Masses of vector and pseudovector hybrid mesons in a chiral symmetric model

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Abstract

We enlarge the chiral model, the so-called extended Linear Sigma Model (eLSM), by including the low-lying hybrid nonet with exotic quantum numbers $J^{PC} = 1^{-+}$ and the nonet of their chiral partners with $J^{PC} = 1^{+-}$ to a global $U(3)_r \times U(3)_l$ chiral symmetry. We use the assignment of the $\pi_1^{hyb} = \pi_1(1600)$ as input to determine the unknown parameters. Then, we compute the lightest vector and pseudovector hybrid masses that could guide ongoing and upcoming experiments in searching for hybrids.

1 Introduction

The investigation of the properties of exotic quarkonia, the so-called hybrids, is extremely interesting and an important step toward the understanding of the nontraditional hadronic states, i.e., those structures beyond the normal meson and baryon, which are allowed in the framework of quantum chromodynamics (QCD) [1–3] and quark model [4, 5]. Hybrids are colour singlets and constitute of quark-antiquark pair and gluonic degree of freedom. In Lattice QCD, a rich spectrum of hybrid states are predicted below 5 GeV [6–8], but there are still no predominantly hybrid states assigned to be one of the listed mesons in the PDG [9]. Quite interestingly, recent results by COMPASS concerning the confirmation of the state $\pi_1(1600)$ with exotic quantum numbers 1^{-+} led to a revival of interest in this topic [10].

In this work, we investigate vector hybrids by enlarging the extended Linear Sigma Model (eLSM) [11]. In particular, we make predictions for a nonet of exotic hybrids with quantum numbers $J^{PC} = 1^{-+}$. Moreover, we also make predictions for the nonet of their chiral partners, with quantum numbers $J^{PC} = 1^{+-}$.

The eLSM has shown to be able to describe various hadronic masses and decays below 1.8 GeV, as the fit in Ref. [11] confirms, hence it represents a solid basis to investigate states that go beyond the simple $\bar{q}q$ picture. In the past, various non-conventional mesons were studied in the eLSM. Namely, the scalar glueball is automatically present in the eLSM as a dilaton and is coupled to light mesons: it represents an important element of the model due to the requirement of dilatation invariance (as well as its anomalous breaking) [12]. The eLSM has been used to study the pseudoscalar glueball [13–17], the first excited pseudoscalar glueball [18, 19], and hybrids [20]. Moreover, the connection and compatibility with chiral perturbation theory [21], as well as the extention to charmed mesons [22–29] and the inclusion of baryons in the so-called mirror assignment [30, 31] were performed.

In the present study, we extend the eLSM to hybrids by constructing the chiral multiplet for hybrid nonets with $J^{PC} = 1^{-+}$ and $J^{PC} = 1^{+-}$ and determine the interaction terms which satisfy chiral symmetry. Consequently, the spontaneous symmetry breaking is responsible for mass differences between the 1^{+-} crypto-exotic hybrids and the lower-lying 1^{-+}. We work out the masses of vector and pseudovector hybrid mesons.

2 Hybrid mesons in the chiral model

In this section, we enlarge the eLSM Lagrangian by including hybrid mesons in the case of $N_f = 3$

$$\mathcal{L}_{eLSM}^{\text{with hybrids}} = \mathcal{L}_{eLSM} + \mathcal{L}_{eLSM}^{\text{hybrid}} \tag{1}$$

where $\mathcal{L}_{eLSM}$ is the standard of the eLSM Lagrangain, which are constructed under chiral and dilatation symmetries, as well as their explicit and spontaneous breaking features (for more details see Refs. [11]).
We introduce the hybrids in the eLSM as:

$$\begin{aligned}\mathcal{L}_{eLSM}^{\text{hybrid}} &= \mathcal{L}_{eLSM}^{\text{hybrid-quadratic}} + \mathcal{L}_{eLSM}^{\text{hybrid-linear}} \\ &= \mathcal{L}_{eLSM}^{\text{hybrid-kin}} + \mathcal{L}_{eLSM}^{\text{hybrid-mass}} + \mathcal{L}_{eLSM}^{\text{hybrid-linear}}\end{aligned} \tag{2}$$

where the $\mathcal{L}_{eLSM}^{\text{hybrid-kin}}$ and $\mathcal{L}_{eLSM}^{\text{hybrid-linear}}$ terms are described in details in Ref. [20]. The masses of hybrids can be extracted from the following mass term

$$\begin{aligned}\mathcal{L}_{eLSM}^{\text{hybrid-mass}} =& m_1^{hyb,2}\frac{G^2}{G_0^2}\mathrm{Tr}\left(L_\mu^{hyb,2} + R_\mu^{hyb,2}\right) + \mathrm{Tr}\left(\Delta^{hyb}\left(L_\mu^{hyb,2} + R_\mu^{hyb,2}\right)\right) \\ &+ \frac{h_1^{hyb}}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}\left(L_\mu^{hyb,2} + R_\mu^{hyb,2}\right) + h_2^{hyb}\mathrm{Tr}[\left|L_\mu^{hyb}\Phi\right|^2 + \left|\Phi R_\mu^{hyb}\right|^2] \\ &+ 2h_3^{nyb}\mathrm{Tr}(L_\mu^{hyb}\Phi R^{hyb,\mu}\Phi^\dagger)\,.\end{aligned} \tag{3}$$

which satisfies both chiral and dilatation invariance. G is the dilaton field and G_0 its vacuum's expectation value. The multiplet of the scalar and pseudoscalar mesons, Φ, is defined as

$$\Phi = S + iP = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} + i\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}, \tag{4}$$

and transforms under chiral transformations $U_L(3) \times U_R(3)$: $\Phi \to U_L \Phi U_R^\dagger$, where U_L and U_R are $U(3)$, under parity $\Phi \to \Phi^\dagger$ and under charge conjugation $\Phi \to \Phi^t$.
(i) The scalar fields are $\{a_0(1450), K_0^*(1430), \sigma_N, \sigma_S\}$ with quantum number $J^{PC} = 0^{++}$ [9], and lie above 1 GeV [11], where the non-strange bare field $\sigma_N \equiv \left|\bar{u}u + \bar{d}d\right\rangle/\sqrt{2}$ corresponds predominantly to the resonance $f_0(1370)$ and the bare field $\sigma_S \equiv |\bar{s}s\rangle$ predominantly to $f_0(1500)$. Finally, in the eLSM the state $f_0(1710)$ is predominantly a scalar glueball, see details in Ref. [12]. (ii) The pseudoscalar fields are $\{\pi, K, \eta, \eta'\}$ with quantum numbers $J^{PC} = 0^{-+}$ [9], where η and η' arise via the mixing $\eta = \eta_N \cos\theta_p + \eta_S \sin\theta_p$, $\eta' = -\eta_N \sin\theta_p + \eta_S \cos\theta_p$ with $\theta_p \simeq -44.6°$ [11].

We now turn to the right-handed and left-handed, $R_\mu^{hyb}$and L_μ^{hyb}, combinations of exotic hybrid states, which combine the vector fields in the hybrid sector $\Pi_{ij}^{hyb,\mu}$ with the pseudovector fileds in the hybrid sector $B_{ij}^{hyb,\mu}$.
The hybrid sector $\Pi_{ij}^{hyb,\mu}$ is vector currents with one additional gluon with quantum numbers $J^{PC}=1^{-+}$, and is given by

$$\Pi_{ij}^{hyb,\mu}=\frac{1}{\sqrt{2}}\bar{q}_j G^{\mu\nu}\gamma_\nu q_i=\Pi^{hyb,\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_{1,N}^{hyb}+\pi_1^{hyb,0}}{\sqrt{2}} & \pi_1^{hyb+} & K_1^{hyb+} \\ \pi_1^{hyb-} & \frac{\eta_{1,N}^{hyb}+\pi_1^{hyb,0}}{\sqrt{2}} & K_1^{hyb,0} \\ K_1^{hyb,-} & \bar{K}_1^{hyb,0} & \eta_{1,S}^{hyb} \end{pmatrix}^\mu , \tag{5}$$

where the gluonic field tensor $G^{\mu\nu}$ is equal to $\partial^\mu A^\nu - \partial^\mu A^\nu - g_{QCD}[A^\mu, A^\nu]$, and $\Pi^{hyb,\mu}$ contains $\{\pi(1600), K_1(?), \eta_1(?), \eta_1(?)\}$ which only the isovector member corresponds to a physical resonance at the present. The exotic hybrid field π_1 is assigned to $\pi_1(1600)$, (the details of this assignment are given in Ref. [32]). There are not yet candidates for the other members of the nonet, but we shall estimate their masses in Sec. 3.
The pseudovector fields, $B_{ij}^{hyb,\mu}$ in the hybrid sector, after including the gluon field, with quantum numbers $J^{PC}=1^{+-}$, is written as

$$B_{ij}^{hyb,\mu}=\frac{1}{\sqrt{2}}\bar{q}_j G^{\mu\nu}\gamma^5\gamma_\nu q_i=B^{hyb,\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{h_{1N,B}^{hyb}+b_1^{hyb,0}}{\sqrt{2}} & b_1^{hyb,+} & K_{1,B}^{hyb+} \\ b_1^{hyb,+} & \frac{h_{1N,B}^{hyb}-b_1^{hyb,0}}{\sqrt{2}} & K_{1,B}^{hyb0} \\ K_{1,B}^{hyb-} & \bar{K}_{1,B}^{hyb0} & h_{1S,B}^{hyb} \end{pmatrix}^\mu . \tag{6}$$

The nonet $B_{ij}^{hyb,\mu}$ has not yet any experimental candidate. So, all fields $\{b_1(?), K_{1,B}(?), h_1(?), h_1(?)\}$ are unkown yet. In the lattice calculation of Ref. [7], an upper limit of about 2.4 GeV is reported, but lattice simulation still used a quite large pion mass. We estimate the mass of the b_1^{hyb} state, the chiral partner of π_1, to a value of about (or eventually somewhat larger than) 2 GeV. For definiteness, we shall assign it to an hypothetical state $b_1(2000?)$ state. The other member masses of the pseudovector crypto-exotic nonet follow as a consequence of this assumption. One can obtain the right-handed and left-handed currents as follows

$$R_\mu^{hyb}=\Pi^{hyb,\mu}-B_{ij}^{hyb,\mu} \text{and } L_\mu^{hyb}=\Pi^{hyb,\mu}+B_{ij}^{hyb,\mu}$$

and transform as $R_\mu^{hyb}\to U_R R_\mu^{hyb} U_R^\dagger$ and $L_\mu^{hyb}\to U_L L_\mu^{hyb} U_L^\dagger$ and under parity as $R_\mu^{hyb}\to L^{\mu,hyb}$ and $L_\mu^{hyb}\to R^{\mu,hyb}$ as well as under C as $R_\mu^{hyb}\to L^{hyb,\mu,t}$ and $L_\mu^{hyb}\to R^{hyb,\mu,t}$. See Ref. [20] for more details and discussions.

3 Masses of hybrids

Masses of hybrids can be calculated from the expression (3) by taking into account that the multiplet of the scalar and pseudoscalar fields, Φ, has a nonzero condensate or vacuum's expectation value. Consequently, the spontaneous symmetry breaking is reflected from that condensate. Especially relevant is the term h_3^{nyb} which generates a mass difference between the 1^{-+} and 1^{+-} hybrids, after shifting the latter masses upwards (see Ref. [20]). Note, the second term breaks explicitly flavor symmetry (direct contribution to the masses due to nonzero bare quark masses):

$$\Delta^{hyb}=diag\{\delta_N^{hyb},\delta_N^{hyb},\delta_S^{hyb}\}. \tag{7}$$

After a straightforward calculation, the (squared) masses of the 1^{-+} exotic hybrid mesons and the (squared) masses of the cryptoexotic pseudovector hybrid states were obtained as seen in Ref. [20]. Consequently, one can get the (exact) relations as

$$m^2_{b_1^{hyb}} - m^2_{\pi_1} = -2h_3^{hyb}\phi_N^2 \tag{8}$$

$$m^2_{K_{1,B}^{hyb}} - m^2_{K_1} = -\sqrt{2}\phi_N\phi_S h_3^{hyb} \tag{9}$$

$$m^2_{h_{1S}^{hyb}} - m^2_{\eta_{1,S}} = -h_3^{hyb}\phi_S^2 \tag{10}$$

As seen in Eqs. (8-10), the parameter h_3^{hyb} is the only parameter responsible for the mass splitting of the hybrid chiral partners. After fixing all the parameters that appear in the Lagrangian (3) and the square masses equations (see details in Ref. [20]), we obtain the following results (shown in Table 1) for the masses of the vector and pseudovector hybrid mesons:

Resonance	$Mass[MeV]$
Π_1^{hyb}	1600 [input using $\pi_1(1600)$] []
$\eta_{1,N}^{hyb}$	1660
$\eta_{1,S}^{hyb}$	1751
K_1^{hyb}	1707
b_1^{hyb}	2000 [input set as an estimate]
$h_{1N,B}^{hyb}$	2000
$K_{1,B}^{hyb}$	2063
$h_{1S,B}^{hyb}$	2126

Table 1: Masses of the exotic $J^{PC} = 1^{-+}$ and $J^{PC} = 1^{+-}$ hybrid mesons.

4 Conclusion

We have enlarged a chiral model, the so-called eLSM, in the case of $N_f = 3$ by including the hybrid state, the lightest hybrid nonet with $J^{PC} = 1^{-+}$ and of its chiral partner with $J^{PC} = 1^{+-}$, into a chiral multiplet. The eLSM implements the global chiral $U(N_f)_r \times U(N_f)_l$ symmetry and the symmetries of QCD: the discrete T, P, and C symmetries. The global chiral symmetry is broken in several ways: explicitly through non-vanishing quark masses, spontaneously due to the chiral condensate, and at the quantum level due to the chiral anomaly. To our knowledge, this is the first time that a model was constructed, which contains vector and pseudovector hybrid mesons. The resonance π_1^{hyb} is assigned to $\pi_1(1600)$ (with mass 1660^{+15}_{-11} MeV) and b_1^{hyb} is set to 2 GeV. The masses of the other hybrid states are computed and their results are reported in Table 1. Note that our model predicts the mass of the state η_1^{hyb} to be the same as $\pi_1^{hyb} \equiv \pi(1600)$ because of the small mixing of the nonstrange-strange quarks, which is in agreement with the homochiral nature of the chiral multiplet. Moreover, the calculation and the results of the decay widths of the lightest vector and pseudovector hybrid mesons are presented in Ref. [20].

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