

Inclusive Decays of Heavy Quark Hybrids

Abhishek Mohapatra*

Physik Department, Technische Universität München,
James-Frank-Strasse 1, 85748 Garching, Germany

* abhishek.mohapatra@tum.de

January 24, 2022

XXXIII International (ONLINE) Workshop on High Energy Physics
“Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quests”
November 8-12, 2021
doi:[10.21468/SciPostPhysProc.7.01.001](https://doi.org/10.21468/SciPostPhysProc.7.01.001)

Abstract

In order to understand the nature of the XYZ particles, theoretical predictions of the various XYZ decay modes are essential. In this work, we focus on the semi-inclusive decays of heavy quarkonium hybrids into traditional quarkonium in the EFT framework. We begin with weakly coupled potential NRQCD effective theory that describes systems with two heavy quarks and incorporates multipole expansions and use it to develop a Born-Oppenheimer effective theory (BOEFT) to describe the hybrids and compute the semi-inclusive decay rates. We compute both the spin-conserving and spin-flipping decay rates and find that our numerical results of the decay rates are different from the previous studies. We also develop a systematic framework in which the theoretical uncertainty can be systematically improved.

Contents

1	Introduction	2
2	Spectrum	3
2.1	Quarkonium	3
2.2	Hybrids	3
3	Inclusive Decay Rate	5
4	Conclusions	7
	References	8

1 Introduction

The Standard Model (SM) describes hadrons as bound states of quarks and gluons bounded by the strong interactions. Traditionally, the hadrons were classified as mesons that are bound state of quark-antiquark pair or baryons that are bound state of 3-quarks using the quark model. However, the underlying theory of strong interactions, Quantum Chromodynamics (QCD) also allows for existence of complex hadron structures beyond mesons and baryons such as tetraquark (4-quark states), pentaquark (5-quark states), hybrids (hadrons with active gluons) and glueballs (bound state of gluons), which are known as *exotic hadrons* or *exotics*. The so called **XYZ states** are the exotic hadrons in the heavy-quark sector. The XYZ states do not fit the usual charmonium ($c\bar{c}$) or bottomonium ($b\bar{b}$) spectrum and in some cases have exotic quantum numbers which cannot be reproduced by the ordinary hadrons such as charged Z_c and Z_b states. In 2003, the Belle experimental collaboration observed the first exotic state $X(3872)$ [1] and since then, several of the new exotic hadrons in the heavy-quark sector have been observed by the different experimental groups: B-factories (BaBar, Belle and CLEO), τ -charm facilities (CLEO-c, BESIII) and also proton-(anti)proton colliders (CDF, D0, LHCb, ATLAS, CMS) (see the reviews [2, 3] for details on experimental observation).

There are several theoretical models and proposals to understand the nature of the XYZ exotics. One viable and attractive interpretation for at-least some of the XYZ mesons is the quarkonium hybrids, which is a bound state of heavy quark and a heavy antiquark together with gluonic excitation. The other proposals are hadroquarkonium, heavy meson molecule, tetraquark, and diquark-diquark model (see Ref. [2, 3] for review). However, no single proposal can theoretically explain the complete spectrum of the XYZ exotic states. On the other hand, several new exotic states have been observed in experiments for which the masses and the decay rates has been measured (see Ref. [4]). Specifically, several of these exotic states has been discovered from their decays to standard quarkonium. Therefore, a theoretical understanding of the decays of XYZ exotics might be an another avenue for understanding their structure. In this work, our objective is to study the inclusive decays of heavy quark hybrids to traditional quarkonium i.e, $H_m \rightarrow Q_n + X$, where H_m is a low-lying hybrid, Q_n is a low-lying quarkonium state and X denotes other final state particles.

Within the QCD framework, one can use lattice simulations and effective field theories (EFTs) to describe the traditional quarkonium and quarkonium hybrids and compute its spectra. Since, the heavy quarks in quarkonium and heavy-quark hybrids are nonrelativistic, the appropriate framework to use is the nonrelativistic effective theory NRQCD [5, 6]. More specifically, if we are only interested in the dynamics of the two heavy quarks, then the appropriate framework to use is the potential NRQCD effective theory known as pNRQCD [7, 8]. In case of quarkonium hybrids, there are well-separated energy scales: m_Q (mass of heavy quark) $\gg m_Q v$ (relative momentum scale) $\gg \Lambda_{\text{QCD}}$ (energy scale for gluonic excitations) $\gg m_Q v^2$ (dynamics of two heavy quark). The above momentum hierarchy suggests of an energy gap between the gluonic excitations and the excitations of the heavy quark-antiquark pair that has also been confirmed by the lattice data [9, 10]. This justifies the use of effective theory based on Born-Oppenheimer approximation (BOEFT) to describe the hybrids [11–15]. On the other hand, the lattice inputs are essential for determining the static potentials that are used for solving the Schrödinger equation for computing the spectra. Traditionally, the lattice studies of the heavy quark hybrids have mainly focused in the charmonium sector and recently in bottomonium sector [16]. In the charm sector, the recent lattice studies have predicted the existence of a lowest hybrid spin-multiplet $J^{PC} = [(0, 1, 2)^{-+}, 1^{--}]$ at about 4.3 GeV [17–20]. In the bottom sector, the lattice study in Ref. [16], predicted hybrid states with quantum numbers $J^{PC} = [(0, 1, 2)^{-+}, 1^{--}]$ approximately 1500 MeV above the ground-state η_b meson.

In this work, we will use the BOEFT for the hybrids and pNRQCD for the low-lying quarko-

onium states. For computing the decay rates, we perform a matching calculation between BOEFT and pNRQCD to obtain the imaginary terms in the BOEFT potential. In Sec. 2, we compute the quarkonium and the hybrid spectrum, in Sec. 3, we perform the matching calculation and compute the decay rates and we conclude in Sec. 4.

2 Spectrum

2.1 Quarkonium

The conventional quarkonium states ($Q\bar{Q}$) are color singlet bound states of a heavy quark and antiquark in the ground state static potential $V_{\Sigma_g^+}(r)$. The shape of the static potential $V_{\Sigma_g^+}(r)$ is well described by the Cornell potential. The Schrödinger equation describing the quarkonium spectrum is given by

$$\left(-\frac{\nabla^2}{m_Q} + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}(r) \right) \Phi_{(n)}^Q(\mathbf{r}) = E_n^Q \Phi_{(n)}^Q(\mathbf{r}), \quad (1)$$

where m_Q is the heavy quark mass, E_n^Q is the quarkonium energy, $\Phi_{(n)}^Q(\mathbf{r})$ denotes the quarkonium wave-function, the index $(n) = (n, j, l, s)$ denotes the usual set of quarkonium quantum numbers. We use the following form of the static potential $V_{\Sigma_g^+}(r)$ from Ref. [12]

$$V_{\Sigma_g^+}(r) = -\frac{\kappa_g}{r} + \sigma_g r + E_g^{Q\bar{Q}}, \quad (2)$$

where $\kappa_g = 0.489$ and the string tension parameter $\sigma_g = 0.187 \text{ GeV}^2$ are determined from the fit to the lattice data. The constant $E_g^{Q\bar{Q}}$ is different for both charmonium and bottomonium and is determined by comparison to the experimental spin-averaged mass from PDG 2020 [4]

$$E_g^{c\bar{c}} = -0.254 \text{ GeV}, \quad E_g^{b\bar{b}} = -0.195 \text{ GeV}, \quad (3)$$

where we have used the RS-scheme charm and bottom mass: $m_c = 1.477 \text{ GeV}$ and $m_b = 4.863 \text{ GeV}$ to compute the quarkonium spectrum. The quarkonium mass is given by $M_{Q\bar{Q}} = 2m_Q + E_n^Q$ for $Q = (c, b)$, where E_n^Q is the eigenvalue in Eq. (1).

2.2 Hybrids

Hybrids ($Q\bar{Q}g$) are exotic hadrons that are color singlet bound states of a color octet $Q\bar{Q}$ source coupled to gluonic excitations. Therefore, hybrid states are more complicated compared to traditional quarkonium due to presence of active gluons. The energy scale for the gluonic excitations is the nonperturbative energy scale Λ_{QCD} . In the BOEFT description, the nonperturbative gluon dynamics generate a background static potential in which the heavy quark-antiquark pair in the hybrids binds together. In the static limit ($m_Q \rightarrow \infty$), the hybrid spectrum is composed of the static energies, which are characterized by the representation of the $D_{\infty h}$ cylindrical symmetry group just like in diatomic molecules. The hybrid static energies are nonperturbative quantities that are generally computed on the lattice. In the short-distance limit $r \rightarrow 0$, where \mathbf{r} is the relative coordinate of $Q\bar{Q}$, the hybrid static energies are degenerate and quantum numbers are characterized by representations of spherical symmetry group $O(3) \times C$ instead of $D_{\infty h}$ [7, 13, 14]. We focus here on the low-lying hybrids coming from Σ_u^- and Π_u static potentials and we closely follow the notations in Ref. [13].

The BOEFT Lagrangian that describes the hybrid states is given by

$$L_{\text{BOEFT}} = \int_{\mathbf{R}} \int_{\mathbf{r}} \sum_{\kappa} \sum_{\lambda\lambda'} \left[\Psi_{\kappa\lambda}^{\dagger}(\mathbf{r}, \mathbf{R}, t) \left\{ i\partial_t - V_{\kappa\lambda\lambda'}(r) + P_{\kappa\lambda}^{i\dagger} \frac{\nabla_{\mathbf{r}}^2}{m_Q} P_{\kappa\lambda'}^i \right\} \Psi_{\kappa\lambda'}(\mathbf{r}, \mathbf{R}, t) \right. \\ \left. + \Psi_{\kappa\lambda}^{\dagger}(\mathbf{r}) \Delta V(r) \delta_{\lambda\lambda'} \Psi_{\kappa\lambda'}(\mathbf{r}, \mathbf{R}, t) \right] + \dots, \quad (4)$$

where $\int_{\mathbf{R}} \equiv \int d^3\mathbf{R}$, \mathbf{r} and \mathbf{R} are the relative and center-of-mass coordinates of the heavy-quark-antiquark pair, the quantum number κ is $\kappa \equiv K^{PC}$, with K defined as the angular momentum of the gluonic degrees of freedom, $\Psi_{\kappa\lambda}$ denotes the hybrid field (or the wave-function) and the ellipses represent higher order terms in the multipole expansion. $P_{\kappa\lambda}^i$ (i denotes vector index) is the projection operator that projects the gluonic degrees of freedom to an eigenstate of $\mathbf{K} \cdot \hat{\mathbf{r}}$ with eigenvalue λ . These projection operators correctly reproduce the hybrid quantum numbers in $D_{\infty h}$ representation. For low-lying hybrids coming from Σ_u^- and Π_u static potentials, the gluon quantum number $\kappa = 1^{+-}$. Therefore, for our purpose, the projectors $P_{1\lambda}^i$ are given by $P_{10}^i = \hat{r}_0^i(\theta, \phi) = \hat{r}^i$ for projecting onto the Σ_u^- state and $P_{1\pm 1}^i = \hat{\theta}_{\pm 1}^i(\theta, \phi) = (\hat{\theta}^i \pm i\hat{\phi}^i)/\sqrt{2}$ for projecting onto the two components of the Π_u^{\pm} state, where $\hat{\theta}$ and $\hat{\phi}$ are the usual spherical unit vectors. In Eq. (4), the BOEFT potential $V_{\kappa\lambda\lambda'}(r)$ can be expanded in $1/m_Q$ as

$$V_{\kappa\lambda\lambda'}(r) = E_{\kappa\lambda}^{(0)}(r) \delta_{\lambda\lambda'} + \frac{V_{\kappa\lambda\lambda'}^{(1)}(r)}{m_Q} + \dots \quad (5)$$

where $E_{\kappa\lambda}^{(0)}(r)$ denotes the static potential, and $V_{\kappa\lambda\lambda'}^{(1)}(r)$ can be written as sum of spin-dependent and spin-independent pieces [15]. The effective potential ΔV in Eq. (4) (that is treated as a perturbation) is responsible for producing transitions to standard quarkonium states and the form will be determined by performing a matching calculation in section 3. From now on, we ignore the subscript κ (as $\kappa = 1^{+-}$ for our purpose) and we write the hybrid wave-function as

$$\Psi_{\lambda}^{(m)}(\mathbf{r}) \equiv \Psi_{\lambda}^{(m,j,l,s)}(\mathbf{r}) = \psi_{\lambda}^{(m)}(r) \Phi_{(jls)}^{\lambda}(\theta, \phi), \quad (6)$$

where m is the principal quantum number, the quantum number j is the eigenvalue of the total angular momentum operator: $\mathbf{J} = \mathbf{L} + \mathbf{S}$, where \mathbf{S} is the $Q\bar{Q}$ spin, $\mathbf{L} = \mathbf{L}_{Q\bar{Q}} + \mathbf{K}$, where \mathbf{K} is the gluon angular momentum, and $\mathbf{L}_{Q\bar{Q}}$ is the angular momentum operator of the two heavy quarks. We use the notation $m(L_{Q\bar{Q}})_L$ to denote the hybrid state.

At leading order, the equations of motion for the fields $\Psi_{\lambda}^{(m)}(\mathbf{r})$ that follow from Eq. (4) are the set of coupled Schrödinger equations which are given by

$$\sum_{\lambda=0,\pm 1} \hat{\mathbf{r}}_{\lambda}^* \cdot \left(-\frac{\nabla_{\mathbf{r}}^2}{m_Q} + E_{\lambda}^{(0)}(r) \right) \hat{\mathbf{r}}_{\lambda} \Psi_{\lambda}^{(m)}(\mathbf{r}) = \mathcal{E}_m \Psi_{\lambda}^{(m)}(\mathbf{r}), \quad (7)$$

where \mathcal{E}_m is the eigenvalue. The hybrid mass is given by $M_{Q\bar{Q}g} = 2m_Q + \mathcal{E}_m$ for $Q = (c, b)$. Since, there are projection operators on both side of $\nabla_{\mathbf{r}}^2$ in Eq. (7), the contributions from Σ_u^- and Π_u potentials mix together that results in pairs of solutions with same angular momentum quantum number but opposite parity [13]. The static potentials that we use for computing the hybrid spectrum is split into a short-distance part and long-distance part [13]:

$$E_{\lambda}^{(0)}(r) = \begin{cases} V_o^{\text{RS}}(v_f) + \Lambda_H^{\text{RS}}(v_f) + b_{\lambda} r^2, & r < 0.25 \text{ fm} \\ \mathcal{V}(r), & r > 0.25 \text{ fm} \end{cases} \quad (8)$$

where for the short-distance part ($r < 0.25$ fm) we have used the RS-scheme octet potential $V_o^{\text{RS}}(r)$ up to order α_s^3 in perturbation theory and the RS-scheme gluelump mass $\Lambda_H^{\text{RS}} = 0.87(15)$

GeV at the renormalon subtraction scale $\nu_f = 1$ GeV [10, 21]. The RS-scheme octet potential is given by [10, 21]

$$V_o^{RS}(r, \nu_f, \mu) = V_o(r, \mu) - \delta V_o^{RS}(\nu_f), \quad (9)$$

with

$$V_o(r, \mu) = \left(\frac{C_A}{2} - C_F \right) \frac{\alpha_{V_o}(\mu)}{r}, \quad (10)$$

$$\delta V_o^{RS}(\nu_f) = \sum_{n=1}^{\infty} N_{V_o} \nu_f \left(\frac{\beta_0}{2\pi} \right)^n \alpha_s^{n+1}(\nu_f) \sum_{k=0}^{\infty} c_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}. \quad (11)$$

where μ denotes the energy scale, $N_{V_o} = 0.114001$, the parameters b and c_k are defined in Ref. [10]. The form of α_{V_o} up to order α_s^3 in perturbation theory is given in Ref. [22]

The long-distance ($r > 0.25$ fm) part of the potential $\mathcal{V}(r)$ is given by

$$\mathcal{V}(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4. \quad (12)$$

The above form of the long-distance potential $\mathcal{V}(r)$ is chosen so as to reproduce the short and long distance behavior of the Cornell potential. The parameters b_λ in Eq. (8) and a_1 , a_2 , a_3 and a_4 in Eq. (12) are different for both Σ_u^- and Π_u static potentials. The parameters are determined by performing a fit to the lattice data in Refs. [9, 10] and demanding that the short-range and the long-range pieces in Eq. (8) are continuous upto first derivatives (see Ref. [13] for details). The result for the spectrum is given in Table III of Ref. [13].

3 Inclusive Decay Rate

We want to compute the inclusive decay rate of low-lying quarkonium hybrids decaying to traditional quarkonium i.e, $H_m \rightarrow Q_n + X$, where H_m is a low-lying hybrid, Q_n is a low-lying quarkonium state and X denotes other final state particles. We denote the energy (mass) difference by $\Delta E = \mathcal{E}_m - E_n^Q \gtrsim 1$ GeV, which for low-lying hybrid and quarkonium states satisfy the following hierarchy of energy scales: $m_Q v \gg \Delta E \gg \Lambda_{\text{QCD}} \gg m_Q v^2$. This implies that the relevant theory at the energy scale ΔE is the weakly coupled pNRQCD effective theory which is obtained from NRQCD by integrating out gluons with momentum and energy of order $\sim m_Q v$ and quarks with energy of order $\sim m_Q v$. In order to describe the dynamics of two heavy quarks in the hybrids that happens at energy scale $m_Q v^2$, we will use the Born-Oppenheimer effective theory (BOEFT). Hence, starting with pNRQCD effective theory, we integrate out gluons with 4-momentum of order $\sim \Delta E$ and $\sim \Lambda_{\text{QCD}}$ in loops and match it to BOEFT that describes system at energy scale $m_Q v^2$. This matching condition leads to imaginary terms in the BOEFT potential which is related to the hybrid decay rate by the optical theorem.

The pNRQCD Lagrangian up to next-to-leading-order (NLO) in multipole expansion or in $1/m_Q$ is given by

$$L_{\text{pNRQCD}} = \int_{\mathbf{R}} \int_{\mathbf{r}} \left(\text{Tr} [S^\dagger (i\partial_0 - h_s) S + O^\dagger (iD_0 - h_o) O] + g \text{Tr} [S^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger \mathbf{r} \cdot \mathbf{E} S] \right. \\ \left. + \frac{g C_F}{m_Q} \text{Tr} [S^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} O + O^\dagger (\mathbf{S}_1 - \mathbf{S}_2) \cdot \mathbf{B} S] + \dots \right), \quad (13)$$

where $\int_{\mathbf{R}} \equiv \int d^3\mathbf{R}$, S and O denotes the singlet and the octet fields and ellipsis represents higher order terms as well as terms including light quarks and gluons. The singlet and octet Hamiltonian densities are given by

$$h_s = -\nabla_r^2/m_Q + V_s(r), \quad h_o = -\nabla_r^2/m_Q + V_o(r), \quad (14)$$

where $V_s(r)$ and $V_o(r)$ are the perturbative singlet and octet potentials. For our purpose, we use the following form for $V_s(r)$ and $V_o(r)$

$$V_s(r) = -\frac{\kappa_g}{r} + E_Q^s, \quad V_o(r) = V_o^{RS}(r), \quad (15)$$

where $V_o^{RS}(r)$ is given by Eq. (9), $\kappa_g = 0.489$, and the constant E_Q^s for $Q = (c, b)$ is chosen so as to reproduce the spin-averaged 1s charmonium and 1s bottomonium mass.

The $\mathbf{r} \cdot \mathbf{E}$ vertex in Eq. (13) is responsible for the spin-conserving decays of hybrid whereas the $\mathbf{S} \cdot \mathbf{B}/m_Q$ vertex is responsible for the spin-flipping decays of hybrid (spin-0 hybrid decaying to spin-1 quarkonium and vice versa). Therefore, the spin-flipping decays are suppressed by powers of the heavy quark mass due to heavy quark spin symmetry.

Beginning with pNRQCD, we integrate out gluons with 4-momentum $\sim \Delta E$ and $\sim \Lambda_{\text{QCD}}$ in two steps, and obtain the BOEFT theory that describes system at energy scale $m_Q v^2$. We perform this by implementing the matching condition wherein we compute the two-point Green's function in both the theories and equate them. For spin-preserving decays of hybrid to quarkonium, the two-point function in pNRQCD is expanded up to $\mathcal{O}(r^2)$ in the multipole expansion using the NLO pNRQCD Lagrangian in Eq. (13) which is equated to the corresponding two-point function in BOEFT computed using the Lagrangian in Eq. (4). For the spin-flipping decay of hybrids, the two-point function is expanded up to $\mathcal{O}(1/m_Q^2)$ using the pNRQCD Lagrangian in Eq. (13). After implementing this matching condition, we obtain the the following form of the effective potential ΔV in Eq. (4) for the spin-preserving decays

$$\Delta V = -\frac{ig^2 T_F}{3 N_c} \int_0^\infty dt e^{i\Lambda t} e^{ih_o t/2} r^k e^{-ih_s t} r^k e^{ih_o t/2} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} |\mathbf{k}| e^{-i|\mathbf{k}|t}. \quad (16)$$

In case of spin-flipping decays, the $\mathbf{r} \cdot \mathbf{E}$ vertex is replaced by the $\mathbf{S} \cdot \mathbf{B}/m$ vertex. Thus, using the optical theorem, the (spin-conserving) decay rate of the hybrids for the process $H_m \rightarrow Q_n + X$ is given by $\Gamma_{H_m \rightarrow Q_n} = -2\langle H_m | \text{Im} \Delta V | H_m \rangle$ (see details of the calculation in Ref. [23])

$$\Gamma(m \rightarrow n) = \sum_{n'} \left| \int d^3 \mathbf{r} \Phi_{(n')}^{s\dagger}(\mathbf{r}) \Phi_{(n)}^Q(\mathbf{r}) \right|^2 \Gamma_{mn'}, \quad (17)$$

where in the above expression we have included the overlap between the quarkonium ($\Phi_{(n)}^Q$) and the Coulomb singlet ($\Phi_{(n')}^s$) wave-functions (we use compact notation (n) and (n') to denote the set of quantum numbers for quarkonium and singlet state) and $\Gamma_{mn'}$ is given by

$$\begin{aligned} \Gamma_{mn'} &= \text{Re} \frac{8\pi\alpha_s T_F}{3N_c} \int_r \int_0^\infty dt \Psi_{(m)}^{i\dagger}(\mathbf{r}) e^{i\Lambda t} e^{ih_o t/2} r^k e^{-ih_s t} r^k e^{ih_o t/2} \Psi_{(m)}^i(\mathbf{r}) \int \frac{d^3 \mathbf{k}}{(2\pi)^3} |\mathbf{k}| e^{-i|\mathbf{k}|t}, \\ &= \frac{4\alpha_s T_F}{3N_c} \int \frac{d^3 \mathbf{l}}{(2\pi)^3} \int \frac{d^3 \mathbf{l}'}{(2\pi)^3} \int_r \int_{r'} \int_{r''} \int_{r'''} \left[\Psi_{(m)}^{i\dagger}(\mathbf{r}) \Phi_l^o(\mathbf{r}) \right] \left[\Phi_{l'}^{o\dagger}(\mathbf{r}') r'^i \Phi_{(n')}^s(\mathbf{r}') \right] \\ &\quad \times \left[\Phi_{(n')}^{s\dagger}(\mathbf{r}'') r''^i \Phi_{l'}^o(\mathbf{r}'') \right] \left[\Phi_{l''}^{o\dagger}(\mathbf{r}''') \Psi_{(m)}^i(\mathbf{r}''') \right] (\Lambda_{\text{glue}} + E_l^o/2 + E_{l'}^o/2 - E_n^s)^3, \quad (18) \end{aligned}$$

where $\int_r \equiv \int d^3 \mathbf{r}$, α_s is evaluated at the scale $\Delta E = \mathcal{E}_m - E_n^Q$, $\Psi_{(m)}^{i\dagger}$ is the hybrid wave-function given in Eq. (6) (i is the vector index, (m) denotes the set of quantum numbers), Φ_l^o is the octet wave-function, E_l^o is the octet energy, E_n^s is the singlet energy, and $\Lambda_{\text{glue}} = 0.87(15) \text{ GeV}$ in RS-scheme. In order to obtain the second line from the first line in Eq. (18), we expand the singlet (h_s) and octet (h_o) Hamiltonians in terms of their eigenfunctions $\Phi_{(n')}^s$ and Φ_l^o , which satisfy

$$h_s(\mathbf{r}, \mathbf{p}) \Phi_{(n')}^s(\mathbf{r}) = E_{n'}^s \Phi_{(n')}^s(\mathbf{r}), \quad h_o(\mathbf{r}, \mathbf{p}) \Phi_l^o(\mathbf{r}) = E_l^o \Phi_l^o(\mathbf{r}). \quad (19)$$

For computing the decay rates using Eq. (17), we need the octet and the singlet wave-functions. We use the singlet and octet potentials in Eq. (15) to compute Φ_l^o and $\Phi_{(n')}^s$.

Suppose, we assume: the singlet and quarkonium energies satisfy $E_n^Q \approx E_n^s$, overlap between hybrid and octet wave-functions $\int d^3r \Psi_{(m)}^{i\dagger}(\mathbf{r})\Phi_l^o(\mathbf{r})$ is nonzero only for hybrid energy \mathcal{E}_m : $\mathcal{E}_m = E_l^o + \Lambda_{\text{glue}}$ (where ignoring the $b_\lambda r^2$ term in Eq. (8)) such that the cubic factor within the integrand in Eq. (18) is replaced by a constant ΔE , and the overlap function of quarkonium and singlet wave-function satisfy $\int d^3r \Phi_{(n')}^{s\dagger}(\mathbf{r})\Phi_{(n)}^Q(\mathbf{r}) \approx 1$ for $(n') = (n)$, then Eq. (17) is simplified to

$$\Gamma^{\text{sim}}(m \rightarrow n) \approx \frac{4\alpha_s (\Delta E) T_F}{3N_c} \left[\int_r \Psi_{(m)}^{i\dagger}(\mathbf{r}) r^j \Phi_{(n)}^Q(\mathbf{r}) \right] \left[\int_r \Psi_{(m)}^{i\dagger}(\mathbf{r}) r^j \Phi_{(n)}^Q(\mathbf{r}) \right]^\dagger \Delta E^3, \quad (20)$$

The simplified decay rate in Eq. (20) is identical to Eq. (17) in Ref. [12] and Eq. (62) in Ref. [24]. However, in Ref. [12], the authors only consider the diagonal elements (where they contract the index i and j in Eq. (20)) instead of the full tensor structure. This led to a selection rule that hybrids with $L = L_{Q\bar{Q}}$ does not decay. This is incorrect as such decays are allowed by considering the tensor structure of the matrix element in Eq. (20). The results for the spin-conserving and spin-flipping decay rates are shown in tables 1 and 2. The spin-flipping decay rates in table 2 are suppressed by the two-powers of the heavy quark mass m .

In both tables 1 and 2, we see that for most of the cases, the values of the hybrid decay rate from the general expression involving overlap functions of octet and singlet state in Eq. (17) differs from that obtained from the simplified expression in Eq. (20) even after considering the error bars. We find that this difference is mainly due to contributions from the cubic factor within the integrand and the Coulomb singlet wave-functions in Eq. (18). Therefore, this raises the questions on the validity of the approximations that were used to obtain the simplified expression in Eq. (20). Specifically, We find that the approximation about the singlet and quarkonium energy $E_n^Q \approx E_n^s$ is only valid for 1s charmonium and bottomonium states and the overlap between hybrid and octet wave-function $\int d^3r \Psi_{(m)}^{i\dagger}(\mathbf{r})\Phi_l^o(\mathbf{r})$ is nonzero over wide range of octet energies, if we don't assume $\mathcal{E}_m \approx E_l^o + \Lambda_{\text{glue}}$ (see Ref. [23]).

4 Conclusions

In this work, we study the inclusive decays of heavy quark hybrids to traditional quarkonium by using the framework of Born-Oppenheimer effective field theory. We have derived an expression of the decay rate given in Eq. (17) that depends on the overlap functions of hybrid, octet, Coulomb singlet and quarkonium wave-functions. The values of the spin-conserving and the spin-flipping decay rates of hybrids are shown in tables 1 and 2. We also find that using certain assumptions, our expression for the decay rate in Eq. (17) reduces to a simplified expression given by Eq. (20) that was earlier derived in Refs. [12, 24]. However, the difference in the values of the decay rate from Eqs. (17) and (20) shown in tables 1 and 2 raises questions on the validity of those approximations.

Acknowledgements

We thank Nora Brambilla, Wai-kin Lai, and Antonio Vairo for advice and collaboration on this work. This work has been supported by the DFG Project-ID 196253076 TRR 110 and the NSFC through funds provided to the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD".

$m(L_{Q\bar{Q}})_L \rightarrow nL'$	ΔE (GeV)	$\alpha_s(\Delta E)$	Γ^{sim} (MeV)	Γ (MeV)
charmonium hybrid decay				
$1p_0 \rightarrow 1s$	1.522	0.298	327^{+137}_{-71}	117^{+49}_{-25}
$1p_0 \rightarrow 2s$	0.912	0.381	194^{+118}_{-53}	71^{+43}_{-19}
$2p_0 \rightarrow 1s$	1.986	0.269	45^{+16}_{-9}	15^{+5}_{-3}
$1p_1 \rightarrow 1s$	1.218	0.329	156^{+76}_{-37}	146^{+71}_{-35}
$2p_1 \rightarrow 1s$	1.599	0.292	65^{+27}_{-14}	9^{+4}_{-2}
$2(s/d)_1 \rightarrow 1p$	1.013	0.361	113^{+63}_{-29}	7^{+4}_{-2}
$4(s/d)_1 \rightarrow 1p$	1.381	0.311	99^{+44}_{-22}	8^{+4}_{-2}
bottomonium hybrid decay				
$1p_0 \rightarrow 1s$	1.622	0.290	69^{+28}_{-14}	102^{+41}_{-22}
$1p_0 \rightarrow 2s$	1.055	0.353	159^{+86}_{-40}	20^{+11}_{-5}
$2p_0 \rightarrow 1s$	1.909	0.273	34^{+12}_{-7}	15^{+5}_{-3}
$2p_0 \rightarrow 2s$	1.342	0.315	42^{+19}_{-10}	63^{+29}_{-14}
$3p_0 \rightarrow 1s$	2.174	0.261	19^{+6}_{-4}	12^{+4}_{-2}
$3p_0 \rightarrow 2s$	1.607	0.291	20^{+4}_{-8}	25^{+10}_{-5}
$4p_0 \rightarrow 1s$	2.421	0.251	12^{+4}_{-2}	7^{+2}_{-1}
$4p_0 \rightarrow 2s$	1.854	0.276	11^{+4}_{-2}	30^{+11}_{-6}
$1p_1 \rightarrow 1s$	1.404	0.309	29^{+13}_{-7}	80^{+35}_{-18}
$2p_1 \rightarrow 1s$	1.617	0.291	28^{+11}_{-6}	26^{+11}_{-6}
$3p_1 \rightarrow 1s$	1.828	0.277	22^{+8}_{-4}	16^{+6}_{-3}
$2(s/d)_1 \rightarrow 1p$	1.068	0.351	15^{+8}_{-4}	163^{+87}_{-41}
$3(s/d)_1 \rightarrow 1p$	1.264	0.324	73^{+35}_{-17}	90^{+43}_{-21}
$3(s/d)_1 \rightarrow 2p$	0.907	0.383	22^{+14}_{-6}	83^{+51}_{-23}
$4(s/d)_1 \rightarrow 1p$	1.300	0.320	155^{+72}_{-36}	103^{+48}_{-24}

Table 1: Preliminary results for the spin-conserving inclusive decay rate for hybrids decays to quarkonium states: $H_m \rightarrow Q_n + X$ due to $\mathbf{r} \cdot \mathbf{E}$ vertex in Eq. (13). The hybrid states are denoted by $m(L_{Q\bar{Q}})_L$ whereas the quarkonium states are denoted by nL' . The decay rate in fourth column is computed using Eq. (20) and in last column using Eq. (17). The values of $\alpha_s(\Delta E)$ are obtained using the RUNDEC code [25]. The upper error bar is from changing the scale to $\Delta E/2$ in α_s while the lower error bar is from changing the scale to $2\Delta E$ in α_s .

References

- [1] S. K. Choi *et al.* [Belle], “Observation of a narrow charmonium-like state in exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ decays,” *Phys. Rev. Lett.* **91**, 262001 (2003) doi:10.1103/PhysRevLett.91.262001.
- [2] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo and C. Z. Yuan, “The XYZ states: experimental and theoretical status and perspectives,” *Phys. Rept.* **873**, 1-154 (2020) doi:10.1016/j.physrep.2020.05.001.
- [3] S. L. Olsen, “A New Hadron Spectroscopy,” *Front. Phys. (Beijing)* **10**, no.2, 121-154 (2015) doi:10.1007/S11467-014-0449-6.
- [4] P. A. Zyla *et al.* [Particle Data Group], “Review of Particle Physics,” *PTEP* **2020**, no.8, 083C01 (2020) doi:10.1093/ptep/ptaa104.
- [5] W. E. Caswell and G. P. Lepage, “Effective Lagrangians for Bound State Problems in QED, QCD, and Other Field Theories,” *Phys. Lett. B* **167**, 437-442 (1986) doi:10.1016/0370-2693(86)91297-9.

$m(L_{Q\bar{Q}})_L \rightarrow NL'$	ΔE (GeV)	$\alpha_s(\Delta E)$	Γ^{sim} (MeV) (1 \rightarrow 0)	Γ^{sim} (MeV) (0 \rightarrow 1)	Γ (MeV) (1 \rightarrow 0)	Γ (MeV) (0 \rightarrow 1)
Charmonium hybrid decay						
$1p_0 \rightarrow 1p$	1.096	0.347	45.54 ^{+23.90} _{-11.37}	136.62 ^{+71.69} _{-34.10}	0.07 ^{+0.04} _{-0.02}	0.22 ^{+0.11} _{-0.05}
$2p_0 \rightarrow 1p$	1.560	0.295	1.66 ^{+0.69} _{-0.35}	4.98 ^{+2.06} _{-1.06}	0.06 ^{+0.02} _{-0.01}	0.18 ^{+0.07} _{-0.04}
$2p_0 \rightarrow 2p$	1.087	0.348	44.17 ^{+23.33} _{-11.07}	132.51 ^{+69.98} _{-33.20}	0.14 ^{+0.07} _{-0.04}	0.43 ^{+0.22} _{-0.11}
$3p_0 \rightarrow 1p$	1.979	0.270	0.73 ^{+0.26} _{-0.14}	2.18 ^{+0.77} _{-0.43}	0.07 ^{+0.02} _{-0.01}	0.21 ^{+0.07} _{-0.04}
$2p_1 \rightarrow 1p$	1.173	0.335	5.09 ^{+2.54} _{-1.23}	15.26 ^{+7.61} _{-3.69}	0.07 ^{+0.04} _{-0.02}	0.21 ^{+0.11} _{-0.05}
$3p_1 \rightarrow 1p$	1.542	0.296	2.05 ^{+0.86} _{-0.44}	6.16 ^{+2.57} _{-1.32}	0.07 ^{+0.03} _{-0.02}	0.22 ^{+0.09} _{-0.05}
$3p_1 \rightarrow 2p$	1.068	0.351	3.71 ^{+1.99} _{-0.94}	11.13 ^{+2.81} _{-5.96}	0.18 ^{+0.09} _{-0.04}	0.53 ^{+0.29} _{-0.13}
$1(s/d)_1 \rightarrow 1s$	1.087	0.348	34.53 ^{+18.23} _{-8.65}	103.60 ^{+54.70} _{-25.95}	11.37 ^{+6.00} _{-2.85}	34.11 ^{+18.00} _{-8.54}
$2(s/d)_1 \rightarrow 1s$	1.439	0.305	15.45 ^{+6.72} _{-3.42}	46.35 ^{+20.16} _{-10.26}	0.18 ^{+0.09} _{-0.04}	0.53 ^{+0.29} _{-0.13}
$3(s/d)_1 \rightarrow 1s$	1.744	0.282	0.20 ^{+0.08} _{-0.04}	0.59 ^{+0.23} _{-0.12}	0.51 ^{+0.20} _{-0.11}	1.53 ^{+0.59} _{-0.32}
Bottomonium hybrid decay						
$1p_0 \rightarrow 1p$	1.157	0.338	4.25 ^{+2.14} _{-1.03}	12.74 ^{+6.42} _{-3.10}	0.95 ^{+0.48} _{-0.23}	2.84 ^{+1.43} _{-0.69}
$2p_0 \rightarrow 1p$	1.444	0.305	0.82 ^{+0.36} _{-0.18}	2.46 ^{+1.07} _{-0.54}	0.11 ^{+0.05} _{-0.02}	0.34 ^{+0.15} _{-0.07}
$2p_0 \rightarrow 2p$	1.086	0.348	3.11 ^{+1.64} _{-0.78}	9.33 ^{+4.93} _{-2.34}	0.11 ^{+0.06} _{-0.03}	0.32 ^{+0.17} _{-0.08}
$3p_0 \rightarrow 1p$	1.708	0.285	0.32 ^{+0.12} _{-0.07}	0.95 ^{+0.37} _{-0.20}	0.14 ^{+0.05} _{-0.03}	0.41 ^{+0.16} _{-0.09}
$3p_0 \rightarrow 2p$	1.351	0.314	0.60 ^{+0.27} _{-0.14}	1.81 ^{+0.82} _{-0.41}	0.13 ^{+0.06} _{-0.03}	0.38 ^{+0.17} _{-0.09}
$4p_0 \rightarrow 1p$	1.955	0.271	0.16 ^{+0.06} _{-0.03}	0.47 ^{+0.17} _{-0.09}	0.03 ^{+0.01} _{-0.01}	0.10 ^{+0.03} _{-0.02}
$4p_0 \rightarrow 2p$	1.598	0.292	0.28 ^{+0.11} _{-0.06}	0.84 ^{+0.34} _{-0.18}	0.02 ^{+0.01} _{-0.003}	0.05 ^{+0.02} _{-0.01}
$1p_1 \rightarrow 1p$	0.938	0.376	1.84 ^{+1.09} _{-0.49}	5.51 ^{+3.28} _{-1.48}	1.21 ^{+0.72} _{-0.32}	3.63 ^{+2.16} _{-0.97}
$2p_1 \rightarrow 1p$	1.152	0.338	1.10 ^{+0.56} _{-0.27}	3.30 ^{+1.67} _{-0.80}	0.05 ^{+0.03} _{-0.01}	0.15 ^{+0.08} _{-0.04}
$3p_1 \rightarrow 1p$	1.362	0.313	0.60 ^{+0.27} _{-0.14}	1.80 ^{+0.81} _{-0.41}	0.12 ^{+0.06} _{-0.03}	0.37 ^{+0.17} _{-0.08}
$3p_1 \rightarrow 2p$	1.005	0.362	0.66 ^{+0.37} _{-0.17}	1.98 ^{+1.11} _{-0.52}	0.11 ^{+0.06} _{-0.03}	0.34 ^{+0.19} _{-0.09}
$1(s/d)_1 \rightarrow 1s$	1.343	0.315	2.89 ^{+1.31} _{-0.66}	8.66 ^{+3.94} _{-1.97}	3.51 ^{+1.59} _{-0.80}	10.52 ^{+4.78} _{-2.40}
$2(s/d)_1 \rightarrow 1s$	1.534	0.297	2.59 ^{+1.08} _{-0.56}	7.76 ^{+3.25} _{-1.67}	1.73 ^{+0.72} _{-0.37}	5.18 ^{+2.17} _{-1.12}
$2(s/d)_1 \rightarrow 2s$	0.967	0.369	0.09 ^{+0.05} _{-0.02}	0.28 ^{+0.16} _{-0.07}	0.61 ^{+0.35} _{-0.16}	1.82 ^{+1.05} _{-0.48}
$3(s/d)_1 \rightarrow 1s$	1.730	0.283	1.05 ^{+0.41} _{-0.22}	3.14 ^{+1.22} _{-0.65}	0.63 ^{+0.24} _{-0.13}	1.89 ^{+0.73} _{-0.39}
$3(s/d)_1 \rightarrow 2s$	1.163	0.337	0.08 ^{+0.04} _{-0.02}	0.25 ^{+0.12} _{-0.06}	0.08 ^{+0.04} _{-0.02}	0.23 ^{+0.11} _{-0.06}
$4(s/d)_1 \rightarrow 1s$	1.765	0.281	0.87 ^{+0.33} _{-0.18}	2.60 ^{+0.99} _{-0.53}	0.37 ^{+0.14} _{-0.08}	1.11 ^{+0.42} _{-0.23}

Table 2: Preliminary results for the spin-flipping inclusive decay rate for hybrids decays to traditional quarkonium states: $H_m \rightarrow Q_n + X$ due to due to $\mathbf{S} \cdot \mathbf{B}/m_Q$ vertex in Eq. (13). The decay rate in fourth and fifth column is computed using Eq. (20) and in last two column using Eq. (17). The notation (1 \rightarrow 0) denotes spin-1 hybrid decaying into spin-0 quarkonium while (0 \rightarrow 1) denotes spin-0 hybrid decaying into spin-1 quarkonium.

- [6] G. T. Bodwin, E. Braaten and G. P. Lepage, “Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium,” Phys. Rev. D **51**, 1125-1171 (1995) [erratum: Phys. Rev. D **55**, 5853 (1997)] doi:10.1103/PhysRevD.55.5853.
- [7] N. Brambilla, A. Pineda, J. Soto and A. Vairo, “Potential NRQCD: An Effective theory for heavy quarkonium,” Nucl. Phys. B **566**, 275 (2000) doi:10.1016/S0550-3213(99)00693-8.
- [8] N. Brambilla, A. Pineda, J. Soto and A. Vairo, “Effective Field Theories for Heavy Quarkonium,” Rev. Mod. Phys. **77**, 1423 (2005) doi:10.1103/RevModPhys.77.1423.
- [9] K. J. Juge, J. Kuti and C. J. Morningstar, “Gluon excitations of the static quark potential and the hybrid quarkonium spectrum,” Nucl. Phys. B Proc. Suppl. **63**, 326-331 (1998) doi:10.1016/S0920-5632(97)00759-7.

- [10] G. S. Bali and A. Pineda, “QCD phenomenology of static sources and gluonic excitations at short distances,” *Phys. Rev. D* **69**, 094001 (2004) doi:10.1103/PhysRevD.69.094001.
- [11] E. Braaten, C. Langmack and D. H. Smith, “Born-Oppenheimer Approximation for the XYZ Mesons,” *Phys. Rev. D* **90**, no.1, 014044 (2014) doi:10.1103/PhysRevD.90.014044.
- [12] R. Oncala and J. Soto, “Heavy Quarkonium Hybrids: Spectrum, Decay and Mixing,” *Phys. Rev. D* **96**, no.1, 014004 (2017) doi:10.1103/PhysRevD.96.014004.
- [13] M. Berwein, N. Brambilla, J. Tarrús Castellà and A. Vairo, “Quarkonium Hybrids with Nonrelativistic Effective Field Theories,” *Phys. Rev. D* **92**, no.11, 114019 (2015) doi:10.1103/PhysRevD.92.114019.
- [14] N. Brambilla, G. Krein, J. Tarrús Castellà and A. Vairo, “Born-Oppenheimer approximation in an effective field theory language,” *Phys. Rev. D* **97**, no.1, 016016 (2018) doi:10.1103/PhysRevD.97.016016.
- [15] N. Brambilla, W. K. Lai, J. Segovia, J. Tarrús Castellà and A. Vairo, “Spin structure of heavy-quark hybrids,” *Phys. Rev. D* **99**, no.1, 014017 (2019) [erratum: *Phys. Rev. D* **101**, no.9, 099902 (2020)] doi:10.1103/PhysRevD.99.014017.
- [16] S. M. Ryan *et al.* [Hadron Spectrum], “Excited and exotic bottomonium spectroscopy from lattice QCD,” *JHEP* **02**, 214 (2021) doi:10.1007/JHEP02(2021)214
- [17] G. S. Bali, S. Collins and C. Ehmman, “Charmonium spectroscopy and mixing with light quark and open charm states from $n_F=2$ lattice QCD,” *Phys. Rev. D* **84**, 094506 (2011) doi:10.1103/PhysRevD.84.094506.
- [18] L. Liu *et al.* [Hadron Spectrum], “Excited and exotic charmonium spectroscopy from lattice QCD,” *JHEP* **07**, 126 (2012) doi:10.1007/JHEP07(2012)126.
- [19] G. K. C. Cheung *et al.* [Hadron Spectrum], “Excited and exotic charmonium, D_s and D meson spectra for two light quark masses from lattice QCD,” *JHEP* **12**, 089 (2016) doi:10.1007/JHEP12(2016)089
- [20] G. Ray and C. McNeile, “Determination of hybrid charmonium meson masses,” [arXiv:2110.14101 [hep-lat]].
- [21] A. Pineda, “Determination of the bottom quark mass from the $\Upsilon(1S)$ system,” *JHEP* **06**, 022 (2001) doi:10.1088/1126-6708/2001/06/022.
- [22] B. A. Kniehl, A. A. Penin, Y. Schroder, V. A. Smirnov and M. Steinhauser, “Two-loop static QCD potential for general colour state,” *Phys. Lett. B* **607**, 96-100 (2005) doi:10.1016/j.physletb.2004.12.024.
- [23] N. Brambilla, W. K. Lai, A. Mohapatra, and A. Vairo, In preparation, preprint-no. TUM-EFT 164/21.
- [24] J. T. Castellà and E. Passemar, “Exotic to standard bottomonium transitions,” *Phys. Rev. D* **104**, no.3, 034019 (2021) doi:10.1103/PhysRevD.104.034019.
- [25] K. G. Chetyrkin, J. H. Kuhn and M. Steinhauser, “RunDec: A Mathematica package for running and decoupling of the strong coupling and quark masses,” *Comput. Phys. Commun.* **133**, 43-65 (2000) doi:10.1016/S0010-4655(00)00155-7.