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The Scalar and Tensor Glueball in Production and Decay

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Abstract

Evidence for the scalar and the tensor glueball is reported. The evidence stems from an analysis of BESIII data on radiative J/ψ data into $\pi^0\pi^0$, K_SK_S, $\eta\eta$, and $\phi\omega$ [1]. The coupled-channel analysis is contrained by a large number of further data. The scalar intensity is described by ten scalar isoscalar mesons, covering the range from $f_0(500)$ to $f_0(2330)$. Five resonances are interpreted as mainly-singlet states in SU(3), five as mainly-octet states. The mainly-singlet resonances are produced over the full mass range, the production of octet states is limited to the 1500 to 2100 MeV mass range and shows a large peak. The peak is interpreted as scalar glueball. Its mass, width and yield are determined to $\mathrm{M_{glueball}} = (1865 \pm 25)$ MeV, $\Gamma_{\mathrm{glueball}} = (370 \pm 50^{+30}_{-20})$ MeV, $\mathrm{Y}_{J/\psi\to\gamma \mathrm{G}_0} = (5.8 \pm 1.0) \cdot 10^{-3}$. The study of the decays of the scalar mesons identifies significant glueball fractions [2]. The tensor wave shows the $f_2(1270)$ and $f_2'(1525)$ and a small enhancement at $M = 2210 \pm 40$ MeV, $\Gamma = (355^{+60}_{-30})$ MeV [3]. An interpretation of these data is suggested.

1 Introduction

Nearly 50 years ago, Fritzsch and Gell-Mann proposed a new theory of strong interactions: Quantum Chromo Dynamics (QCD) was born [4, 5]. The new theory predicted not only $q\bar{q}$ mesons and qqq baryons but also allowed for the existence of quark-less particles called glueballs. Their existence is a direct consequence of the nonabelian nature of QCD and of confinement. First quantitative estimates of glueball masses were given in a bag model [6]. More reliable are calculations on a lattice where the scalar glueball is predicted to have a mass in the 1500 to 1800 MeV range [7–10]. Analytic approximations to QCD predict the scalar glueball at 1850 to 1980 MeV [11–13]. The tensor glueball is expected to have higher mass, with a mass gap of about 600 MeV. QCD sum rules predict a scalar glueball at about 1780 MeV and a tensor glueball 100 MeV higher [14]. We thus expect the mass of the scalar glueball to be between 1500 and 2000 MeV and a tensor glueball mass in the 1900 to 2600 MeV range. The mass of the pseudoscalar glueball is expected slightly above the tensor glueball.

Glueballs are embedded into the spectrum of isoscalar mesons. The scalar and tensor glueball have isospin $I = 0$, positive G-parity (decaying into an even number of pions), their parity P and their C-parity are positive, and their total spin J is 0 or 2: $(I^G)J^{PC} = (0^+)0^{++}$ or

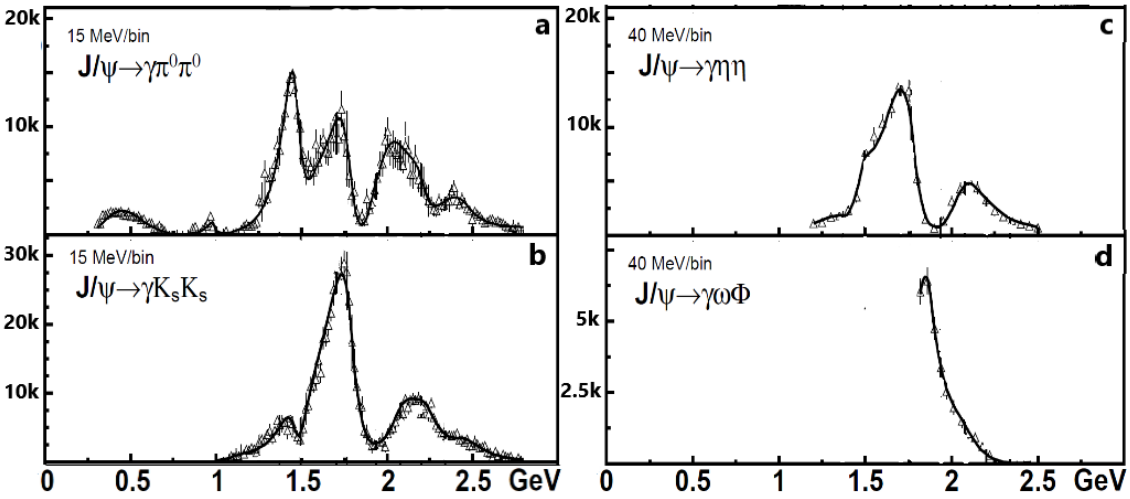

Figure 1: Number of events in the S-wave as functions of the two-meson invariant mass from the reactions $J/\psi \to \gamma\ \pi^0\pi^0$ (a), K_SK_S (b), $\eta\eta$ (c), $\phi\omega$ (d). (a) and (b) are based on the analysis of $1.3 \cdot 10^9$ J/ψ decays, (c) and (d) on $0.225 \cdot 10^9$ J/ψ decays. Adapted from [1].

$(0^+)2^{++}$. Glueballs have the same quantum numbers and may mix with them. Most claims for the scalar glueball are based on the observation of three scalar isoscalar resonances, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. In this mass range, two isoscalar tensor mesons are known, $f_2(1270)$ and $f_2'(1525)$ where $f_2(1270)$ consists mainly of light quarks ($n\bar{n}$) and $f_2'(1525)$ of strange quarks ($s\bar{s}$). Amsler and Close [15, 16] interpreted these three scalar mesons as mixed states of an $n\bar{n}$, $s\bar{s}$ and the scalar glueball (gg). Several authors suggested similar mixing schemes all based on the three resonances $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ (see [17] and refs. therein).

In this contribution, I present the results on a coupled-channel analysis of BESIII data on radiative J/ψ decays into $\pi^0\pi^0$ [18], K_sK_s [19], $\eta\eta$ [20], and $\omega\phi$ [21]. The results on $J/\psi \to \gamma 2\pi^+2\pi^-$ [22, 23] and $J/\psi \to \gamma\omega\omega$ [24] were included in the interpretation of the results. The analysis was constrained by a large number of further data: from the GAMS collaboration on the charge-exchange reactions $\pi^-p \to \pi^0\pi^0 n, \eta\eta n$ and $\eta\eta' n$ at 100 GeV/c in a mass range up to 3 GeV, BNL data on $\pi^-p \to K_SK_S n$, the CERN-Munich data on $\pi\pi \to \pi\pi$ elastic scattering, the low-mass $\pi\pi$ interactions from the K_{e4} of charged Kaons, and by 15 Dalitz plots on $\bar{p}N$ annihilation. The references to these data can be found elsewhere [1].

2 Radiative J/ψ decays

Radiative J/ψ decays are the prime reaction for searching for glueballs. Lattice gauge calculations predict a branching ratio for radiative J/ψ decays to produce the scalar glueball of $(3.8 \pm 0.9)10^{-3}$ [25] and the tensor glueball with a branching ratio of $(11 \pm 2)10^{-3}$ [26]. This is a significant fraction of all radiative J/ψ decays, (8.8±1.1)%.

The fit to the data – shown in Fig. 1 – requires five pairs of close-by isoscalar resonances. Their masses and widths are given in Table 1. Most resonances have been reported before: the five lower-mass resonances are included in the Meson Summary Table of the Review of Particle Physics [27], four states are not considered to be established, one is “new”. The agreement

Table 1: Pole masses and widths (in GeV) of scalar mesons. The small numbers give RPP values for comparison.

Name	$f_0(980)$	$f_0(1500)$	$f_0(1770)$	$f_0(2100)$	$f_0(2330)$
M [MeV]	1.014±0.008 0.990±0.020	1.483 ± 0.015 1.506 ± 0.006	1.765±0.015	2.075±0.020 $2.086^{+0.020}_{-0.024}$	2.340±0.020 ~2.330
Γ [MeV]	0.071±0.010 0.010→0.100	0.116±0.012 0.112±0.009	0.180±0.020	0.260±0.025 $0.284^{+0.060}_{-0.032}$	0.165±0.025 0.250±0.020
Name	$f_0(500)$	$f_0(1370)$	$f_0(1710)$	$f_0(2020)$	$f_0(2200)$
M [MeV]	0.410±0.020 0.400→0.550	1.370±0.040 1.200→1.500	1.700±0.018 1.704±0.012	1.925±0.025 1.992±0.016	2.200±0.025 2.187±0.014
Γ [MeV]	0.480±0.030 0.400→0.700	0.390±0.040 0.100→0.500	0.255±0.025 0.123±0.018	0.320±0.035 0.442±0.060	0.150±0.030 ~ 0.200

between our values and those reported earlier is rather good.

Oller has interpreted the $f_0(500)$ as mainly singlet state in SU(3), $f_0(980)$ as mainly octet state [28] (see also [29]). The interference between $f_0(1370)$ and $f_0(1500)$ in Fig. 2 (left) reveals a repetition of this pattern: $f_0(1370)$ is a singlet, $f_0(1500)$ is an octet state.

We now assume that the upper states in Table 1 are singlet states, the lower ones octet states. In Fig. 2 (right) we plot the squared meson masses as a function of a consecutive number. A linear relation is found with a slope of 1.1 GeV^{-2}. The separation is equal to the $\eta' - \eta$ mass square separation but reversed: the mainly singlet states are lower in mass than the mainly octet states. This pattern is expected for instant-induced interactions [30]. These states could have a glueball component; then they certainly have at least a singlet component. We define high-mass states (H) as resonances that have a mainly-octet $q\bar{q}$ configuration but that may additionally have a glueball component. The low-mass states (L) are mainly-singlet states.

3 The scalar glueball

Table 2 lists the yields of scalar mesons in radiative J/ψ decays in units of 10^{-5}. RPP numbers are also given for comparison but with two digits only, statistical and systematic uncertainties are added quadratically. The CERN-Munich data on elastic $\pi\pi$ scattering extend up to 1.9 GeV only; the missing intensity can hence be given only up to this mass.

The missing intensity is compared with the $\rho\rho$ and $\omega\omega$ yield in radiative J/ψ decays. The J/ψ yields for $f_0(1750)$ reported in the RPP should be compared to our sum for the yields of $f_0(1710)$ and $f_0(1770)$. The RPP presents yields for $f_0(2100)$ and $f_0(2200)$; they should be compared to the yields of our three high-mass states. The $J/\psi \to \gamma 4\pi$ yield [22, 23] is distributed among these three states.

Figure 3 (left) presents the total yield of H and L scalar mesons in radiative J/ψ decays. Both distributions show a significant yield at about 1900 MeV. The production of mainly-octet scalar mesons is surprising. The production is strong, it could be due to a singlet $q\bar{q}$ component but this hypothesis does not explain the peak structure. We assign the production of high-mass scalar mesons to their glueball component. Obviously, H and L scalar mesons have a glueball component of similar strength in their wave function.

Table 2: J/ψ radiative decay rates in units of 10^{-5} units. Mostly, small numbers represent the RPP values. Only the 4π decay modes gives our estimates derived from [22, 23]. The small numbers are given with two digits only; statistical and systematic errors are added quadratically. They are from RPP values or from Refs. [22, 23]. The missing intensities in parentheses are our estimates. The branching ratios for the $K\bar{K}$ decay are calculated from K_SK_S by multiplication with a factor 4. The RPP lists decays of $f_0(1710)$, $f_0(1750)$ and $f_0(1800)$. These values should be compared to the sum of our yields for $f_0(1710)$ and $f_0(1770)$. In the BES analysis [19], two scalar resonances are used, $f_0(1710)$ and $f_0(1790)$. The $K\bar{K}$ intensity is assigned to $f_0(1710)$. Similarly, the yield of the three high-mass states should be compared to the RPP values for $f_0(2100)$ or $f_0(2200)$.

$BR_{J/\psi\to\gamma f_0\to}$	$\gamma\pi\pi$	$\gamma K\bar{K}$	$\gamma\eta\eta$	$\gamma\eta\eta'$	$\gamma\omega\phi$	missing		total
						$\gamma 4\pi$	$\gamma\omega\omega$	
$f_0(2330)$	4±2	2.5±0.5	1.5±0.4					8±3
		20±3						
$f_0(2200)$	5±2	5±5	0.7±0.4			(38±13)		49±17
$f_0(2100)/f_0(2200)$	62±10	109^{+8}_{-19}	$11.0^{+6.5}_{-3.0}$			115±41		
$f_0(2100)$	20±8	32±20	18±15			(38±13)		108±25
$f_0(2020)$	42±10	55±25	10±10			(38±13)		145±32
$f_0(1770)$	24±8	60±20	7±1	2.5±1.1	22±4	65±15		181±26
$f_0(1750)$	38±5	99^{+10}_{-6}	24^{+12}_{-7}		25±6	97±18	31±10	
$f_0(1710)$	6±2	23±8	12±4	6.5±2.5	1±1	7±3		56±10
$f_0(1500)$	9.0±1.7	3±1	1.1±0.4	1.2±0.5	∼0	33±8		47±9
	10.9±2.4	2.9±1.2	$1.7^{+0.6}_{-1.4}$	$6.4^{+1.0}_{-2.2}$		36±9		
$f_0(1370)$	38±10	13±4	3.5±1	0.9±0.3	∼0	14±5		69±12
		42±15				27±9		
$f_0(980)$	1.3±0.2	0.8±0.3	∼0	∼0	∼0	∼0		2.1±0.4
$f_0(500)$	105±20	5±5	4±3	∼0	∼0	∼0		114±21

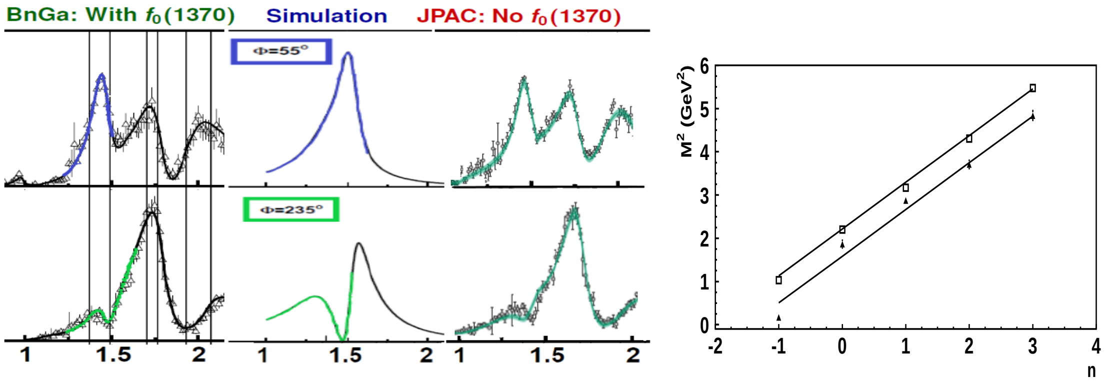

Figure 2: Left: Interference between $f_0(1370)$ and $f_0(1500)$: The BESIII data on $\pi\pi$ and $K\bar{K}$ are shown with the BnGa fit (left) and the JPAC fit (right). In the center, the interference of two Breit-Wigner amplitudes with masses and widths given in Table 1 is shown. A phase difference between the $\pi\pi$ and $K\bar{K}$ decay modes of 180° is required to reproduce the phase difference. One state is singlet in SU(3), the other one octet. Right: Squared masses of scalar isoscalar mesons as functions of a consecutive number n. The higher-mass states are interpreted as mainly-octet, the lower-mass states as mainly-singlet states. Figures from [1,2].

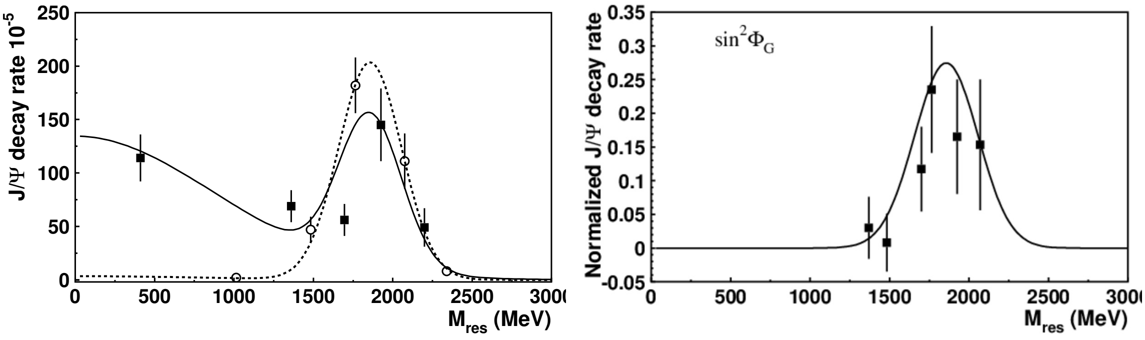

Figure 3: Left: Yield of radiatively produced scalar isoscalar "octet" mesons (open circles) and "singlet" (full squares) mesons. Right: Glueball component in the wave function.

The wave function of scalar states can be cast into the form

$$f_0^{\rm nH}(xxx) = \left(n\bar{n}\cos\varphi_{\rm n}^{\rm s} - s\bar{s}\sin\varphi_{\rm n}^{\rm s}\right)\cos\phi_{\rm nH}^{G} + G\sin\phi_{\rm nH}^{G}$$
$$f_0^{\rm nL}(xxx) = \left(n\bar{n}\sin\varphi_{\rm n}^{\rm s} + s\bar{s}\cos\varphi_{\rm n}^{\rm s}\right)\cos\phi_{\rm nL}^{G} + G\sin\phi_{\rm nL}^{G}$$

$\varphi_{\rm n}^{\rm s}$ is the scalar mixing angle, $\phi_{\rm nH}^{G}$ and $\phi_{\rm nL}^{G}$ are the meson-glueball mixing angles of the high-mass state H and of the low-mass state L in the nth nonet. The fractional glueball content of a meson is given by $\sin^2\phi_{\rm nH}^{G}$ or $\sin^2\phi_{\rm nL}^{G}$.

The $q\bar{q}$ part of the wave function of a scalar meson couples to the final states with the SU(3) structure constant γ_α and with a decay coupling strength c_n. The γ_α are shown in Fig. 4 as functions of the scalar mixing angle. The γ_α of a $q\bar{q}$ are, of course, identical for a singlet meson and a glueball. The glueball contents of all scalar mesons couple to the final states with one coupling constant c_G.

The coupling of a meson in nonet n to the final state α is given by

$$g_\alpha^{\rm n} = \quad c_{\rm n}\gamma_\alpha^{q} + c_G\gamma_\alpha^{G}$$

The fractional contributions of the glueball to the wave functions of scalar statesw were determined by a fit the coupling constants to the values derived from the PWA of the BESIII data:

$f_0(1370)$	$f_0(1500)$	$f_0(1710)$	$f_0(1770)$	$f_0(2020)$	$f_0(2100)$
(5±4)%	< 5%	(12±6)%	(25±10)%	(16±9)%	(17±8)%

The glueball is seen to be distributed among several states. The sum of the fractional contribution amounts (78±18)%. Another 10% could come be from the two higher mass states $f_0(2200)$ and $f_0(2330)$. Figure 3 exhibits the fractional contribution of the scalar mesons to the glueball. The contributions can be described by a Breit-Wigner function with mass and width $M = 1865$ MeV, $\Gamma = 370$ MeV, the area is normalized to one. Nearly the full glueball is assigned to the observed scalar states.

Further evidence for the glueball nature of the peak in Fig. 3 can be derived from a comparison of J/ψ radiative decays with the decay $\bar{B}_s \to J/\psi f_0$. Figure 5 shows the form factor [31] from production of scalar mesons in $J/\psi \to \gamma f_0$ and $\bar{B}_s \to J/\psi f_0$ decays [32,33]. The squared form factors are proportional to the yield.

The LHCb data demonstrate that the production of high-mass scalar states is strongly suppressed. The $f_0(980)$ is produced abundantly, there is some $f_0(1500)$ intensity but little production of scalar mesons above this mass. The $s\bar{s} \to f_0$ yield dies out rapidly with increasing mass. In contrast, two gluons couple strongly to high-mass scalar mesons. The difference is particularly large for the $f_0(1710)/f_0(1770)$ resonances in their $K\bar{K}$ decay. These two resonances decay strongly into $K\bar{K}$ but are not produced with $s\bar{s}$ in the initial state, only via two gluons.

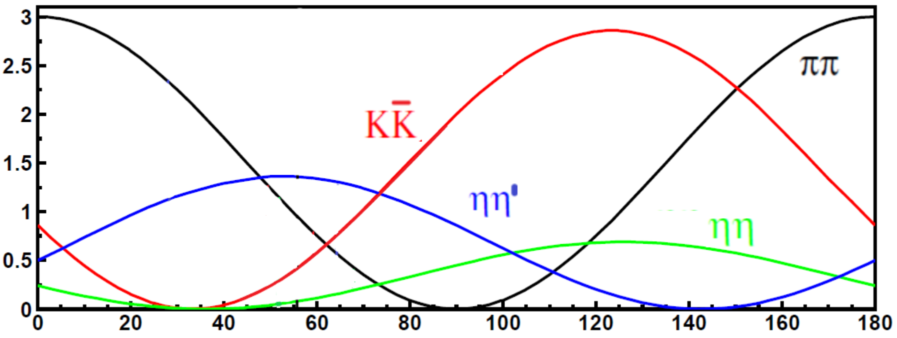

Figure 4: The SU(3) structure constants as functions of the mixing angle $\alpha = \varphi - 90°$. For $\alpha = 90°$, it is a $s\bar{s}$ state, for $\alpha = 0$, the meson is a $n\bar{n}$. Figure adapted from [2].

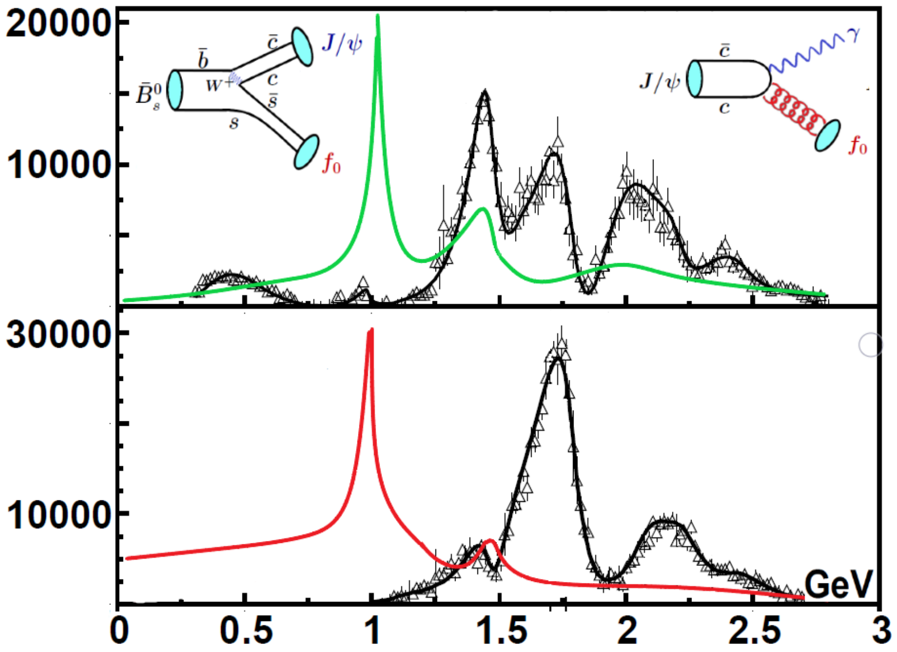

Figure 5: The BESIII data on $J/\psi \to \gamma\pi^0\pi^0$ and K_sK_s and pion and kaon form factor derived from LHCb data on $\bar{B}_s \to J/\psi\pi^+\pi^-$ and K^+K^-.

4 The tensor glueball

With a scalar glueball at 1865 MeV and its large yield in radiative J/ψ decays we must expect the tensor glueball with an even larger yield. The experimental mass distributions in the D-wave show large peaks due to $f_2(1270)$ and $f_2'(1525)$. In addition, there is a small but wide enhancement at $M = 2210 \pm 40$ MeV, $\Gamma = (355^{+60}_{-30})$ MeV. This could be the desired tensor glueball. To have the large expected yield, the resonance should have large unobserved decay modes. Certainly, significant more work is required to decide if this is the tensor glueball.

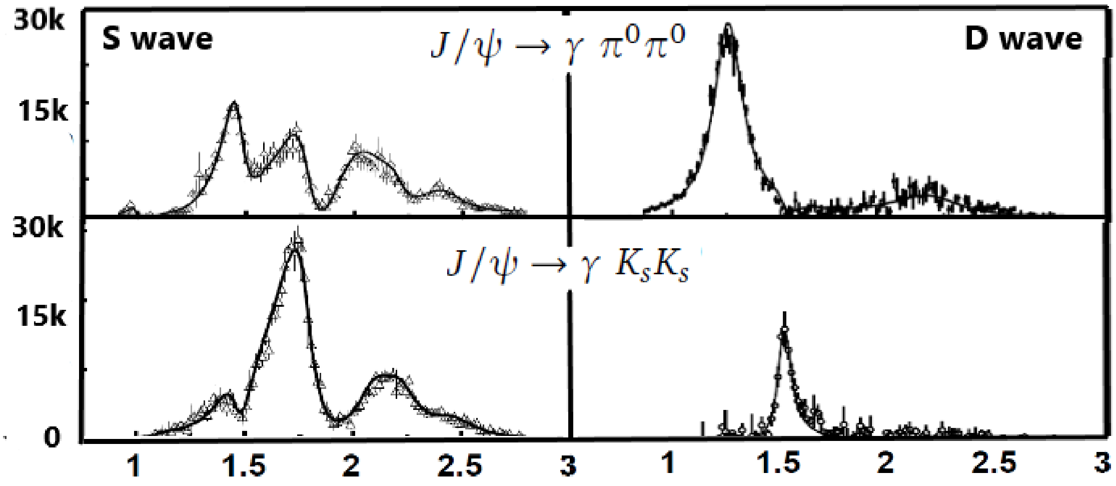

Figure 6: The scalar and tensor intensities in radiative J/ψ decays to $\pi^0\pi^0$ and K_sK_s.

5 Conclusion

In radiative J/ψ decays mainly-octet and mainly-singlet scalar mesons are produced abundantly. The yield of scalar mesons shows a peak structure; mainly-octet mesons are produced with no background, mainly-singlet mesons above a smooth background. The peak is fit with a Breit-Wigner shape with a pole at $M = (1865 \pm 25) - i(185 \pm 25^{+15}_{-10})\,MeV$. The yield is determined to $\mathrm{Y}_{J/\psi\to\gamma G_0} = (5.8 \pm 1.0)\cdot 10^{-3}$. The peak is interpreted as scalar glueball because of the following reasons:

1. Its mass is consistent with QCD predictions.
2. It is produced abundantly in radiative J/ψ decays where glueballs are expected.
3. The yield in radiative J/ψ decays is consistent with QCD predictions.
4. Several scalar mesons contribute to the glueball. Their decay modes require a glueball contribution.
5. The contributions of the observed scalar mesons to the glueball add up to (78±18)%. Higher-mass states are expected to contribute about 10%. The sum is nearly compatible with one, the full glueball is thus identified.
6. In decays $\bar{B}_s \to J/\psi$ plus scalar mesons, the reaction proceeds via $s\bar{s}$ in the initial state and scalar mesons of higher mass are produced at most weakly. In radiative J/ψ the reaction proceeds via two gluons in the initial state. Here, the yield of high-mass scalar mesons is siginicantly larger: the high mass scalar mesons are produced by two gluons and not by $s\bar{s}$ in the initial state.

The search for the tensor glueball in radiative J/ψ decays revealed a several 100 MeV wide peak of little intensity. This could be the tensor glueball but further studies are certainly required to establish its nature.

Acknowledgements

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