6 \alpha_4 + 10\hat{\alpha}_6 \alpha_4^2 \tag{78}$$

The coupling κ of this model is a dimensionful coupling, and is expected to be parametrically of the order of the heavy mass scale M_H . Thus, $\frac{\kappa}{M_H}$ is of $\mathcal{O}(1)$ and is kept throughout the matching procedure consistently.

The Feynrules file for the UV model, saved at two_scalars.fr, is shown below.

```
1 (* --- Contents of Feynrules file two_scalars.fr --- *)
2 M$ModelName = "two_scalars";
                                 *)
3 (* **** Particle classes
                            ****
4 M$ClassesDescription = {
5 S[1] == {ClassName -> phiH, SelfConjugate -> True, Mass -> MH,
           FullName -> "heavy"},
6
7 S[2] == {ClassName -> phi, SelfConjugate -> True, Mass -> mL,
8 FullName -> "light"}
9 };
10 (* ****
                          ***** *)
            Parameters
11 M$Parameters = {
12 MH == {ParameterType -> Internal, ComplexParameter -> False},
13 mL == {ParameterType -> Internal, ComplexParameter -> False},
```

```
14 V == {ParameterType -> Internal, ComplexParameter -> False},
15 lambda0 == {ParameterType -> Internal, ComplexParameter -> False},
16 kappa == {ParameterType -> Internal, ComplexParameter -> False},
17 lambda2 == {ParameterType -> Internal, ComplexParameter -> False}
18 };
19 (* ****
                            **** *)
              Lagrangian
20 Ltot := Block[{mu},
   + 1/2 * del[phi,mu] * del[phi,mu] + 1/2 *del[phiH,mu] * del[phiH,
21
     mul
    - 1/2 * MH<sup>2</sup> * phiH<sup>2</sup> - 1/2 * mL<sup>2</sup> * phi<sup>2</sup>
22
   - lambda0 / 24 * phi^4 - kappa / 2 * phi^2 * phiH
23
    - lambda2 / 4 * phi^2 * phiH^2
24
25 ];
```

Note that we use the keyword FullName to characterize each field as "heavy" or "light". This is mandatory: matchmaker uses this keyword to distinguish between fields that are integrated out and those that are light and are also present in the EFT. Also note that all the parameters that are used in the Lagrangian, masses as well as couplings, must be declared. In this example all parameters are real.

The Feynrules file for the EFT model, saved at one_scalar.fr, is:

```
1 (* --- Contents of Feynrules file for the EFT model one_scalar.fr ---
      *)
2 M$ModelName = "one_scalar";
3 (* **** Particle classes **** *)
4 M$ClassesDescription = {
s S[2] == {ClassName -> phi, SelfConjugate -> True, Mass -> 0,
           FullName -> "light"}
6
7 };
8 (* *****
             Parameters
                         **** *)
9 M$Parameters = {
10 alpha4kin == {ParameterType -> Internal, ComplexParameter -> False},
n alpha2mass == {ParameterType -> Internal, ComplexParameter -> False},
12 alpha4 == {ParameterType -> Internal, ComplexParameter -> False},
13 alpha6 == {ParameterType -> Internal, ComplexParameter -> False},
14 alpha6Rtilde == {ParameterType -> Internal, ComplexParameter -> False
     }.
15 alpha6Rhat == {ParameterType -> Internal, ComplexParameter -> False}
16 };
                          **** *)
17 (* ****
             Lagrangian
18 Ltot := Block [{mu,mu2},
  1/2 * alpha4kin * del[phi,mu] * del[phi,mu]
19
    - 1/2 * alpha2mass * phi^2
20
    - alpha4/24 * phi^4
21
   - alpha6 * phi^6/720
22
   - alpha6Rtilde/24 * phi^3 * del[del[phi,mu],mu]
23
    - alpha6Rhat/2 * del[del[phi,mu],mu] * del[del[phi,mu2],mu2]
24
25 ];
```

Note that we have included WCs (denoted by alpha) also for the kinetic and mass terms (squared), as well as for all operators that are redundant solely due to the equations of motion. In order for matchmakereft to perform the reduction to the physical basis, we need to provide a set of relations that express the redundant WCs in terms of the irreducible ones, see Eq. (78). This is done at one_scalar.red:

```
alpha2mass -> alpha2mass + alpha6Rhat * alpha2mass^2
```

Note that **only** the WCs corresponding to physical operators, among those appearing in the EFT Lagrangian and defined in the file one_scalar.fr, must be present on the left hand side of the replacement rules in this file. The WCs corresponding to redundant operators appear only on the right hand side. When these rules are used, both redundant and non-redundant WCs have been matched and are known as functions of the parameters of the UV theory. The rules are therefore instructions on how to update the non-redundant WCs, to include the effect of the redundant ones.

With these files prepared we are ready to proceed with matching. In the matching directory, where two_scalars.fr,one_scalar.fr,one_scalar.red are present, we can run matchmakereft:

```
>matchmakereft
```

upon which we enter the python interface

Welcome to matchmakereft Please refer to arXiv:2112.10787 when using this code

matchmakereft >

We first need to create the matchmaker models, i.e. the directories with all the necessary information for the UV and the EFT models. We do this by

```
matchmakereft> create_model two_scalars.fr
```

which has the response

```
Creating model two_scalars_MM. This might take some time depending on
the complexity of the model
Model two_scalars_MM created
It took 7 seconds to create it.
```

We can now observe that the directory two_scalars_MM is created. We proceed with creating the EFT model

```
matchmakereft> create_model one_scalar.fr
Creating model one_scalar_MM. This might take some time depending on
the complexity of the model
Model one_scalar_MM created
It took 7 seconds to create it.
```

The one_scalar_MM directory is now created as well, and we are ready for the matching calculation. This is performed by the match_model_to_eft command:

matchmakereft> match_model_to_eft two_scalars_MM one_scalar_MM

Upon completion, the results of the matching are stored in the UV model directory, in this case two_scalars_MM. The file two_scalars_MM/MatchingProblems.dat contains troubleshooting information in case the matching procedure failed. In our case it is an empty list, indicating no problems:

```
problist = {}
```

The result of the matching procedure is in two_scalars_MM/MatchingResults.dat, a Mathematica file with a list of lists of replacement rules. The off-shell matching gives the following results for the WCs of the Green basis. At tree level the non-vanishing contributions are,

$$\alpha_2^{(0)} = m_L^2, (79)$$

$$\alpha_4^{(0)} = \lambda_0 - \frac{4\kappa^2 m_L^2}{M_H^4} - \frac{3\kappa^2}{M_H^2},$$
(80)

$$\alpha_{6}^{(0)} = \frac{1}{M_{H}^{2}} \left(\frac{60\kappa^{4}}{M_{H}^{4}} - \frac{20\lambda_{0}\kappa^{2}}{M_{H}^{2}} + \frac{45\lambda_{2}\kappa^{2}}{M_{H}^{2}} \right),$$
(81)

$$\tilde{\alpha}_{6}^{(0)} = \frac{4\kappa^2}{M_H^4}.$$
(82)

At one loop we get (we define $L_M \equiv \log(\frac{\mu^2}{M_H^2})$)

$$\alpha_{2}^{(1)} = -\frac{1}{16\pi^{2}} \left(1 + L_{M}\right) \left(\frac{\kappa^{2} m_{L}^{4}}{M_{H}^{4}} + \frac{\kappa^{2} m_{L}^{2}}{M_{H}^{2}} + \frac{1}{2} \lambda_{2} M_{H}^{2} + \kappa^{2}\right), \tag{83}$$

$$\alpha_{4k}^{(1)} = 1 + \frac{1}{16\pi^2} \left(\frac{5\kappa^2 m_L^2}{2M_H^4} + \frac{\kappa^2}{2M_H^2} \right) + \frac{1}{16\pi^2} \frac{\kappa^2 m_L^2}{M_H^4} L_M, \tag{84}$$

$$\begin{aligned} \alpha_{4}^{(1)} &= \frac{1}{16\pi^{2}} \left(\frac{48\kappa^{4}m_{L}^{2}}{M_{H}^{6}} + \frac{24\lambda_{2}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} - \frac{12\lambda_{0}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} + \frac{18\kappa^{4}}{M_{H}^{4}} + \frac{18\lambda_{2}\kappa^{2}}{M_{H}^{2}} - \frac{6\lambda_{0}\kappa^{2}}{M_{H}^{2}} \right) \tag{85} \\ &+ \frac{1}{16\pi^{2}} \left(\frac{36\kappa^{4}m_{L}^{2}}{M_{H}^{6}} + \frac{18\lambda_{2}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} - \frac{12\lambda_{0}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} + \frac{12\kappa^{4}}{M_{H}^{4}} + \frac{12\lambda_{2}\kappa^{2}}{M_{H}^{2}} - \frac{6\lambda_{0}\kappa^{2}}{M_{H}^{2}} - \frac{3\lambda_{2}^{2}}{2} \right) L_{M}, \end{aligned}$$

$$\begin{aligned} \alpha_{6}^{(1)} &= \frac{1}{16\pi^{2}M_{H}^{2}} \left(-\frac{1290\kappa^{6}}{M_{H}^{6}} + \frac{720\lambda_{0}\kappa^{4}}{M_{H}^{4}} - \frac{1665\lambda_{2}\kappa^{4}}{M_{H}^{4}} + \frac{360\lambda_{0}\lambda_{2}\kappa^{2}}{M_{H}^{2}} - \frac{90\lambda_{0}^{2}\kappa^{2}}{M_{H}^{2}} - \frac{495\lambda_{2}^{2}\kappa^{2}}{M_{H}^{2}} + \frac{15\lambda_{2}^{3}}{2} \right) \\ &+ \frac{1}{16\pi^{2}M_{H}^{2}} \left(-\frac{810\kappa^{6}}{M_{H}^{6}} + \frac{540\lambda_{0}\kappa^{4}}{M_{H}^{4}} - \frac{945\lambda_{2}\kappa^{4}}{M_{H}^{4}} + \frac{270\lambda_{0}\lambda_{2}\kappa^{2}}{M_{H}^{2}} - \frac{90\lambda_{0}^{2}\kappa^{2}}{M_{H}^{2}} - \frac{270\lambda_{2}^{2}\kappa^{2}}{M_{H}^{2}} \right) L_{M}, \end{aligned}$$

$$(87)$$

$$(88)$$

$$\tilde{\alpha}_{6}^{(1)} = \frac{1}{16\pi^{2}M_{H}^{2}} \left(-\frac{107\kappa^{4}}{3M_{H}^{4}} + \frac{9\lambda_{0}\kappa^{2}}{M_{H}^{2}} - \frac{77\lambda_{2}\kappa^{2}}{3M_{H}^{2}} + \frac{\lambda_{2}^{2}}{3} \right) + \frac{1}{16\pi^{2}M_{H}^{2}} \left(-\frac{14\kappa^{4}}{M_{H}^{4}} + \frac{2\lambda_{0}\kappa^{2}}{M_{H}^{2}} - \frac{16\lambda_{2}\kappa^{2}}{M_{H}^{2}} \right) L_{M},$$
(89)

$$\hat{\alpha}_{6}^{(1)} = -\frac{\kappa^2}{96\pi^2 M_H^4}.$$
(90)

As can be seen in the equations above, the kinetic operator receives a correction and therefore ϕ is no longer canonically normalized. A field redefinition is needed to obtain a canonically normalized theory on which we can apply the corresponding redundancies to go to the physical basis. Matchmakereft does these two processes (canonical normalization and going to the physical basis) automatically. The resulting WCs in the physical basis read, up to one loop order

$$\alpha_{2} = m_{L}^{2} - \frac{1}{16\pi^{2}} \left(\frac{11\kappa^{2}m_{L}^{4}}{3M_{H}^{4}} + \frac{3\kappa^{2}m_{L}^{2}}{2M_{H}^{2}} + \frac{1}{2}\lambda_{2}M_{H}^{2} + \kappa^{2} \right) - \frac{1}{16\pi^{2}} \left(\frac{2\kappa^{2}m_{L}^{4}}{M_{H}^{4}} + \frac{\kappa^{2}m_{L}^{2}}{M_{H}^{2}} + \frac{1}{2}\lambda_{2}M_{H}^{2} + \kappa^{2} \right) L_{M},$$

$$(91)$$

$$4\kappa^{2}m^{2} - 3\kappa^{2}$$

$$\alpha_{4} = \lambda_{0} - \frac{4\kappa m_{L}}{M_{H}^{4}} - \frac{3\kappa^{2}}{M_{H}^{2}}$$
(92)
$$1 \left(332\kappa^{4}m_{L}^{2} 80\lambda_{0}\kappa^{2}m_{L}^{2} 149\lambda_{2}\kappa^{2}m_{L}^{2} \lambda_{2}^{2}m_{L}^{2} 25\kappa^{4} 7\lambda_{0}\kappa^{2} 20\lambda_{2}\kappa^{2} \right)$$

$$+ \frac{1}{16\pi^{2}} \left(\frac{3M_{H}^{6}}{3M_{H}^{6}} - \frac{3M_{H}^{4}}{3M_{H}^{4}} + \frac{3M_{H}^{4}}{3M_{H}^{4}} - \frac{3M_{H}^{2}}{3M_{H}^{2}} + \frac{3M_{H}^{2}}{M_{H}^{4}} - \frac{3M_{H}^{2}}{3M_{H}^{2}} + \frac{M_{H}^{2}}{M_{H}^{2}} + \frac{M_{H}^{2}}{M_{H}^{2}} \right)$$

$$+ \frac{1}{16\pi^{2}} \left(\frac{60\kappa^{4}m_{L}^{2}}{M_{H}^{6}} - \frac{16\lambda_{0}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} + \frac{34\lambda_{2}\kappa^{2}m_{L}^{2}}{M_{H}^{4}} + \frac{16\kappa^{4}}{M_{H}^{4}} - \frac{6\lambda_{0}\kappa^{2}}{M_{H}^{2}} + \frac{14\lambda_{2}\kappa^{2}}{M_{H}^{2}} - \frac{3\lambda_{2}^{2}}{2} \right) L_{M},$$

$$\alpha_{6} = \frac{1}{M_{H}^{2}} \left(\frac{60\kappa^{4}}{M_{H}^{4}} - \frac{20\lambda_{0}\kappa^{2}}{M_{H}^{2}} + \frac{45\lambda_{2}\kappa^{2}}{M_{H}^{2}} \right)$$

$$+ \frac{1}{16\pi^{2}} \left(\frac{-\frac{1560\kappa^{6}m_{L}^{2}}{M_{H}^{4}} + \frac{440\lambda_{0}\kappa^{4}m_{L}^{2}}{M_{H}^{6}} - \frac{1635\lambda_{2}\kappa^{4}m_{L}^{2}}{2M_{H}^{6}} - \frac{2320\kappa^{6}}{M_{H}^{6}} + \frac{3610\lambda_{0}\kappa^{4}}{3M_{H}^{4}} \right)$$

$$- \frac{4955\lambda_{2}\kappa^{4}}{2M_{H}^{4}} - \frac{410\lambda_{0}^{2}\kappa^{2}}{3M_{H}^{2}} - \frac{490\lambda_{2}^{2}\kappa^{2}}{M_{H}^{2}} + \frac{1465\lambda_{0}\lambda_{2}\kappa^{2}}{3M_{H}^{2}} + \frac{15\lambda_{2}^{3}}{2} - \frac{5\lambda_{0}\lambda_{2}^{2}}{3} \right)$$

$$+ \frac{1}{16\pi^{2}}M_{H}^{2}} \left(-\frac{960\kappa^{6}m_{L}^{2}}{M_{H}^{8}} + \frac{320\lambda_{0}\kappa^{4}m_{L}^{2}}{M_{H}^{6}} - \frac{495\lambda_{2}\kappa^{4}m_{L}^{2}}{3M_{H}^{2}} - \frac{1260\kappa^{6}}{M_{H}^{6}} + \frac{760\lambda_{0}\kappa^{4}}{M_{H}^{4}} \right)$$

$$- \frac{1425\lambda_{2}\kappa^{4}}{M_{H}^{4}} - \frac{100\lambda_{0}^{2}\kappa^{2}}{M_{H}^{2}} - \frac{240\lambda_{2}^{2}\kappa^{2}}{M_{H}^{2}} + \frac{350\lambda_{0}\lambda_{2}\kappa^{2}}{M_{H}^{2}} \right) L_{M}.$$

$$(93)$$

Next, we would like to compute the RGE equations for both models. For each model we need a new pair of Feynrule files as a starting point. We start for the RGEs for the UV model. The corresponding UV model, located in the file rge_two_scalars_uv.fr is the same as two_scalars.fr with the very crucial difference that the heavy field H is also declared as "light" here.

```
1 S[1] == {ClassName-> H, SelfConjugate->True, Mass->MH,
2 FullName->"light"}
```

The target model is now at rge_two_scalars_eft.fr. It consists of an EFT Lagrangian with the field *H* considered as a light field:

```
1 (* --- Contents of rge_two_scalars_eft.fr --- *)
2 M$ModelName = "rge_two_scalars_eft";
3 (* **** Particle classes **** *)
4 M$ClassesDescription = {
s S[1] == {ClassName -> phiH, SelfConjugate -> True, Mass -> 0,
           FullName -> "light"},
6
7 S[2] == {ClassName -> phi,SelfConjugate -> True,Mass -> 0,
           FullName -> "light"}
8
9 };
10 (* ****
             Parameters
                          ***** *)
11 M$Parameters = {
12 alpha4kinphi == {ParameterType -> Internal, ComplexParameter -> False
     }.
13 alpha4kinH == {ParameterType -> Internal, ComplexParameter -> False},
14 alpha2MH == {ParameterType -> Internal, ComplexParameter -> False},
15 alpha2ML == {ParameterType -> Internal, ComplexParameter -> False},
```

```
16 alpha4 == {ParameterType -> Internal, ComplexParameter -> False},
17 alphaV == {ParameterType -> Internal, ComplexParameter -> False},
18 alpha1 == {ParameterType -> Internal, ComplexParameter -> False},
19 alpha4H == {ParameterType -> Internal, ComplexParameter -> False},
20 alpha2 == {ParameterType -> Internal, ComplexParameter -> False},
21 alpha3 == {ParameterType -> Internal, ComplexParameter -> False}
22 };
23 (* ****
              Lagrangian
                             **** *)
24 Ltot := Block[{mu},
   alphaV * phiH
25
   +1/2 * alpha4kinphi* del[phi,mu] * del[phi,mu]
26
   + 1/2 * alpha4kinH * del[phiH,mu] * del[phiH,mu]
27
    - 1/2 * alpha2MH * phiH^2
28
    -1/2 * alpha2ML * phi<sup>2</sup>
29
    - alpha4 / 24 * phi^4
30
    - alpha1 / 2 * phi^2 * phiH
31
    - alpha2 / 4 * phi^2 * phiH^2
32
    -alpha3 * phiH^3
33
   -alpha4H * phiH^4
34
35];
```

Note the presence of the new interaction terms Φ^3 , Φ^4 : all possible operators of dimension up to 4, compatible with the symmetries should appear here. Also note that we now have a WC also for the kinetic term of the Φ field.

We don't need a rge_two_scalars_eft.red file at all, since all operators of dimension four present are physical. We can now create the two models with

```
matchmakereft> create_model rge_two_scalars_uv.fr
```

and

matchmakereft> create_model rge_two_scalars_eft.fr

and then we can proceed with the RGE computation via

```
matchmakereft> compute_rge_model_to_eft rge_two_scalars_uv_MM
    rge_two_scalars_eft_MM
```

We then get, as before, inside the rge_two_scalars_uv_MM directory an empty problem file MatchingProblems.dat, as well as a MatchingResults.dat file. Moreover, there is another file produced, RGEResult.dat which contains the explicit form of the beta functions for our UV model.

```
1 RGEResult = {
2 \[Beta][alphaV] -> -1/32*(kappa*mL^2)/Pi^2
3 \[Beta][alpha4kinphi] -> 0,
4 \[Beta][alpha4kinH] -> 0,
5 \[Beta][alpha2MH] -> kappa^2/(16*Pi^2)+(lambda2*mL^2)/(16*Pi^2),
6 \[Beta][alpha2ML] -> kappa^2/(8*Pi^2)+(lambda2*MH^2)/(16*Pi^2) +
7 (lambda0*mL^2)/(16*Pi^2),
8 \[Beta][alpha4] -> (3*lambda0^2)/(16*Pi^2) +
9 (3*lambda2^2)/(16*Pi^2),
10 \[Beta][alpha1] ->(lambda0*kappa)/(16*Pi^2) + (kappa*lambda2)/(4*Pi^2),
11 \[Beta][alpha2] -> (lambda0*lambda2)/(16*Pi^2),
12 \[Beta][alpha3] -> (kappa*lambda2)/(32*Pi^2),
13 \[Beta][alpha4H] -> lambda2^2/(128*Pi^2)}
```

Let us now touch upon the topic of tadpole contributions, which will also explain the reason for the $\alpha_V \Phi$ operator defined in line 25 of rge_two_scalars_eft.fr. In our model there are tadpole contributions to many 1LPI diagrams, due to the operator $\kappa \phi^2 \Phi$. They would vanish, up to one loop, if the mass of the light field was set to zero, but are non-vanishing otherwise. The resulting finite contributions are taken into account by our matching procedure. However, the corresponding poles, which contribute to the beta functions of m_L and κ are disregarded, during the RGE computation. However, there is an easy way to account for them. By adding the $\alpha_V \Phi$ term in the Lagrangian of rge_two_scalars_eft.fr, we induce the computation of the one-point function beta-function: as we see from line 2 of RGEResult.dat, we have

$$\beta(\alpha_V) = -\frac{1}{32\pi^2} \kappa m_L^2. \tag{94}$$

If, instead of working with tadpole contributions, we would have shifted the field $\Phi \rightarrow \Phi + V$ we would have induced an explicit linear term $VM_H^2\Phi$ and, due to the operators $\phi^2\Phi$ and $\phi^2\Phi^2$, we would modify the mass term for the light scalar, m_L and the κ coupling:

$$\tilde{m}_L^2 = m_L^2 - \kappa V + \frac{1}{2} \kappa V^2 \quad , \quad \tilde{\kappa} = \kappa - \lambda_2 V.$$
(95)

By setting the tree-level contribution of *V* such that it cancels loop corrections order by order, we could eliminate tadpoles from the theory completely. Instead, here, we include their finite part in the calculation, and we absorb the pole in the renormalization of m_L^2 and κ . We should therefore modify the beta functions we read from RGEResult.dat, to account for the tadpole pole, by

$$\delta\beta(M_L^2) = \kappa\beta(V) = \frac{\kappa}{M_H^2}\beta(\alpha_V) = -\frac{\kappa^2}{16\pi^2}\frac{m_L^2}{M_H^2},$$
(96)

$$\delta\beta(\kappa) = \lambda_2\beta(V) = \frac{\lambda_2}{M_H^2}\beta(\alpha_V) = -\frac{\kappa\lambda_2}{16\pi^2}\frac{m_L^2}{M_H^2}.$$
(97)

Reading the results of RGEResult.dat and adding these contributions we get

$$\beta(m_L^2) = \frac{\lambda_2 M_H^2}{16\pi^2} + \frac{\kappa^2}{8\pi^2} + \frac{\lambda_0 m_L^2}{16\pi^2} - \frac{\kappa^2}{16\pi^2} \frac{m_L^2}{M_H^2},\tag{98}$$

$$\beta(m_H^2) = \frac{\kappa^2}{16\pi^2} + \frac{\lambda_2 m_L^2}{16\pi^2},$$
(99)

$$\beta(\lambda_0) = \frac{3\lambda_0^2}{16\pi^2} + \frac{3\lambda_2^2}{16\pi^2},\tag{100}$$

$$\beta(\kappa) = \frac{\lambda_0 \kappa}{16\pi^2} + \frac{\lambda_2 \kappa}{4\pi^2} - \frac{\kappa \lambda_2}{16\pi^2} \frac{m_L^2}{M_H^2},\tag{101}$$

$$\beta(\lambda_2) = \frac{\lambda_2^2}{4\pi^2} + \frac{\lambda_0 \lambda_2}{16\pi^2}.$$
(102)

We can perform the same procedure, but now with the EFT model. We create the model via the following rge_one_scalar_uv.fr file

```
1 M$ModelName = "rge_one_scalar_uv";
2 (* **** Particle classes **** *)
3 M$ClassesDescription = {
4 S[2] == {ClassName -> phi,SelfConjugate -> True,Mass -> mL,
5 FullName -> "light"}
6 };
7 (* ***** Parameters ***** *)
8 M$Parameters = {
9 mL == {ParameterType -> Internal, ComplexParameter -> False},
10 a4 == {ParameterType -> Internal, ComplexParameter -> False},
11 a6 == {ParameterType -> Internal, ComplexParameter -> False}
2 };
```

```
13 (* ***** Lagrangian ***** *)
14 Ltot := Block[{mu,mu2},
15  1/2 * del[phi,mu] * del[phi,mu] - 1/2 * mL^2 * phi^2
16  -a4/24 phi^4 - a6 * phi^6/720
17 ];
18
```

which contains the physical operators of the EFT model. Note that we have changed the names of the couplings to something other than alphaXXX⁷. We also create rge_one_scalar_eft.fr and rge_one_scalar_eft.red that are identical with one_scalar.fr and one_scalar.red defined above. Once again, we can create the two models and run compute_rge_model_to_eft. The result is

$$\beta(\alpha_2) = \frac{\alpha_2 \alpha_4}{16\pi^2},\tag{103}$$

$$\beta(\alpha_4) = \frac{3\alpha_4^2}{16\pi^2} + \frac{\alpha_2\alpha_6}{16\pi^2},\tag{104}$$

$$\beta(\alpha_6) = \frac{15\alpha_4 \alpha_6}{16\pi^2}.$$
 (105)

We can now check whether the matching conditions and the RGE equations are consistent with each other: if we match at $\mu = M_H$ and we evolve all couplings using the RGEs of the UV and the EFT model, to a lower scale Q, we should find the same expressions as when we match directly at $\mu = Q$. Let's see how this works in the case of the mass coefficient, α_2 . The matching condition for α_2 , at scale $\mu = M_H$, gives, see Eq. (91),

$$\alpha_2(M_H) = m_L^2(M_H) - \frac{1}{16\pi^2} \left(\frac{11\kappa^2 m_L^4}{3M_H^4} + \frac{3\kappa^2 m_L^2}{2M_H^2} + \frac{1}{2}\lambda_2 M_H^2 + \kappa^2 \right).$$
(106)

We can use Eq. (103) to evolve α_2 from M_H to a lower scale Q. In the leading-log approximation it reads,

$$\alpha_2(Q) = \alpha_2(M_H) + \frac{1}{2}L_Q\beta(\alpha_2) = \alpha_2(M_H) + \frac{1}{2}L_Q\frac{\alpha_2\alpha_4}{16\pi^2},$$
(107)

where $L_Q \equiv \log(\frac{Q^2}{M_H^2})$. Replacing the tree-level matched values of α_2 and α_4 we get

$$\alpha_2(Q) = m_L^2(M_H) - \frac{1}{16\pi^2} \left(\frac{11\kappa^2 m_L^4}{3M_H^4} + \frac{3\kappa^2 m_L^2}{2M_H^2} + \frac{1}{2}\lambda_2 M_H^2 + \kappa^2 \right)$$
(108)

$$-\frac{L_Q}{16\pi^2} \left(\frac{2\kappa^2 m_L^4}{M_H^4} + \frac{3\kappa^2 m_L^2}{2M_H^2} - \frac{\lambda_0 m_L^2}{2} \right).$$
(109)

If the matching condition for α_2 had no tree-level contribution, this would be the full expression for $\alpha_2(Q)$. In our case, however, there is a tree-level contribution, $m_L^2(M_H)$. We need to use the RGE equation of the UV model, for m_L , Eq. (98) to evolve it to the scale Q. We then get

$$\alpha_2(Q) = m_L^2(Q) - \frac{1}{16\pi^2} \left(\frac{11\kappa^2 m_L^4}{3M_H^4} + \frac{3\kappa^2 m_L^2}{2M_H^2} + \frac{1}{2}\lambda_2 M_H^2 + \kappa^2 \right)$$
(110)

$$-\frac{L_Q}{16\pi^2} \left(\frac{2\kappa^2 m_L^4}{M_H^4} + \frac{3\kappa^2 m_L^2}{2M_H^2} + \frac{\lambda_2 M_H^2}{2} + 2\kappa^2 \right).$$
(111)

Had we match directly at the scale $\mu = Q$ using Eq. (91), we would have found exactly the same result.

⁷If alphaXXX names are used then matchmakereft automatically changes them to WCXXX.

Similarly for α_4 , matching at $\mu = M_H$ and evolving to $\mu = Q$, with the help of the RGEs for both the UV and the EFT couplings, gives

$$\begin{aligned} \alpha_4(Q) &= \lambda_0 - \frac{4\kappa^2 m_L^2}{M_H^4} - \frac{3\kappa^2}{M_H^2} + \frac{1}{16\pi^2} \left(\frac{332\kappa^4 m_L^2}{3M_H^6} + \frac{149\lambda_2 \kappa^2 m_L^2}{3M_H^4} - \frac{80\lambda_0 \kappa^2 m_L^2}{3M_H^4} \right) \\ &- \frac{\lambda_2^2 m_L^2}{3M_H^2} + \frac{25\kappa^4}{M_H^4} + \frac{20\lambda_2 \kappa^2}{M_H^2} - \frac{7\lambda_0 \kappa^2}{M_H^2} \right) + \frac{1}{16\pi^2} L_Q \left(\frac{60\kappa^4 m_L^2}{M_H^6} + \frac{34\lambda_2 \kappa^2 m_L^2}{M_H^4} \right) \\ &- \frac{16\lambda_0 \kappa^2 m_L^2}{M_H^4} + \frac{16\kappa^4}{M_H^4} + \frac{14\lambda_2 \kappa^2}{M_H^2} - \frac{6\lambda_0 \kappa^2}{M_H^2} - \frac{3\lambda_2^2}{2} \right), \end{aligned}$$
(112)

in agreement with what we would get by matching directly at $\mu = Q$, see Eq. (93).

D **SMEFT Green basis**

Here we present a Green basis of the dimension 6 SMEFT including two and four-fermion evanescent operators. As far as possible, we will follow the notation and the conventions used in the model file included with matchmakereft. We present first the renormalizable SM Lagrangian, which reads

$$\mathcal{L}_{SM} = -\frac{1}{4} G^{A}_{\mu\nu} G^{A\mu\nu} - \frac{1}{4} W^{I}_{\mu\nu} W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_{\mu}H)^{\dagger} D^{\mu}H - m^{2}H^{\dagger}H - \lambda (H^{\dagger}H)^{2} + i [\bar{\ell} \not\!\!\!D \ell + \bar{e} \not\!\!\!D e + \bar{q} \not\!\!D q + \bar{u} \not\!\!D u + \bar{d} \not\!\!D d] - [\bar{\ell} Y_{e} e H + \bar{q} Y_{u} u \ddot{H} + \bar{q} Y_{d} dH + \text{h.c.}].$$
(113)

Hereinafter, we omit flavor and gauge indices whenever possible. Otherwise, we use i, j, k, l, \dots as flavor indices and A, B, C, \ldots and I, J, K, \ldots for the adjoint representation of SU(3) and SU(2), respectively. We will use on the other hand a, b, c, \ldots and r, s, t, \ldots for the fundamental representation of the color and the electroweak group, respectively. In the model file coming along with matchmakereft we use a slightly different notation for the couplings in the renormalizable Lagrangian. For instance,

$$Y_u \rightarrow \text{alphaOlambdau}, \ \lambda \rightarrow \text{alphaOlambda}, \ m^2 \rightarrow \text{alphaOmuH2}, \ \dots$$
 (114)

We refer the reader to the model file SMEFT_Green_Bpreserving.fr for more details.

The doublet \tilde{H} is defined by $\tilde{H} = i\sigma^2 H^*$ as usual and we assume the following definition for the covariant derivative

$$D_{\mu}q = (\partial_{\mu} - ig_3 T^A G^A_{\mu} - ig_2 \frac{\sigma^I}{2} W^I_{\mu} - ig_1 Y B_{\mu})q, \qquad (115)$$

where $T^A = \lambda^A/2$ with λ^A and σ^I the Gell-Mann and Pauli matrices, respectively. Correspondingly, the field strength tensors are

$$G^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + g_3 f^{ABC} G^B_\mu G^C_\nu, \qquad (116)$$

$$W^{I}_{\mu\nu} = \partial_{\mu}W^{I}_{\nu} - \partial_{\nu}W^{I}_{\mu} + g_{2}\epsilon^{IJK}W^{J}_{\mu}W^{K}_{\nu}, \qquad (117)$$

$$B_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}, \qquad (118)$$

and their covariant derivatives

$$(D_{\rho}G_{\mu\nu})^{A} = \partial_{\rho}G_{\mu\nu}^{A} + g_{3}f^{ABC}G_{\rho}^{B}G_{\mu\nu}^{C}, \qquad (119)$$

$$(D_{\rho}G_{\mu\nu})^{A} = \partial_{\rho}G_{\mu\nu}^{A} + g_{3}f^{ABC}G_{\rho}^{B}G_{\mu\nu}^{C},$$

$$(D_{\rho}W_{\mu\nu})^{I} = \partial_{\rho}W_{\mu\nu}^{I} + g_{2}\epsilon^{IJK}W_{\rho}^{J}W_{\mu\nu}^{K},$$

$$(120)$$

$$(D_{\rho}B_{\mu\nu}) = \partial_{\rho}B_{\mu\nu}, \qquad (121)$$

with f^{ABC} and ϵ^{IJK} the SU(3) and SU(2) structure constants, respectively.

The *B*-number preserving dimension 6 Green basis is presented in Tables 1-9. We follow the convention of denoting operators in the Warsaw basis [68] by \mathcal{O} , whereas redundant and evanescent operators are denoted by \mathcal{R} and \mathcal{E} , respectively. We should stress that in our case, redundant and evanescent operators do not vanish when using equations of motion or going to D = 4 dimensions, being reduced to the operators in the physical basis in this case. We follow the notation of [64] regarding redundant operators and include evanescent operators with two and four fermions as well as possibly featuring charge conjugation. We use the shorthand notation $\gamma^{\mu_1...\mu_n} \equiv \gamma^{\mu_1}...\gamma^{\mu_n}$ with no (anti)symmetrization. In the case of four fermion operators with charge conjugation we do not include for the moment structures with three gammas, which we plan to include in the near future.

We denote by $iCPV = \epsilon_{0123} \in \{1, -1\}$ with $e^{\alpha\beta\mu\nu}$ the Levi-Civita tensor, in such a way that for D = 4 e.g.

$$\sigma^{\mu\nu}\epsilon_{\mu\nu\rho\sigma} = 2iCPV i\sigma_{\rho\sigma}\gamma_5, \tag{122}$$

with $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^{\mu}, \gamma^{\nu}]$ and

$$\gamma_5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 = i CPV \frac{i}{4!} \epsilon_{\mu\nu\alpha\beta} \gamma^\mu \gamma^\nu \gamma^\alpha \gamma^\beta.$$
(123)

At any rate, dual tensors are defined by

$$\tilde{X}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} X^{\mu\nu}, \quad \text{with} \quad X = G, W, B.$$
(124)

We follow [27] for our definitions of evanescent operators, with the difference that we do not define them to be zero. Considering at most three gamma matrices, they read

$$E_{LR}^{(2)} = P_L \gamma^{\mu\nu} P_L \otimes P_R \gamma_{\mu\nu} P_R = 4(1 + a_{\rm ev} \epsilon) P_L \otimes P_R,, \qquad (125)$$

$$S_{RL}^{(2)} = P_R \gamma^{\mu\nu} P_R \otimes P_L \gamma_{\mu\nu} P_L = 4(1 + a'_{ev} \epsilon) P_R \otimes P_L, \qquad (126)$$

$$E_{LL}^{(3)} = P_R \gamma^{\mu\nu\lambda} P_L \otimes P_R \gamma_{\mu\nu\lambda} P_L = 4(4 - b_{\rm ev}\epsilon) P_R \gamma^{\mu} P_L \otimes P_R \gamma_{\mu} P_L, \qquad (127)$$

$$P_{RR}^{(3)} = P_L \gamma^{\mu\nu\lambda} P_R \otimes P_L \gamma_{\mu\nu\lambda} P_R = 4(4 - b'_{ev} \epsilon) P_L \gamma^{\mu} P_R \otimes P_L \gamma_{\mu} P_R,$$
(128)

$$E_{LR}^{(3)} = P_R \gamma^{\mu\nu\lambda} P_L \otimes P_L \gamma_{\mu\nu\lambda} P_R = 4(1 + c_{\rm ev}\epsilon) P_R \gamma^{\mu} P_L \otimes P_L \gamma_{\mu} P_R,$$
(129)

$$E_{RL}^{(3)} = P_L \gamma^{\mu\nu\lambda} P_R \otimes P_R \gamma_{\mu\nu\lambda} P_L = 4(1 + c'_{ev}\epsilon) P_L \gamma^{\mu} P_R \otimes P_R \gamma_{\mu} P_L,$$
(130)

while

$$P_L \gamma^{\mu\nu} P_L \otimes P_L \gamma_{\mu\nu} P_L = (4 - 2\epsilon) P_L \otimes P_L - P_L \sigma^{\mu\nu} P_L \otimes P_L \sigma_{\mu\nu} P_L, \tag{131}$$

$$P_R \gamma^{\mu\nu} P_R \otimes P_R \gamma_{\mu\nu} P_R = (4 - 2\epsilon) P_R \otimes P_R - P_R \sigma^{\mu\nu} P_R \otimes P_R \sigma_{\mu\nu} P_R, \tag{132}$$

assuming the following basis of the space of Lorentz singlets and pseudo-singlets in D = 4

$$\left\{ \Gamma_{1}^{i} \otimes \Gamma_{2}^{i} \mid i = 1, \dots, 10 \right\} = \left\{ P_{L} \otimes P_{L}, P_{R} \otimes P_{R}, P_{L} \otimes P_{R}, P_{R} \otimes P_{L}, P_{R} \gamma^{\mu} P_{L} \otimes P_{R} \gamma_{\mu} P_{L}, P_{L} \gamma^{\mu} P_{R} \otimes P_{L} \gamma_{\mu} P_{R}, P_{R} \gamma^{\mu} P_{L} \otimes P_{L} \gamma_{\mu} P_{R}, P_{L} \gamma^{\mu} P_{R} \otimes P_{R} \gamma_{\mu} P_{L}, P_{L} \sigma^{\mu\nu} P_{L} \otimes P_{L} \sigma_{\mu\nu} P_{L}, P_{R} \sigma^{\mu\nu} P_{R} \otimes P_{R} \sigma_{\mu\nu} P_{R} \right\}.$$

$$(133)$$

In order to be consistent with flavor, we further assume that $a_{ev} = a'_{ev}$, $c_{ev} = c'_{ev}$. Therefore, at the end of the day the one-loop matching will be dependent on four free parameters: iCPV, aEV, bEV and cEV. However, physical observables computed with the obtained dimension-6 EFT can not depend on such parameters. One final important remark should be made. In the model file .fr of the SMEFT used by matchmakereft to do the matching, the operator of the Warsaw basis $\mathcal{O}_{\ell equ}^{(3)}$ is replaced by $\mathcal{E}_{\ell equ}^{[2]}$ since matchmakereft and FORM use always $\gamma^{\mu\nu}$ instead of its antisymmetric version $\sigma^{\mu\nu}$. After doing the matching, such operator is reduced onto the one present in the Warsaw basis by the relation (we use α , β and γ for the WCs of physical, redundant and evanescent operators, respectively)

$$\left(\alpha_{\ell equ}^{(3)}\right)_{ijkl} = \left(\gamma_{\ell equ}^{[2]}\right)_{ijkl} - \left(1 - \frac{\epsilon}{4}\right) \left(\gamma_{\ell equ}^{[2]}\right)_{ilkj} - \frac{1}{8} \left(\gamma_{\ell uqe}\right)_{ilkj} - \frac{1}{8} \left(\gamma_{ue\ell q}^{c}\right)_{ljik} - \left(1 - \frac{\epsilon}{4}\right) \left(\gamma_{ue\ell q}^{c}\right)_{ljik}.$$

$$(134)$$

| | X ³ | | X^2H^2 | | H^2D^4 |
|---|--|---|---|---|---|
| \mathcal{O}_{3G} | $f^{ABC}G^{A u}_\mu G^{B ho}_ u G^{C\mu}_ ho$ | \mathcal{O}_{HG} | $G^A_{\mu u}G^{A\mu u}(H^\dagger H)$ | \mathcal{R}_{DH} | $(D_{\mu}D^{\mu}H)^{\dagger}(D_{\nu}D^{\nu}H)$ |
| $\mathcal{O}_{\widetilde{3G}}$ | $f^{ABC}\widetilde{G}^{A u}_{\mu}G^{B ho}_{ u}G^{C\mu}_{ ho}$ | $\mathcal{O}_{_{H\widetilde{G}}}$ | $\widetilde{G}^A_{\mu u}G^{A\mu u}(H^\dagger H)$ | | H^4D^2 |
| $\mathcal{O}_{3W} \ \mathcal{O}_{\widetilde{3W}}$ | $\epsilon^{IJK} W^{I u}_{\mu} W^{J ho}_{ u} W^{J ho}_{ ho} W^{K\mu}_{ ho} \ \epsilon^{IJK} \widetilde{W}^{I u}_{\mu} W^{J ho}_{ u} W^{J ho}_{ u} W^{K\mu}_{ ho}$ | $\mathcal{O}_{HW} \ \mathcal{O}_{H\widetilde{W}}$ | $W^{I}_{\mu u}W^{I\mu u}(H^{\dagger}H) \ \widetilde{W}^{I}_{\mu u}W^{I\mu u}(H^{\dagger}H)$ | $\mathcal{O}_{H\square} \ \mathcal{O}_{HD}$ | $(H^\dagger H) \square (H^\dagger H) \ (H^\dagger D^\mu H)^\dagger (H^\dagger D_\mu H)$ |
| 3W | X^2D^2 | \mathcal{O}_{HB} | $B_{\mu\nu}B^{\mu\nu}(H^{\dagger}H)$ | \mathcal{R}'_{HD} | $(H^{\dagger}H)(D_{\mu}H)^{\dagger}(D^{\mu}H)$ |
| \mathcal{R}_{2G} | $-\frac{1}{2}(D_{\mu}G^{A\mu\nu})(D^{\rho}G^{A}_{\rho\nu})$ | $\mathcal{O}_{_{H\widetilde{B}}}$ | $\widetilde{B}_{\mu u}B^{\mu u}(H^{\dagger}H)$ | \mathcal{R}''_{HD} | $(H^{\dagger}H)D_{\mu}(H^{\dagger}\mathrm{i}\overleftrightarrow{D}^{\mu}H)$ |
| \mathcal{R}_{2W} | $-\frac{1}{2}(D_{\mu}W^{I\mu\nu})(D^{\rho}W^{I}_{\rho\nu})$ | \mathcal{O}_{HWB} | $W^I_{\mu u}B^{\mu u}(H^\dagger\sigma^I H)$ | | H^6 |
| \mathcal{R}_{2B} | $-rac{1}{2}(\partial_{\mu}B^{\mu\nu})(\partial^{\rho}B_{\rho\nu})$ | $\mathcal{O}_{H\widetilde{W}B}$ | $\widetilde{W}^{I}_{\mu u}B^{\mu u}(H^{\dagger}\sigma^{I}H)$ | \mathcal{O}_{H} | $(H^{\dagger}H)^3$ |
| | | | $H^2 X D^2$ | | |
| | | \mathcal{R}_{WDH} | $D_{\nu}W^{I\mu\nu}(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}^{I}H)$ | | |
| | | \mathcal{R}_{BDH} | $\partial_{\nu}B^{\mu\nu}(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ | | |

Table 1: Physical and redundant bosonic operators.

| | $\psi^2 D^3$ | | $\psi^2 X D$ | | $\psi^2 D H^2$ |
|-------------------------|--|------------------------------------|--|---|---|
| \mathcal{R}_{qD} | $\frac{\mathrm{i}}{2}\overline{q}\left\{ D_{\mu}D^{\mu},D^{\mu} ight\} q$ | \mathcal{R}_{Gq} | $(\overline{q}T^A\gamma^\mu q)D^ u G^A_{\mu u}$ | $\mathcal{O}_{Hq}^{(1)}$ | $(\overline{q}\gamma^{\mu}q)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ |
| \mathcal{R}_{uD}^{qD} | $\frac{\mathrm{i}}{2}\overline{u}\left\{ D_{\mu}D^{\mu},\not\!\!\!D\right\} u$ | \mathcal{R}'_{Gq} | $\frac{1}{2}(\overline{q}T^A\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}q)G^A_{\mu\nu}$ | $\mathcal{R}_{Hq}^{\prime(1)}$ | $(\overline{q} \operatorname{i} \overleftrightarrow{p} q)(H^{\dagger}H)$ |
| \mathcal{R}_{dD} | $\frac{\overline{i}}{2}\overline{d}\left\{D_{\mu}D^{\mu},\not\!\!\!D\right\}d$ | $\mathcal{R}'_{\widetilde{G}q}$ | $\frac{1}{2}(\overline{q}T^{A}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}q)\widetilde{G}^{A}_{\mu\nu}$ | $\mathcal{R}_{Hq}^{\prime\prime(1)}$ | $(\overline{q}\gamma^{\mu}q)\partial_{\mu}(H^{\dagger}H)$ |
| $\mathcal{R}_{\ell D}$ | $rac{\mathrm{i}}{2}\overline{\ell}\left\{ D_{\mu}D^{\mu},D\!\!\!/\right\} \ell$ | \mathcal{R}_{Wq} | $(\overline{q}\sigma^{I}\gamma^{\mu}q)D^{\nu}W^{I}_{\mu u}$ | $\mathcal{O}_{Hq}^{(3)}$ | $(\overline{q}\sigma^{I}\gamma^{\mu}q)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}^{I}H)$ |
| \mathcal{R}_{eD} | $rac{\mathrm{i}}{2}\overline{e}\left\{ D_{\mu}D^{\mu},D\!\!\!/\right\} e$ | \mathcal{R}'_{Wq} | $\frac{1}{2}(\overline{q}\sigma^{I}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}q)W^{I}_{\mu\nu}$ | $\mathcal{R}_{Hq}^{\prime(3)}$ | $(\overline{q} i \overleftrightarrow{p}^{I} q)(H^{\dagger} \sigma^{I} H)$ |
| | $^{2}HD^{2}+$ h.c. | $\mathcal{R}'_{\widetilde{W}q}$ | $\frac{1}{2}(\overline{q}\sigma^{I}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}q)\widetilde{W}_{\mu\nu}^{I}$ | $\mathcal{R}_{Hq}^{\prime\prime(3)}$ | $(\overline{q}\sigma^{I}\gamma^{\mu}q)D_{\mu}(H^{\dagger}\sigma^{I}H)$ |
| \mathcal{R}_{uHD1} | $(\overline{q}u)D_{\mu}D^{\mu}\widetilde{H}$ | \mathcal{R}_{Bq} | $(\overline{q}\gamma^{\mu}q)\partial^{\nu}B_{\mu u}$ | \mathcal{O}_{Hu} | $(\overline{u}\gamma^{\mu}u)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ |
| \mathcal{R}_{uHD2} | $(\overline{q}\mathrm{i}\sigma_{\mu\nu}D^{\mu}u)D^{\nu}\widetilde{H}$ | \mathcal{R}_{Bq}' | $\frac{1}{2}(\overline{q}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}q)B_{\mu\nu}$ | \mathcal{R}'_{Hu} | $(\overline{u} \mathrm{i} \overleftrightarrow{D} u)(H^{\dagger}H)$ |
| \mathcal{R}_{uHD3} | $(\overline{q}D_{\mu}D^{\mu}u)\widetilde{H}$ | $\mathcal{R}'_{\widetilde{B}q}$ | $\frac{1}{2}(\overline{q}\gamma^{\mu}i\overleftarrow{D}^{\nu}q)\widetilde{B}_{\mu\nu}$ | \mathcal{R}''_{Hu} | $(\overline{u}\gamma^{\mu}u)\partial_{\mu}(H^{\dagger}H)$ |
| \mathcal{R}_{uHD4} | $(\overline{q}D_{\mu}u)D^{\mu}\widetilde{H}$ | \mathcal{R}_{Gu}^{T} | $(\overline{u}T^A\gamma^\mu u)D^\nu G^A_{\mu u}$ | \mathcal{O}_{Hd} | $(\overline{d}\gamma^{\mu}d)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ |
| \mathcal{R}_{dHD1} | $(\overline{q}d)D_{\mu}D^{\mu}H$ | \mathcal{R}_{Gu}' | $\frac{1}{2}(\overline{u}T^A\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}u)G^A_{\mu\nu}$ | \mathcal{R}'_{Hd} | $(\overline{d} i \overleftrightarrow{p} d)(H^{\dagger}H)$ |
| \mathcal{R}_{dHD2} | $(\overline{q}\mathrm{i}\sigma_{\mu\nu}D^{\mu}d)D^{\nu}H$ | $\mathcal{R}'_{\widetilde{G}u}$ | $\frac{1}{2}(\overline{u}T^{A}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}u)\widetilde{G}^{A}_{\mu\nu}$ | $\mathcal{R}_{Hd}^{\prime\prime}$ | $(\overline{d}\gamma^{\mu}d)\partial_{\mu}(H^{\dagger}H)$ |
| \mathcal{R}_{dHD3} | $(\overline{q}D_{\mu}D^{\mu}d)H$ | \mathcal{R}_{Bu} | $(\overline{u}\gamma^{\mu}u)\partial^{\nu}B_{\mu\nu}$ | $\mathcal{O}_{Hud} \ \mathcal{O}_{H\ell}^{(1)}$ | $(\overline{u}\gamma^{\mu}d)(\widetilde{H}^{\dagger}\mathrm{i}D_{\mu}H)$ |
| \mathcal{R}_{dHD4} | $(\overline{q}D_{\mu}d)D^{\mu}H$ | \mathcal{R}'_{Bu} | $\frac{1}{2}(\overline{u}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}u)B_{\mu\nu}$ | $\mathcal{O}_{H\ell}^{(1)}$ | $(\overline{\ell}\gamma^{\mu}\ell)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ |
| \mathcal{R}_{eHD1} | $(\overline{\ell} e)D_{\mu}D^{\mu}H$ | $\mathcal{R}'_{\widetilde{B}u}$ | $\frac{1}{2}(\overline{u}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}u)\widetilde{B}_{\mu\nu}$ | $\mathcal{R}_{H\ell}^{\prime(1)}$ | $(\overline{\ell}i\overleftrightarrow{D}\ell)(H^{\dagger}H)$ |
| \mathcal{R}_{eHD2} | $(\bar{\ell} \mathrm{i}\sigma_{\mu\nu}D^{\mu}e)D^{\nu}H$ | \mathcal{R}_{Gd} | $(\overline{d}T^A\gamma^\mu d)D^\nu G^A_{\mu\nu}$ | $\mathcal{R}_{H\ell}^{\prime\prime(1)}$ | $(\bar{\ell}\gamma^{\mu}\ell)\partial_{\mu}(H^{\dagger}H)$ |
| \mathcal{R}_{eHD3} | $(\overline{\ell} D_\mu D^\mu e) H$ | \mathcal{R}_{Gd}' | $\frac{1}{2}(\overline{d}T^A\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}d)G^A_{\mu\nu}$ | $\mathcal{O}_{H\ell}^{(3)}$ | $(\overline{\ell}\sigma^{I}\gamma^{\mu}\ell)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}^{I}H)$ |
| \mathcal{R}_{eHD4} | $(\overline{\ell}D_{\mu}e)D^{\mu}H$ | $\mathcal{R}'_{\widetilde{G}d}$ | $\frac{1}{2}(\overline{d}T^A\gamma^\mu \mathrm{i}\overleftrightarrow{D}^\nu d)\widetilde{G}^A_{\mu\nu}$ | ${\cal R}_{H\ell}^{\prime(3)}$ | $(\overline{\ell}\mathrm{i}\overleftrightarrow{D}^{I}\ell)(H^{\dagger}\sigma^{I}H)$ |
| | $b^2 XH + h.c.$ | \mathcal{R}_{Bd} | $(\overline{d}\gamma^{\mu}d)\partial^{\nu}B_{\mu\nu}$ | ${\cal R}^{\prime\prime(3)}_{H\ell}$ | $(\bar{\ell}\sigma^{I}\gamma^{\mu}\ell)D_{\mu}(H^{\dagger}\sigma^{I}H)$ |
| \mathcal{O}_{uG} | $(\overline{q}T^A\sigma^{\mu\nu}u)\widetilde{H}G^A_{\mu\nu}$ | \mathcal{R}_{Bd}' | $\frac{1}{2}(\overline{d}\gamma^{\mu}i\overleftrightarrow{D}^{\nu}d)B_{\mu\nu}$ | \mathcal{O}_{He} | $(\overline{e}\gamma^{\mu}e)(H^{\dagger}\mathrm{i}\overleftrightarrow{D}_{\mu}H)$ |
| \mathcal{O}_{uW} | $(\overline{q}\sigma^{\mu\nu}u)\sigma^{I}\widetilde{H}W^{I}_{\mu\nu}$ | $\mathcal{R}'_{\widetilde{B}d}$ | $\frac{1}{2}(\overline{d}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}d)\widetilde{B}_{\mu\nu}$ | \mathcal{R}'_{He} | $(\overline{e} i \overleftrightarrow{p} e)(H^{\dagger}H)$ |
| \mathcal{O}_{uB} | $(\overline{q}\sigma^{\mu\nu}u)\widetilde{H}B_{\mu\nu}$ | $\mathcal{R}_{W\ell}^{-1}$ | $(\overline{\ell}\sigma^{I}\gamma^{\mu}\ell)D^{\nu}W^{I}_{\mu u}$ | \mathcal{R}_{He}'' | $(\overline{e}\gamma^{\mu}e)\partial_{\mu}(H^{\dagger}H)$ |
| \mathcal{O}_{dG} | $(\overline{q}T^A\sigma^{\mu\nu}d)HG^A_{\mu\nu}$ | | $\frac{1}{2}(\overline{\ell}\sigma^{I}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}\ell)W^{I}_{\mu\nu}$ | | $\psi^2 H^3 + \text{h.c.}$ |
| \mathcal{O}_{dW} | $(\overline{q}\sigma^{\mu\nu}d)\sigma^{I}HW^{I}_{\mu\nu}$ | $\mathcal{R}'_{\widetilde{W}\ell}$ | $\frac{1}{2}(\overline{\ell}\sigma^{I}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}\ell)\widetilde{W}^{I}_{\mu\nu}$ | \mathcal{O}_{uH} | $(H^{\dagger}H)\overline{q}\widetilde{H}u$ |
| \mathcal{O}_{dB} | $(\overline{q}\sigma^{\mu\nu}d)HB_{\mu\nu}$ | $\mathcal{R}_{B\ell}$ | $(\overline{\ell}\gamma^{\mu}\ell)\partial^{\nu}B_{\mu\nu}$ | \mathcal{O}_{dH} | $(H^{\dagger}H)\overline{q}Hd$ |
| \mathcal{O}_{eW} | $(\overline{\ell}\sigma^{\mu u}e)\sigma^{I}HW^{I}_{\mu u}$ | $\mathcal{R}'_{B\ell}$ | $\frac{1}{2}(\ell\gamma^{\mu}iD^{\nu}\ell)B_{\mu\nu}$ | \mathcal{O}_{eH} | $(H^{\dagger}H)\overline{\ell}He$ |
| \mathcal{O}_{eB} | $(\bar{\ell}\sigma^{\mu\nu}e)HB_{\mu\nu}$ | $\mathcal{R}'_{\widetilde{B}\ell}$ | $\frac{1}{2}(\overline{\ell}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}\ell)\widetilde{B}_{\mu\nu}$ | | |
| | | \mathcal{R}_{Be} | $(\overline{e}\gamma^{\mu}e)\partial^{\nu}B_{\mu\nu}$ | | |
| | | \mathcal{R}_{Be}' | $\frac{1}{2}(\overline{e}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}e)B_{\mu\nu}$ | | |
| | | $\mathcal{R}'_{\widetilde{B}e}$ | $\frac{1}{2}(\overline{e}\gamma^{\mu}\mathrm{i}\overleftrightarrow{D}^{\nu}e)\widetilde{B}_{\mu\nu}$ | | |

 Table 2: Physical and redundant operators with two fermions.

| | Four-quark | F | our-lepton | | Semileptonic |
|--|--|--------------------------|--|--|--|
| $\mathcal{O}_{qq}^{(1)}$ | $(\overline{q}\gamma^{\mu}q)(\overline{q}\gamma_{\mu}q)$ | $\mathcal{O}_{\ell\ell}$ | $(\bar\ell\gamma^\mu\ell)(\bar\ell\gamma_\mu\ell)$ | $\mathcal{O}_{\ell q}^{(1)}$ | $(\overline{\ell}\gamma^\mu\ell)(\overline{q}\gamma_\mu q)$ |
| $\mathcal{O}_{qq}^{(3)}$ | $(\overline{q}\gamma^\mu\sigma^I q)(\overline{q}\gamma_\mu\sigma^I q)$ | \mathcal{O}_{ee} | $(\overline{e}\gamma^{\mu}e)(\overline{e}\gamma_{\mu}e)$ | $\mathcal{O}_{\ell q}^{(1)}\ \mathcal{O}_{\ell q}^{(3)}$ | $(\overline{\ell}\gamma^\mu\sigma^I\ell)(\overline{q}\gamma_\mu\sigma^Iq)$ |
| \mathcal{O}_{uu} | $(\overline{u}\gamma^{\mu}u)(\overline{u}\gamma_{\mu}u)$ | $\mathcal{O}_{\ell e}$ | $(\overline{\ell}\gamma^{\mu}\ell)(\overline{e}\gamma_{\mu}e)$ | \mathcal{O}_{eu} | $(\overline{e}\gamma^{\mu}e)(\overline{u}\gamma_{\mu}u)$ |
| \mathcal{O}_{dd} | $(\overline{d}\gamma^{\mu}d)(\overline{d}\gamma_{\mu}d)$ | | | \mathcal{O}_{ed} | $(\overline{e}\gamma^{\mu}e)(\overline{d}\gamma_{\mu}d)$ |
| $\mathcal{O}_{dd} \ \mathcal{O}_{ud}^{(1)}$ | $(\overline{u}\gamma^{\mu}u)(\overline{d}\gamma_{\mu}d)$ | | | \mathcal{O}_{qe} | $(\overline{q}\gamma^{\mu}q)(\overline{e}\gamma_{\mu}e)$ |
| $\mathcal{O}_{ud}^{(8)}$ $\mathcal{O}^{(1)}$ | $(\overline{u}\gamma^{\mu}T^{A}u)(\overline{d}\gamma_{\mu}T^{A}d)$ | | | $\mathcal{O}_{\ell u}$ | $(\overline{\ell}\gamma^{\mu}\ell)(\overline{u}\gamma_{\mu}u)$ |
| $\mathcal{O}_{au}^{(1)}$ | $(\overline{q}\gamma^{\mu}q)(\overline{u}\gamma_{\mu}u)$ | | | $\mathcal{O}_{\ell d}^{\circ}$ | $(\overline{\ell}\gamma^{\mu}\ell)(\overline{d}\gamma^{\mu}\mu)$ |
| $\mathcal{O}_{qu}^{(8)}$ | $(\overline{q}\gamma^{\mu}T^{A}q)(\overline{u}\gamma_{\mu}T^{A}u)$ | | | $\mathcal{O}_{\ell e d a}$ | $(\overline{\ell}e)(\overline{d}q)$ |
| $\mathcal{O}_{qu}^{(1)} \mathcal{O}_{ad}^{(1)}$ | $(\overline{q}\gamma^{\mu}q)(\overline{d}\gamma_{\mu}d)$ | | | $\mathcal{O}_{\ell equ}^{(1)}$ | $(\overline{\ell}_r e)\epsilon_{rs}(\overline{q}_s u)$ |
| $\mathcal{O}_{qd}^{(8)}$ | $(\overline{q}\gamma^{\mu}T^{A}q)(\overline{d}\gamma_{\mu}T^{A}d)$ | | | $\mathcal{O}_{\ell equ}^{(3)}$ | $(\bar{\ell}_r \sigma^{\mu\nu} e) \epsilon_{rs} (\bar{q}_s \sigma_{\mu\nu} u)$ |
| $\mathcal{O}_{qd}^{(1)}$ | $(\overline{q}_r u)\epsilon_{rs}(\overline{q}_s d)$ | | | 1 | |
| $\mathcal{O}_{quqd}^{(2)}$ $\mathcal{O}_{quqd}^{(8)}$ | $(\overline{q}_r T^A u) \epsilon_{rs} (\overline{q}_s T^A d)$ | | | | |

 Table 3: Baryon and lepton number conserving operators with four fermions.

| | $\Psi^2 XH$ + h.c. | | Ψ^2 | XD | |
|--------------------|---|---------------------------------|---|------------------------------------|---|
| \mathcal{E}_{uG} | $ar{q}T^A\sigma^{\mu u}u\widetilde{H}\widetilde{G}^A_{\mu u}$ | \mathcal{E}_{Gq} | $\bar{q}T^{A}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})qD_{\rho}\widetilde{G}^{A}_{\mu\nu}$ | \mathcal{E}_{Gd} | $\bar{d}T^{A}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})dD_{\rho}\widetilde{G}^{A}_{\mu\nu}$ |
| \mathcal{E}_{uW} | $ar{q}\sigma^{I}\sigma^{\mu u}u\widetilde{H}\widetilde{W}^{I}_{\mu u}$ | \mathcal{E}_{Gq}' | $i\bar{q}(T^A\sigma^{\mu\nu}\not\!$ | \mathcal{E}_{Gd}' | $i\bar{d}(T^A\sigma^{\mu\nu}D\!\!\!/ - \overleftrightarrow{D}\sigma^{\mu\nu}T^A)dG^A_{\mu\nu}$ |
| \mathcal{E}_{uB} | $ar{q}\sigma^{\mu u}u\widetilde{H}\widetilde{B}_{\mu u}$ | $\mathcal{E}_{\widetilde{G}q}'$ | $iar{q}(T^A\sigma^{\mu u}D\!\!\!/-\overleftarrow{D}\sigma^{\mu u}T^A)q\widetilde{G}^A_{\mu u}$ | $\mathcal{E}_{\widetilde{G}d}'$ | $i\bar{d}(T^A\sigma^{\mu\nu}\not\!$ |
| \mathcal{E}_{dG} | $ar{q}T^A\sigma^{\mu u}dH\widetilde{G}^A_{\mu u}$ | \mathcal{E}_{Wq} | $\bar{q}\sigma^{I}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})qD_{\rho}\widetilde{W}^{I}_{\mu\nu}$ | \mathcal{E}_{Bd} | $\bar{d}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})d\partial_{\rho}\widetilde{B}_{\mu\nu}$ |
| \mathcal{E}_{dW} | $ar{q}\sigma^{I}\sigma^{\mu u}dH\widetilde{W}^{I}_{\mu u}$ | \mathcal{E}'_{Wq} | $i\bar{q}(\sigma^{I}\sigma^{\mu\nu}\not{D}-\not{D}\sigma^{\mu\nu}\sigma^{I})qW^{I}_{\mu\nu}$ | \mathcal{E}_{Bd}' | $i\bar{d}(\sigma^{\mu\nu}\not\!$ |
| \mathcal{E}_{dB} | $ar{q}\sigma^{\mu u}dH\widetilde{B}_{\mu u}$ | $\mathcal{E}'_{\widetilde{W}q}$ | $i \bar{q} (\sigma^I \sigma^{\mu \nu} D - D \sigma^{\mu \nu} \sigma^I) q \widetilde{W}^I_{\mu \nu}$ | $\mathcal{E}'_{\widetilde{B}d}$ | $i\bar{d}(\sigma^{\mu\nu}\not\!$ |
| \mathcal{E}_{eW} | $ar{\ell}\sigma^{I}\sigma^{\mu u}$ e $H\widetilde{W}^{I}_{\mu u}$ | \mathcal{E}_{Bq} | $\bar{q}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})q\partial_{\rho}\widetilde{B}_{\mu\nu}$ | $\mathcal{E}_{W\ell}$ | $\bar{\ell}\sigma^{I}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})\ell D_{\rho}\widetilde{W}^{I}_{\mu\nu}$ |
| \mathcal{E}_{eB} | $ar{\ell}\sigma^{\mu u}$ e $H\widetilde{B}_{\mu u}$ | \mathcal{E}_{Bq}' | $i\bar{q}(\sigma^{\mu\nu}\not\!$ | $\mathcal{E}'_{W\ell}$ | $i\bar{\ell}(\sigma^{I}\sigma^{\mu\nu}\not{D}-\overleftarrow{\not{D}}\sigma^{\mu\nu}\sigma^{I})\ell W^{I}_{\mu\nu}$ |
| | $\psi^2 H D^2 + \text{h.c.}$ | $\mathcal{E}'_{\widetilde{B}q}$ | $\mathrm{i}ar{q}(\sigma^{\mu u}\not\!$ | $\mathcal{E}'_{\widetilde{W}\ell}$ | $\mathrm{i}\bar{\ell}(\sigma^{I}\sigma^{\mu\nu}\mathcal{D}-\overleftarrow{\mathcal{D}}\sigma^{\mu\nu}\sigma^{I})\ell\widetilde{W}_{\mu\nu}^{I}$ |
| \mathcal{E}_{uH} | $ar{q}\sigma^{\mu u}D^{ ho}uD^{\sigma}\widetilde{H}\epsilon_{\mu u ho\sigma}$ | \mathcal{E}_{Gu} | $\bar{u}T^{A}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})uD_{\rho}\widetilde{G}^{A}_{\mu\nu}$ | $\mathcal{E}_{B\ell}$ | $\bar{\ell}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})\ell\partial_{\rho}\widetilde{B}_{\mu\nu}$ |
| \mathcal{E}_{dH} | $ar{q}\sigma^{\mu u}D^{ ho}dD^{\sigma}H\epsilon_{\mu u ho\sigma}$ | \mathcal{E}_{Gu}' | $i\bar{u}(T^A\sigma^{\mu\nu}\not{D}-\overleftarrow{D}\sigma^{\mu\nu}T^A)uG^A_{\mu\nu}$ | $\mathcal{E}_{B\ell}'$ | $i\bar{\ell}(\sigma^{\mu\nu}\not\!$ |
| \mathcal{E}_{eH} | $\bar{\ell}\sigma^{\mu u}D^{ ho}eD^{\sigma}H\epsilon_{\mu u ho\sigma}$ | $\mathcal{E}'_{\widetilde{G}u}$ | $i\bar{u}(T^A\sigma^{\mu\nu}D\!\!\!/-\overleftrightarrow{D}\sigma^{\mu\nu}T^A)u\widetilde{G}^A_{\mu\nu}$ | $\mathcal{E}'_{\widetilde{B}\ell}$ | $\mathrm{i}\bar{\ell}(\sigma^{\mu\nu}\not\!\!\!D - \overleftarrow{\not\!\!\!\!D}\sigma^{\mu\nu})\ell\widetilde{B}_{\mu\nu}$ |
| | | \mathcal{E}_{Bu} | $\bar{u}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})u\partial_{\rho}\widetilde{B}_{\mu\nu}$ | \mathcal{E}_{Be}^{-1} | $\bar{e}(\sigma^{\mu\nu}\gamma^{\rho}+\gamma^{\rho}\sigma^{\mu\nu})e\partial_{\rho}\widetilde{B}_{\mu\nu}$ |
| | | \mathcal{E}_{Bu}' | $i\bar{u}(\sigma^{\mu\nu}D - D\sigma^{\mu\nu})uB_{\mu\nu}$ | \mathcal{E}_{Be}' | $i\bar{e}(\sigma^{\mu\nu}\not{D}-\not{D}\sigma^{\mu\nu})eB_{\mu\nu}$ |
| l | | $\mathcal{E}'_{\widetilde{B}u}$ | $i\bar{u}(\sigma^{\mu\nu}\not\!$ | $\mathcal{E}'_{\widetilde{B}e}$ | $i\bar{e}(\sigma^{\mu\nu}\not\!$ |

Table 4: Evanescent operators with two fermions.

| | ĪRĪL | | R R R | | ĪLĀR |
|---|---|--|---|---|---|
| $\mathcal{E}_{qu} \ \mathcal{E}_{qu}^{(8)}$ | $(ar q u)(ar u q) \ (ar q T^A u)(ar u T^A q)$ | $\mathcal{E}^{(8)}_{uu} \ \mathcal{E}^{[3]}_{uu}$ | $egin{aligned} & (ar{u}\gamma^{\mu}T^{A}u)(ar{u}\gamma_{\mu}T^{A}u) \ & (ar{u}\gamma^{\mu u ho}u)(ar{u}\gamma_{\mu u ho}u) \end{aligned}$ | ${f {\cal E}_{qu}^{[3]}} \ {f {\cal E}_{qu}^{[3](8)}} \ {f {\cal E}_{qu}^{[3](8)}} \ {f {\cal E}_{qu}^{[3]}}$ | $(ar q \gamma^{\mu u ho} q) (ar u \gamma_{\mu u ho} u) \ (ar q \gamma^{\mu u ho} T^A q) (ar u \gamma_{\mu u ho} T^A u)$ |
| $\mathcal{E}_{qd} \ \mathcal{E}_{qd}^{(8)}$ | $(ar q d)(ar d q) \ (ar q T^A d)(ar d T^A q)$ | ${\cal E}^{[3](8)}_{uu} \ {\cal E}^{(8)}_{dd}$ | $egin{aligned} & (ar{u}\gamma^{\mu u ho}T^A u)(ar{u}\gamma_{\mu u ho}T^A u) \ & (ar{d}\gamma^{\mu}T^A d)(ar{d}\gamma_{\mu}T^A d) \end{aligned}$ | $\mathcal{E}_{qd}^{[3]} \ \mathcal{E}_{qd}^{[3](8)} \ \mathcal{E}_{qd}^{[3](8)}$ | $egin{aligned} & (ar{q}\gamma^{\mu u ho}q)(ar{d}\gamma_{\mu u ho}d) \ & (ar{q}\gamma^{\mu u ho}T^Aq)(ar{d}\gamma_{\mu u ho}T^Ad) \end{aligned}$ |
| $\mathcal{E}_{qu}^{[2]}$ | $(\bar{q}\gamma^{\mu\nu}u)(\bar{u}\gamma_{\mu\nu}q)$ | $\mathcal{E}_{dd}^{[3]}$ | $(ar{d}\gamma^{\mu u ho}d)(ar{d}\gamma_{\mu u ho}d)$ | | ĪLĪL |
| $\mathcal{E}_{qu}^{[2](8)}\ \mathcal{E}_{qd}^{[2]}\ \mathcal{E}_{qd}^{[2]}\ \mathcal{E}_{qd}^{[2](8)}\ \mathcal{E}_{qd}^{[2](8)}$ | $egin{aligned} &(ar{q}\gamma^{\mu u}T^Au)(ar{u}\gamma_{\mu u}T^Aq)\ &(ar{q}\gamma^{\mu u}d)(ar{d}\gamma_{\mu u}q)\ &(ar{q}\gamma^{\mu u}T^Ad)(ar{d}\gamma_{\mu u}T^Aq) \end{aligned}$ | $\mathcal{E}_{dd}^{[3](8)}$ \mathcal{E}_{ud} $\mathcal{E}_{ud}^{(8)}$ $\mathcal{E}_{ud}^{(8)}$ | $egin{aligned} & (ar{d}\gamma^{\mu u ho}T^A d)(ar{d}\gamma_{\mu u ho}T^A d) \ & (ar{u}\gamma^\mu d)(ar{d}\gamma_\mu u) \ & (ar{u}\gamma^\mu T^A d)(ar{d}\gamma_\mu T^A u) \end{aligned}$ | $\mathcal{E}_{qq}^{(8)} \ \mathcal{E}_{qq}^{(3,8)} \ \mathcal{E}_{qq}^{(3,8)} \ \mathcal{E}_{qq}^{[3](1)}$ | $egin{aligned} & (ar{q}\gamma^{\mu}T^{A}q)(ar{q}\gamma_{\mu}T^{A}q) \ & (ar{q}\gamma^{\mu}\sigma^{I}T^{A}q)(ar{q}\gamma_{\mu}\sigma^{I}T^{A}q) \ & (ar{q}\gamma^{\mu u ho}q)(ar{q}\gamma_{\mu u ho}q)(ar{q}\gamma_{\mu u ho}q) \end{aligned}$ |
| | ĪRĪR | $\mathcal{E}_{ud}^{[3]}$ | $(\bar{u}\gamma^{\mu\nu\rho}d)(\bar{d}\gamma_{\mu\nu\rho}u)$ | \mathcal{E}_{qq}^{3} | $(ar q \gamma^{\mu u ho} \sigma^I q) (ar q \gamma_{\mu u ho} \sigma^I q)$ |
| $\mathcal{E}^{[2]}_{quqd} \ \mathcal{E}^{[2](8)}_{quqd}$ | $(\bar{q}_r \gamma^{\mu\nu} u) \epsilon_{rs} (\bar{q}_s \gamma_{\mu\nu} d)$ $(\bar{q}_r \gamma^{\mu\nu} T^A u) \epsilon_{rs} (\bar{q}_s \gamma_{\mu\nu} T^A d)$ | $\mathcal{E}_{ud}^{[3](8)} \\ \mathcal{E}_{ud}^{\prime [3]} \\ \mathcal{E}_{ud}^{\prime [3](8)} \\ \mathcal{E}_{ud}^{\prime [3](8)}$ | $egin{aligned} &(ar{u}\gamma^{\mu u ho}T^A d)(ar{d}\gamma_{\mu u ho}T^A u)\ &(ar{u}\gamma^{\mu u ho}u)(ar{d}\gamma_{\mu u ho}d)\ &(ar{u}\gamma^{\mu u ho}T^A u)(ar{d}\gamma_{\mu u ho}T^A d) \end{aligned}$ | $\mathcal{E}_{qq}^{[3](8)} \ \mathcal{E}_{qq}^{[3](3,8)} \ \mathcal{E}_{qq}^{[3](3,8)}$ | $(\bar{q}\gamma^{\mu\nu\rho}T^{A}q)(\bar{q}\gamma_{\mu\nu\rho}T^{A}q)$ $(\bar{q}\gamma^{\mu\nu\rho}\sigma^{I}T^{A}q)(\bar{q}\gamma_{\mu\nu\rho}\sigma^{I}T^{A}q)$ |

Table 5: Evanescent operators with four fermions involving only quarks.

| | ĪRĪL | | R R R R | | L L RR |
|--|--|---------------------------------|--|--|--|
| $\mathcal{E}_{\ell u}$ | $(\bar{\ell}u)(\bar{u}\ell)$ | \mathcal{E}_{eu} | $(\bar{e}\gamma^{\mu}u)(\bar{u}\gamma_{\mu}e)$ | $\mathcal{E}_{\ell q d e}$ | $(\bar{\ell}\gamma^{\mu}q)(\bar{d}\gamma_{\mu}e)$ |
| $\mathcal{E}_{\ell d}$ | $(\bar{\ell}d)(\bar{d}\ell)$ | \mathcal{E}_{ed} | $(ar{e}\gamma^\mu d)(ar{d}\gamma_\mu e)$ | $\mathcal{E}_{\ell q d e} \ \mathcal{E}_{\ell u}^{[3]} \ \mathcal{E}_{\ell u}^{[3]}$ | $(\bar{\ell}\gamma^{\mu\nu ho}\ell)(\bar{u}\gamma_{\mu\nu ho}u)$ |
| \mathcal{E}_{qe} | $(\bar{q}e)(\bar{e}q)$ | $\mathcal{E}_{eu}^{[3]}$ | $(\bar{e}\gamma^{\mu\nu\rho}u)(\bar{u}\gamma_{\mu\nu\rho}e)$ | $\mathcal{E}_{\ell d}^{[3]}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}\ell)(\bar{d}\gamma_{\mu\nu\rho}d)$ |
| $\mathcal{E}_{\ell e d q}^{\lfloor 2 floor}$ | $(\bar{\ell}\gamma^{\mu\nu}e)(\bar{d}\gamma_{\mu\nu}q)$ | $\mathcal{E}_{ed}^{[3]}$ | $(\bar{e}\gamma^{\mu u ho}d)(\bar{d}\gamma_{\mu u ho}e)$ | $\mathcal{E}_{qe}^{[3]}$ | $(ar q \gamma^{\mu u ho} q) (ar e \gamma_{\mu u ho} e)$ |
| $\mathcal{E}_{qe}^{[2]}$ $\mathcal{E}_{\ell e d q}^{[2]}$ $\mathcal{E}_{\ell u}^{[2]}$ | $(\bar{\ell}\gamma^{\mu\nu}u)(\bar{u}\gamma_{\mu\nu}\ell)$ | $\mathcal{E}_{eu}^{\prime [3]}$ | $(\bar{e}\gamma^{\mu\nu\rho}e)(\bar{u}\gamma_{\mu\nu\rho}u)$ | $\mathcal{E}_{qe}^{[3]} \ \mathcal{E}_{\ell q d e}^{[3]}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}q)(\bar{d}\gamma_{\mu\nu\rho}e)$ |
| $\mathcal{E}^{[2]}_{\ell d}$ | $(\bar{\ell}\gamma^{\mu\nu}d)(\bar{d}\gamma_{\mu\nu}\ell)$ | $\mathcal{E}_{ed}^{\prime [3]}$ | $(\bar{e}\gamma^{\mu\nu\rho}e)(\bar{d}\gamma_{\mu\nu\rho}d)$ | | ĪLĪL |
| $\mathcal{E}_{qe}^{[2]}$ | $(\bar{q}\gamma^{\mu\nu}e)(\bar{e}\gamma_{\mu\nu}q)$ | | | $\mathcal{E}_{\ell q}$ | $(ar{\ell}\gamma^\mu q)(ar{q}\gamma_\mu\ell)$ |
| | ĪRĪR | | | $\mathcal{E}_{\ell q}^{(3)}$ | $(ar{\ell}\gamma^\mu\sigma^I q)(ar{q}\gamma_\mu\sigma^I\ell)$ |
| $\mathcal{E}^{[2]}_{\ell equ}$ | $(\bar{\ell}_r \gamma^{\mu\nu} e) \epsilon_{rs} (\bar{q}_s \gamma_{\mu\nu} u)$ | | | $\mathcal{E}^{[3]}_{\ell a}$ | $(\bar{\ell}\gamma^{\mu\nu ho}q)(\bar{q}\gamma_{\mu u ho}\ell)$ |
| \mathcal{E}_{luae} | $(\bar{\ell}_r u)\epsilon_{rs}(\bar{q}_s e)$ | | | $\mathcal{E}_{\ell a}^{3}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^{I}q)(\bar{q}\gamma_{\mu\nu\rho}\sigma^{I}\ell)$ |
| $\mathcal{E}_{\ell u q e} \ \mathcal{E}_{\ell u q e}^{[2]}$ | $(\bar{\ell}_r \gamma^{\mu\nu} u) \epsilon_{rs} (\bar{q}_s \gamma_{\mu\nu} e)$ | | | $\mathcal{E}_{\ell q}^{\prime [3]}$ | $(\bar{\ell}\gamma^{\mu\nu ho}\ell)(\bar{q}\gamma_{\mu\nu ho}q)$ |
| | | | | $ \begin{array}{c} \mathcal{E}_{\ell q}^{[3]} \\ \mathcal{E}_{\ell q}^{3} \\ \mathcal{E}_{\ell q}^{\prime [3]} \\ \mathcal{E}_{\ell q}^{\prime [3]} \\ \mathcal{E}_{\ell q}^{\prime 3} \end{array} $ | $(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^{I}\ell)(\bar{q}\gamma_{\mu\nu\rho}\sigma^{I}q)$ |

Table 6: Semileptonic four-fermion evanescent operators. We use the shorthand notation $\gamma^{\mu_1...\mu_n} \equiv \gamma^{\mu_1}...\gamma^{\mu_n}$ with no (anti)symmetrization.

| | R R R R | | ĪLĪL | | ĪLĀR |
|--|---|-----------------------------------|---|------------------------------|---|
| $\mathcal{E}^{[3]}_{ee}$ | $(ar{e}\gamma^{\mu u ho}e)(ar{e}\gamma_{\mu u ho}e)$ | $\mathcal{E}^{(3)}_{\ell\ell}$ | $(\bar{\ell}\gamma^\mu\sigma^I\ell)(\bar{\ell}\gamma_\mu\sigma^I\ell)$ | $\mathcal{E}^{[3]}_{\ell e}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}l)(\bar{e}\gamma_{\mu\nu\rho}e)$ |
| | ĪRĪL | $\mathcal{E}^{[3]}_{\ell\ell}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}l)(\bar{\ell}\gamma_{\mu\nu\rho}\ell)$ | | |
| $\mathcal{E}^{(3)}_{\ell\ell} \ \mathcal{E}^{[3]}_{\ell\ell} \ \mathcal{E}^{[3]}_{\ell\ell} \ \mathcal{E}^{3}_{\ell\ell}$ | $(\bar{\ell}\gamma^\mu\sigma^I\ell)(\bar{\ell}\gamma_\mu\sigma^I\ell)$ | $\mathcal{E}_{\ell\ell}^{3}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^{I}l)(\bar{\ell}\gamma_{\mu\nu\rho}\sigma^{I}\ell)$ | | |
| $\mathcal{E}_{\ell\ell}^{[3]}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}l)(\bar{\ell}\gamma_{\mu\nu\rho}\ell)$ | | | | |
| $\mathcal{E}_{\ell\ell}^{3}$ | $(\bar{\ell}\gamma^{\mu\nu\rho}\sigma^{I}l)(\bar{\ell}\gamma_{\mu\nu\rho}\sigma^{I}\ell)$ | | | | |

Table 7: Leptonic four-fermion evanescent operators. We use the shorthand notation $\gamma^{\mu_1...\mu_n} \equiv \gamma^{\mu_1}...\gamma^{\mu_n}$ with no (anti)symmetrization.

| | $ar{L}^c L ar{L} L^c$ | | <i>R</i> ^c <i>RRR</i> ^c | | $\bar{R}^c R \bar{L} L^c$ |
|---|---|---|---|--|---|
| $\mathcal{E}^c_{qq} \ \mathcal{E}^{c\prime}_{qq}$ | $(\overline{q^c}_{ar}q_{bs})(ar{q}_{bs}q^c_{ar}) \ (\overline{q^c}_{ar}q_{bs})(ar{q}_{as}q^c_{br})$ | $egin{split} \mathcal{E}^{c}_{uu}\ \mathcal{E}^{c}_{dd} \end{split}$ | $(\overline{u^c}_a u_b)(ar{u}_b u^c_a) \ (\overline{d^c}_a d_b)(ar{d}_b d^c_a)$ | $\mathcal{E}^{c}_{udqq} \ \mathcal{E}^{c[2]}_{udqq}$ | $(\overline{u^{c}}_{a}d_{b})(\bar{q}_{br}\epsilon_{rs}q_{as}^{c})$ $(\overline{u^{c}}_{a}\gamma^{\mu\nu}d_{b})(\bar{q}_{br}\epsilon_{rs}\gamma_{\mu\nu}q_{as}^{c})$ |
| $\mathcal{E}_{qq}^{c[2]}$ | $(\overline{q^c}_{ar}\gamma^{\mu\nu}q_{bs})(\bar{q}_{bs}\gamma_{\mu\nu}q^c_{ar})$ | \mathcal{E}_{ud}^{c} | $(\overline{u^c}_a d_b)(\bar{d}_b u^c_a)$ | | $ar{L}^c Rar{R} L^c$ |
| ${\cal E}^{c\prime [2]}_{qq}$ | $(\overline{q^c}_{ar}\gamma^{\mu\nu}q_{bs})(\bar{q}_{as}\gamma_{\mu\nu}q^c_{br})$ | $ \begin{array}{c} \mathcal{E}_{ud}^{c\prime} \\ \mathcal{E}_{ul}^{c[2]} \\ \mathcal{U}_{du}^{c[2]} \\ \mathcal{E}_{dd}^{c[2]} \\ \mathcal{U}_{ud}^{c\prime} \\ \mathcal{E}_{ud}^{c\prime} \\ \end{array} $ | $(\overline{u^{c}}_{a}d_{b})(\bar{d}_{a}u_{b}^{c})$ $(\overline{u^{c}}_{a}\gamma^{\mu\nu}u_{b})(\bar{u}_{b}\gamma_{\mu\nu}u_{a}^{c})$ $(\overline{d^{c}}_{a}\gamma^{\mu\nu}d_{b})(\bar{d}_{b}\gamma_{\mu\nu}d_{a}^{c})$ $(\overline{u^{c}}_{a}\gamma^{\mu\nu}d_{b})(\bar{d}_{b}\gamma_{\mu\nu}u_{a}^{c})$ $(\overline{u^{c}}_{a}\gamma^{\mu\nu}d_{b})(\bar{d}_{a}\gamma_{\mu\nu}u_{b}^{c})$ | \mathcal{E}_{qu}^{c} \mathcal{E}_{qd}^{c} $\mathcal{E}_{qu}^{c\prime}$ $\mathcal{E}_{qd}^{c\prime}$ | $(\overline{q^{c}}_{a}\gamma^{\mu}u_{b})(\overline{u}_{b}\gamma_{\mu}q_{a}^{c})$ $(\overline{q^{c}}_{a}\gamma^{\mu}d_{b})(\overline{d}_{b}\gamma_{\mu}q_{a}^{c})$ $(\overline{q^{c}}_{a}\gamma^{\mu}u_{b})(\overline{u}_{a}\gamma_{\mu}q_{b}^{c})$ $(\overline{q^{c}}_{a}\gamma^{\mu}d_{b})(\overline{d}_{a}\gamma_{\mu}q_{b}^{c})$ |

Table 8: Evanescent operators with four fermions involving only quarks and featuring charge conjugation. We use the shorthand notation $\gamma^{\mu_1...\mu_n} \equiv \gamma^{\mu_1}...\gamma^{\mu_n}$ with no (anti)symmetrization.

| | $ar{L}^c L ar{L} L^c$ | | $ar{R}^c R ar{R} R^c$ | | $\bar{R}^c R \bar{L} L^c$ |
|---|---|---------------------------|--|-------------------------------------|---|
| $\mathcal{E}^{c}_{\ell\ell}$ | $(\overline{\ell^c}_r \ell_s)(\overline{\ell}_s \ell_r^c)$ | \mathcal{E}_{ee}^{c} | $(\overline{e^c}e)(\overline{e}e^c)$ | \mathcal{E}_{uelq}^{c} | $(\overline{u^c}e)(\overline{\ell}_r\epsilon_{rs}q_s^c)$ |
| $\mathcal{E}^c_{q\ell}$ | $(\overline{q^c}_r\ell_s)(\overline{\ell}_sq^c_r)$ | \mathcal{E}_{eu}^{c} | $(\overline{e^c}u)(\overline{u}e^c)$ | $\mathcal{E}_{ue\ell q}^{c[2]}$ | $(\overline{u^c}\gamma^{\mu\nu}e)(\bar{\ell}_r\gamma_{\mu\nu}\epsilon_{rs}q_s^c)$ |
| $\mathcal{E}^{c\prime}_{q\ell}$ | $(\overline{q^c}_r\ell_s)(\bar{q}_r\ell^c_s)$ | \mathcal{E}_{ed}^{c} | $(\overline{e^c}d)(\overline{d}e^c)$ | | $ar{L}^c R ar{R} L^c$ |
| $\mathcal{E}^{c[2]}_{\ell\ell} \ \mathcal{E}^{c[2]}_{q\ell} \ \mathcal{E}^{c[2]}_{q\ell}$ | $(\overline{\ell^c}_r\gamma^{\mu\nu}\ell_s)(\bar{\ell}_s\gamma_{\mu\nu}q_r^c)$ | $\mathcal{E}^{c[2]}_{ee}$ | $(\overline{e^c}\gamma^{\mu\nu}e)(\bar{e}\gamma_{\mu\nu}e^c)$ | $\mathcal{E}_{\ell e}^{c}$ | $(\overline{\ell^c}\gamma^\mu e)(\bar{e}\gamma_\mu\ell^c)$ |
| $\mathcal{E}^{c[2]}_{q\ell}$ | $(\overline{q^c}_r \gamma^{\mu\nu} \ell_s)(\bar{q}_s \gamma_{\mu\nu} q^c_r)$ | $\mathcal{E}_{eu}^{c[2]}$ | $(\overline{e^c}\gamma^{\mu\nu}u)(\bar{u}\gamma_{\mu\nu}e^c)$ | \mathcal{E}^{c}_{qe} | $(\overline{q^c}\gamma^\mu e)(ar{e}\gamma_\mu q^c)$ |
| $\mathcal{E}^{c\prime [2]}_{q\ell}$ | $(\overline{q^c}_r \gamma^{\mu\nu} \ell_s)(\bar{q}_r \gamma_{\mu\nu} \ell^c_s)$ | $\mathcal{E}_{ed}^{c[2]}$ | $(\overline{e^c}\gamma^{\mu\nu}d)(\overline{d}\gamma_{\mu\nu}e^c)$ | $\mathcal{E}_{\ell u}^{c}$ | $(\overline{\ell^c}\gamma^\mu u)(\bar{u}\gamma_\mu\ell^c)$ |
| | | | | $\mathcal{E}^{c}_{\ell d}$ | $(\overline{\ell^c}\gamma^\mu d)(\bar{d}\gamma_\mu\ell^c)$ |
| | | | | $\mathcal{E}_{qed\ell}^{c^{\circ}}$ | $(\overline{q^c}\gamma^\mu e)(\overline{d}\gamma_\mu\ell^c)$ |

Table 9: Semileptonic and leptonic evanescent operators with four fermions featuring charge conjugation. We use the shorthand notation $\gamma^{\mu_1...\mu_n} \equiv \gamma^{\mu_1}...\gamma^{\mu_n}$ with no (anti)symmetrization.

References

- [1] H. Georgi, *Effective field theory*, Ann. Rev. Nucl. Part. Sci. **43**, 209 (1993), doi:10.1146/annurev.ns.43.120193.001233.
- [2] N. P. Hartland, F. Maltoni, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou and C. Zhang, A Monte Carlo global analysis of the Standard Model Effective Field Theory: the top quark sector, JHEP 04, 100 (2019), doi:10.1007/JHEP04(2019)100, 1901.05965.
- [3] I. Brivio, S. Bruggisser, F. Maltoni, R. Moutafis, T. Plehn, E. Vryonidou, S. Westhoff and C. Zhang, *O new physics, where art thou? A global search in the top sector*, JHEP 02, 131 (2020), doi:10.1007/JHEP02(2020)131, 1910.03606.
- [4] S. Banerjee, R. S. Gupta, J. Y. Reiness, S. Seth and M. Spannowsky, *Towards the ultimate differential SMEFT analysis*, JHEP 09, 170 (2020), doi:10.1007/JHEP09(2020)170, 1912.07628.
- [5] R. Aoude, T. Hurth, S. Renner and W. Shepherd, *The impact of flavour data on global fits of the MFV SMEFT*, JHEP **12**, 113 (2020), doi:10.1007/JHEP12(2020)113, 2003.05432.
- [6] S. Dawson, S. Homiller and S. D. Lane, Putting standard model EFT fits to work, Phys. Rev. D 102(5), 055012 (2020), doi:10.1103/PhysRevD.102.055012, 2007.01296.
- [7] Anisha, S. Das Bakshi, J. Chakrabortty and S. K. Patra, Connecting electroweak-scale observables to BSM physics through EFT and Bayesian statistics, Phys. Rev. D 103(7), 076007 (2021), doi:10.1103/PhysRevD.103.076007, 2010.04088.
- [8] A. Falkowski, M. González-Alonso and O. Naviliat-Cuncic, Comprehensive analysis of beta decays within and beyond the Standard Model, JHEP 04, 126 (2021), doi:10.1007/JHEP04(2021)126, 2010.13797.
- [9] J. Ellis, M. Madigan, K. Mimasu, V. Sanz and T. You, Top, Higgs, Diboson and Electroweak Fit to the Standard Model Effective Field Theory, JHEP 04, 279 (2021), doi:10.1007/JHEP04(2021)279, 2012.02779.
- [10] J. J. Ethier, G. Magni, F. Maltoni, L. Mantani, E. R. Nocera, J. Rojo, E. Slade, E. Vryonidou and C. Zhang, *Combined SMEFT interpretation of Higgs, diboson, and top quark data from the LHC*, JHEP **11**, 089 (2021), doi:10.1007/JHEP11(2021)089, 2105.00006.
- [11] I. Brivio, S. Bruggisser, E. Geoffray, W. Kilian, M. Krämer, M. Luchmann, T. Plehn and B. Summ, *From Models to SMEFT and Back?* (2021), 2108.01094.
- [12] E. d. S. Almeida, A. Alves, O. J. P. Éboli and M. C. Gonzalez-Garcia, *Electroweak legacy of the LHC Run II* (2021), 2108.04828.
- [13] Anisha, S. Das Bakshi, S. Banerjee, A. Biekötter, J. Chakrabortty, S. Kumar Patra and M. Spannowsky, *Effective limits on single scalar extensions in the light of recent LHC data* (2021), 2111.05876.
- [14] I. Brivio and M. Trott, *The Standard Model as an Effective Field Theory*, Phys. Rept. **793**, 1 (2019), doi:10.1016/j.physrep.2018.11.002, 1706.08945.
- [15] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators I: Formalism and lambda Dependence*, JHEP **10**, 087 (2013), doi:10.1007/JHEP10(2013)087, 1308.2627.

- [16] E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Evolution of the Standard Model Dimension Six Operators II: Yukawa Dependence*, JHEP **01**, 035 (2014), doi:10.1007/JHEP01(2014)035, 1310.4838.
- [17] R. Alonso, E. E. Jenkins, A. V. Manohar and M. Trott, Renormalization Group Evolution of the Standard Model Dimension Six Operators III: Gauge Coupling Dependence and Phenomenology, JHEP 04, 159 (2014), doi:10.1007/JHEP04(2014)159, 1312.2014.
- [18] R. Alonso, H.-M. Chang, E. E. Jenkins, A. V. Manohar and B. Shotwell, *Renormalization group evolution of dimension-six baryon number violating operators*, Phys. Lett. B **734**, 302 (2014), doi:10.1016/j.physletb.2014.05.065, 1405.0486.
- [19] C. Grojean, E. E. Jenkins, A. V. Manohar and M. Trott, *Renormalization Group Scaling of Higgs Operators and \Gamma(h -> \gamma \gamma)*, JHEP 04, 016 (2013), doi:10.1007/JHEP04(2013)016, 1301.2588.
- [20] J. Elias-Miro, J. R. Espinosa, E. Masso and A. Pomarol, *Higgs windows to new physics through d=6 operators: constraints and one-loop anomalous dimensions*, JHEP **11**, 066 (2013), doi:10.1007/JHEP11(2013)066, 1308.1879.
- [21] S. Davidson, M. Gorbahn and M. Leak, Majorana neutrino masses in the renormalization group equations for lepton flavor violation, Phys. Rev. D 98(9), 095014 (2018), doi:10.1103/PhysRevD.98.095014, 1807.04283.
- [22] M. Chala and A. Titov, *Neutrino masses in the Standard Model effective field theory*, Phys. Rev. D 104(3), 035002 (2021), doi:10.1103/PhysRevD.104.035002, 2104.08248.
- [23] Y. Liao and X.-D. Ma, Renormalization Group Evolution of Dimension-seven Baryon- and Lepton-number-violating Operators, JHEP 11, 043 (2016), doi:10.1007/JHEP11(2016)043, 1607.07309.
- [24] M. Chala, G. Guedes, M. Ramos and J. Santiago, Towards the renormalisation of the Standard Model effective field theory to dimension eight: Bosonic interactions I, SciPost Phys. 11, 065 (2021), doi:10.21468/SciPostPhys.11.3.065, 2106.05291.
- [25] M. Accettulli Huber and S. De Angelis, *Standard Model EFTs via on-shell methods*, JHEP 11, 221 (2021), doi:10.1007/JHEP11(2021)221, 2108.03669.
- [26] E. E. Jenkins, A. V. Manohar and P. Stoffer, Low-Energy Effective Field Theory below the Electroweak Scale: Operators and Matching, JHEP 03, 016 (2018), doi:10.1007/JHEP03(2018)016, 1709.04486.
- [27] W. Dekens and P. Stoffer, Low-energy effective field theory below the electroweak scale: matching at one loop, JHEP 10, 197 (2019), doi:10.1007/JHEP10(2019)197, 1908.
 05295.
- [28] E. E. Jenkins, A. V. Manohar and P. Stoffer, Low-Energy Effective Field Theory below the Electroweak Scale: Anomalous Dimensions, JHEP 01, 084 (2018), doi:10.1007/JHEP01(2018)084, 1711.05270.
- [29] A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, *DsixTools: The Standard Model Effective Field Theory Toolkit*, Eur. Phys. J. C 77(6), 405 (2017), doi:10.1140/epjc/s10052-017-4967-6, 1704.04504.

- [30] J. Fuentes-Martin, P. Ruiz-Femenia, A. Vicente and J. Virto, *DsixTools 2.0: The Effective Field Theory Toolkit*, Eur. Phys. J. C 81(2), 167 (2021), doi:10.1140/epjc/s10052-020-08778-y, 2010.16341.
- [31] J. Aebischer, J. Kumar and D. M. Straub, Wilson: a Python package for the running and matching of Wilson coefficients above and below the electroweak scale, Eur. Phys. J. C 78(12), 1026 (2018), doi:10.1140/epjc/s10052-018-6492-7, 1804.05033.
- [32] J. de Blas, J. C. Criado, M. Perez-Victoria and J. Santiago, *Effective description of general extensions of the Standard Model: the complete tree-level dictionary*, JHEP 03, 109 (2018), doi:10.1007/JHEP03(2018)109, 1711.10391.
- [33] F. del Aguila, M. Perez-Victoria and J. Santiago, Observable contributions of new exotic quarks to quark mixing, JHEP 09, 011 (2000), doi:10.1088/1126-6708/2000/09/011, hep-ph/0007316.
- [34] F. del Aguila, J. de Blas and M. Perez-Victoria, Effects of new leptons in Electroweak Precision Data, Phys. Rev. D 78, 013010 (2008), doi:10.1103/PhysRevD.78.013010, 0803.4008.
- [35] F. del Aguila, J. de Blas and M. Perez-Victoria, *Electroweak Limits on General New Vector Bosons*, JHEP 09, 033 (2010), doi:10.1007/JHEP09(2010)033, 1005.3998.
- [36] J. de Blas, M. Chala, M. Perez-Victoria and J. Santiago, *Observable Effects of General New Scalar Particles*, JHEP **04**, 078 (2015), doi:10.1007/JHEP04(2015)078, 1412.8480.
- [37] B. Henning, X. Lu and H. Murayama, *How to use the Standard Model effective field theory*, JHEP **01**, 023 (2016), doi:10.1007/JHEP01(2016)023, 1412.1837.
- [38] M. K. Gaillard, *The Effective One Loop Lagrangian With Derivative Couplings*, Nucl. Phys. B **268**, 669 (1986), doi:10.1016/0550-3213(86)90264-6.
- [39] O. Cheyette, Effective Action for the Standard Model With Large Higgs Mass, Nucl. Phys. B 297, 183 (1988), doi:10.1016/0550-3213(88)90205-2.
- [40] A. Drozd, J. Ellis, J. Quevillon and T. You, *The Universal One-Loop Effective Action*, JHEP 03, 180 (2016), doi:10.1007/JHEP03(2016)180, 1512.03003.
- [41] F. del Aguila, Z. Kunszt and J. Santiago, One-loop effective lagrangians after matching, Eur. Phys. J. C 76(5), 244 (2016), doi:10.1140/epjc/s10052-016-4081-1, 1602.00126.
- [42] B. Henning, X. Lu and H. Murayama, One-loop Matching and Running with Covariant Derivative Expansion, JHEP 01, 123 (2018), doi:10.1007/JHEP01(2018)123, 1604. 01019.
- [43] S. A. R. Ellis, J. Quevillon, T. You and Z. Zhang, Mixed heavy-light matching in the Universal One-Loop Effective Action, Phys. Lett. B 762, 166 (2016), doi:10.1016/j.physletb.2016.09.016, 1604.02445.
- [44] J. Fuentes-Martin, J. Portoles and P. Ruiz-Femenia, Integrating out heavy particles with functional methods: a simplified framework, JHEP 09, 156 (2016), doi:10.1007/JHEP09(2016)156, 1607.02142.
- [45] Z. Zhang, Covariant diagrams for one-loop matching, JHEP 05, 152 (2017), doi:10.1007/JHEP05(2017)152, 1610.00710.

- [46] S. A. R. Ellis, J. Quevillon, T. You and Z. Zhang, Extending the Universal One-Loop Effective Action: Heavy-Light Coefficients, JHEP 08, 054 (2017), doi:10.1007/JHEP08(2017)054, 1706.07765.
- [47] M. Krämer, B. Summ and A. Voigt, *Completing the scalar and fermionic Universal One-Loop Effective Action*, JHEP **01**, 079 (2020), doi:10.1007/JHEP01(2020)079, 1908.04798.
- [48] S. A. R. Ellis, J. Quevillon, P. N. H. Vuong, T. You and Z. Zhang, *The Fermionic Universal One-Loop Effective Action*, JHEP **11**, 078 (2020), doi:10.1007/JHEP11(2020)078, 2006. 16260.
- [49] A. Angelescu and P. Huang, Integrating Out New Fermions at One Loop, JHEP 01, 049 (2021), doi:10.1007/JHEP01(2021)049, 2006.16532.
- [50] T. Cohen, X. Lu and Z. Zhang, Functional Prescription for EFT Matching, JHEP 02, 228 (2021), doi:10.1007/JHEP02(2021)228, 2011.02484.
- [51] S. Dittmaier, S. Schuhmacher and M. Stahlhofen, Integrating out heavy fields in the path integral using the background-field method: general formalism, Eur. Phys. J. C 81, 826 (2021), doi:10.1140/epjc/s10052-021-09587-7, 2102.12020.
- [52] T. Cohen, X. Lu and Z. Zhang, *STrEAMlining EFT Matching*, SciPost Phys. **10**(5), 098 (2021), doi:10.21468/SciPostPhys.10.5.098, 2012.07851.
- [53] J. Fuentes-Martin, M. König, J. Pagès, A. E. Thomsen and F. Wilsch, SuperTracer: A Calculator of Functional Supertraces for One-Loop EFT Matching, JHEP 04, 281 (2021), doi:10.1007/JHEP04(2021)281, 2012.08506.
- [54] S. Das Bakshi, J. Chakrabortty and S. K. Patra, *CoDEx: Wilson coefficient calculator connecting SMEFT to UV theory*, Eur. Phys. J. C **79**(1), 21 (2019), doi:10.1140/epjc/s10052-018-6444-2, 1808.04403.
- [55] N. Brambilla, H. S. Chung, V. Shtabovenko and A. Vairo, FeynOnium: Using FeynCalc for automatic calculations in Nonrelativistic Effective Field Theories, JHEP 11, 130 (2020), doi:10.1007/JHEP11(2020)130, 2006.15451.
- [56] G. Uhlrich, F. Mahmoudi and A. Arbey, Automatic extraction of one-loop Wilson coefficients in general BSM scenarios using MARTY-1.4, In European Physical Society Conference on High Energy Physics 2021 (2021), 2110.14515.
- [57] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr and B. Fuks, *FeynRules 2.0 A complete toolbox for tree-level phenomenology*, Comput. Phys. Commun. 185, 2250 (2014), doi:10.1016/j.cpc.2014.04.012, 1310.1921.
- [58] A. V. Manohar, Introduction to Effective Field Theories (2018), doi:10.1093/oso/9780198855743.003.0002, 1804.05863.
- [59] T. Cohen, As Scales Become Separated: Lectures on Effective Field Theory, PoS TASI2018, 011 (2019), 1903.03622.
- [60] P. Nogueira, Automatic Feynman graph generation, J. Comput. Phys. 105, 279 (1993), doi:10.1006/jcph.1993.1074.
- [61] B. Ruijl, T. Ueda and J. Vermaseren, FORM version 4.2 (2017), 1707.06453.
- [62] A. Denner, H. Eck, O. Hahn and J. Kublbeck, Feynman rules for fermion number violating interactions, Nucl. Phys. B 387, 467 (1992), doi:10.1016/0550-3213(92)90169-C.

- [63] A. Denner, H. Eck, O. Hahn and J. Kublbeck, Compact Feynman rules for Majorana fermions, Phys. Lett. B 291, 278 (1992), doi:10.1016/0370-2693(92)91045-B.
- [64] V. Gherardi, D. Marzocca and E. Venturini, *Matching scalar leptoquarks to the SMEFT at one loop*, JHEP 07, 225 (2020), doi:10.1007/JHEP07(2020)225, [Erratum: JHEP 01, 006 (2021)], 2003.12525.
- [65] C. Gnendiger et al., To d, or not to d: recent developments and comparisons of regularization schemes, Eur. Phys. J. C 77(7), 471 (2017), doi:10.1140/epjc/s10052-017-5023-2, 1705.01827.
- [66] M. Chala, G. Guedes, M. Ramos and J. Santiago, *Running in the ALPs*, Eur. Phys. J. C 81(2), 181 (2021), doi:10.1140/epjc/s10052-021-08968-2, 2012.09017.
- [67] M. Chala and J. Santiago, *Positivity bounds in the Standard Model effective field theory beyond tree level* (2021), 2110.01624.
- [68] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, Dimension-Six Terms in the Standard Model Lagrangian, JHEP 10, 085 (2010), doi:10.1007/JHEP10(2010)085, 1008.4884.
- [69] U. Haisch, M. Ruhdorfer, E. Salvioni, E. Venturini and A. Weiler, Singlet night in Feynman-ville: one-loop matching of a real scalar, JHEP 04, 164 (2020), doi:10.1007/JHEP04(2020)164, [Erratum: JHEP 07, 066 (2020)], 2003.05936.
- [70] M. Jiang, N. Craig, Y.-Y. Li and D. Sutherland, *Complete one-loop matching for a singlet scalar in the Standard Model EFT*, JHEP 02, 031 (2019), doi:10.1007/JHEP02(2019)031,
 [Erratum: JHEP 01, 135 (2021)], 1811.08878.
- [71] D. Zhang and S. Zhou, Complete One-loop Matching of the Type-I Seesaw Model onto the Standard Model Effective Field Theory (2021), doi:10.1007/JHEP09(2021)163, 2107. 12133.
- M. S. Bilenky and A. Santamaria, One loop effective Lagrangian for a standard model with a heavy charged scalar singlet, Nucl. Phys. B 420, 47 (1994), doi:10.1016/0550-3213(94)90375-1, hep-ph/9310302.
- [73] A. Crivellin and M. Hoferichter, *Consequences of chirally enhanced explanations of* $(g-2)_{\mu}$ *for* $h \rightarrow \mu\mu$ *and* $Z \rightarrow \mu\mu$, JHEP **07**, 135 (2021), doi:10.1007/JHEP07(2021)135, 2104.03202.
- [74] G. Guedes and J. Santiago, *New leptons with exotic decays: collider limits and dark matter complementarity* (2021), 2107.03429.
- [75] A. Falkowski, B. Fuks, K. Mawatari, K. Mimasu, F. Riva and V. Sanz, Rosetta: an operator basis translator for Standard Model effective field theory, Eur. Phys. J. C 75(12), 583 (2015), doi:10.1140/epjc/s10052-015-3806-x, 1508.05895.
- [76] M. Chala, A. Díaz-Carmona and G. Guedes, A Green's basis for the bosonic SMEFT to dimension 8 (2021), 2112.12724.
- [77] M. Chala and A. Titov, One-loop matching in the SMEFT extended with a sterile neutrino, JHEP 05, 139 (2020), doi:10.1007/JHEP05(2020)139, 2001.07732.