Effects of Hawking evaporation on PBH distributions

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March 17, 2022

Abstract

Primordial black holes (PBHs) lose mass by Hawking evaporation. For sufficiently small PBHs, they may lose a large portion of their formation mass by today, or even evaporate completely if they form with mass $M < M_{\rm crit} \sim 5\times10^{14}$ g. We investigate the effect of this mass loss on extended PBH distributions, showing that the shape of the distribution is significantly changed between formation and today. We reconsider the γ-ray constraints on PBH dark matter in the Milky Way center with a correctly 'evolved' lognormal distribution, and derive a semi-analytic time-dependent distribution which can be used to accurately project monochromatic constraints to extended distribution constraints. We also derive the rate of black hole explosions in the Milky Way per year, finding that although there is a significant number, it is extremely unlikely to find one close enough to Earth to observe. Along with a more careful argument for why monochromatic PBH distributions are unlikely to source an exploding PBH population today, we (unfortunately) conclude that we are unlikely to witness any PBH explosions.

Contents

1 Introduction

Primordial black holes (PBHS) [1–4] are one of the earliest and most intriguing dark matter candidates. With the recent direct observations of black holes [5–7], PBHs could be considered to be back in the limelight as a popular dark matter candidate. However, the fraction $f_{\rm PBH}$ of the dark matter energy density in PBHs is constrained by a wide range of observations across the PBH mass spectrum [8,9]. Typically, these constraints are given for *monochromatic* dark matter distributions, but there has been growing interest in studying extended mass distributions [10–14].

Black holes are understood to have a temperature proportional to the surface gravity at the horizon and so lose mass by Hawking radiation [15,16]. The black holes radiate a thermal spectrum consisting of all particles with a mass below this surface temperature, with emission rate,

$$\frac{d^2N_i}{dt dE} = \frac{1}{2\pi}\sum_{\rm dof}\frac{\Gamma_i(E,M,a^*)}{e^{E'/T}\pm 1}\,, \tag{1}$$

where N_i is the number of particles emitted, Γ_i is the 'greybody factor', E' is the energy of the particle (including the BH spin), a^* is the reduced spin parameter, the sum is over the degrees of freedom of the particle (including color and helicity), and the $\pm$ sign accounts for fermions and bosons respectively. For large black holes, mass loss from Hawking evaporation is negligible over their lifetime. For sufficiently small black holes, however, the effect of Hawking evaporation is large. These PBHs may lose a significant portion of their mass by today, or even evaporate completely (possibly leaving some small remnant behind). Black holes which evaporate exactly with the lifetime of the universe are called 'critical mass' black holes, forming with a mass $M_{\rm crit} \sim 5\times 10^{14}$g. In this paper we will explore the effect that Hawking evaporation has on extended PBH distributions, centered near this critical mass. Throughout, we use the public code BlackHawk [17,18] to calculate lifetimes and emission spectra of the primordial black holes.

First we look at monochromatic PBH distributions at masses just slightly above the critical mass. These could leave behind a sizeable remnant population of tiny black holes today, with masses $M < M_{\rm crit}$. We will argue that this scenario, however, requires a very high level of fine-tuning for the initial black hole mass, and so there is a kind of 'stability' bound disqualifying such a remnant population[1]. Since the bounds on PBH dark matter are always given in terms of the black hole initial mass, this small section offers a more satisfying answer to the question of what comprises the dark matter *today*.

In the second part of this paper, we explore extended mass distributions with central mass near the critical mass. Since black holes of different masses evolve at different rates, these extended mass distributions evolve non-trivially from their formation time until today. That means that a distribution which is e.g. lognormal at PBH formation, has quite a different shape today—we will refer to this as the 'evolved' distribution, which we derive explicitly. Often, constraints on extended distributions are derived by 'adapting' the monochromatic constraints with a kind of interpolation [10]. However, it was pointed out in Ref. [13] that this method does not work for small PBH masses, for the above reason—the distribution changes over time. We show that using the correct evolved distribution, however, allows us to still use the method of Ref. [10] to derive correct constraints. In particular, we rederive the constraints on galactic center γ-rays detected by HESS and Fermi [20–22] for a lognormal extended distribution, showing the rather large effect of properly evolving the PBH distribution (and agreeing with the isotropic γ-ray constraints found numerically in Ref. [13]).

In the final portion of this paper, we investigate the 'exploding' tail of the tiniest black holes in the evolving distribution, and calculate the rate of black hole explosions over time. We find that there are a significant number of black hole explosions in the Milky Way every year—

[1]There are a number of constraints that already exist for monochromatic PBH distributions which form just above the critical mass. However, it is still instructional to consider this argument which so often goes unstated. If, for instance, Hawking radiation proceeded much faster than expected in the Schwarzschild case, this argument would still hold while the aforementioned constraints might not; see, e.g., Ref. [19].

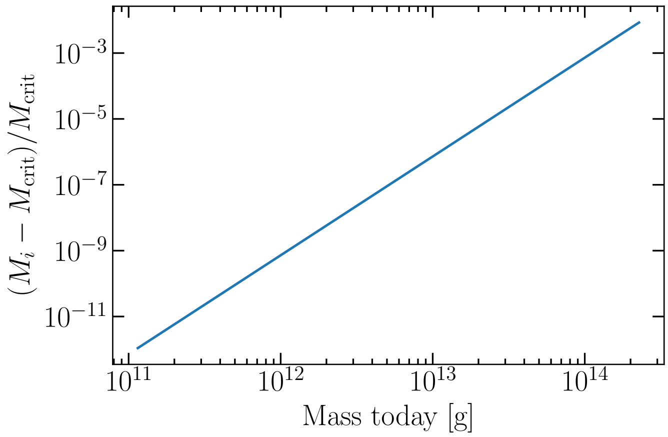

Figure 1: For a range of black hole masses today, we show the relative difference between their mass at formation and the critical black hole mass. This difference is so small that we would need incredible fine-tuning to consistently produce these populations today, far beyond the precision with which we are even able to perform these calculations. Below 10^{11} g, the black hole lifetime is less than a year, making such a scenario even more unreasonable (unless we consider ourselves to be in an extra-special period where all the PBHs are about to explode, any day now).

however, the expected distance to the nearest black hole explosion from Earth is sufficiently large that the photon flux is probably too small to witness one of these transient events, without exceptional luck. Nonetheless, it is interesting to consider the possible signal from such an event—since black holes evaporate with the entire particle spectrum, witnessing such an explosion could have profound science consequences [15, 16, 23, 24].

This paper is structured as follows. In Section 2 we address the question of whether there could be a monochromatic spectrum of PBHs today with masses $m < M_{\rm crit}$. In Section 3, we derive the evolving PBH distribution, before calculating the γ-ray bounds for a lognormal distribution in Section 4. Finally, we examine the PBH explosion rate in Section 5, and conclude in Section 6.

2 The monochromatic stability constraint

It is often stated that 'black holes with masses $m < M_{\rm crit}$ cannot be the dark matter, since they evaporate before today'. This statement is technically true, when considering the mass at formation. But there remains the question—can a monochromatic distribution with mass slightly larger than the critical mass leave behind a sizeable population today of very tiny black holes? Constraints from Hawking emissions already do exist for such a population. However, this question can be answered on more theoretical grounds, without reference to specific observations.

Consider the scenario where there is a remnant population today of black holes of masses $m < M_{\rm crit}$. If the black holes had mass 1.1×10^{11} g, the mass of these black holes at formation would have been 7.4×10^{14} g (very close to the critical mass). However, if the population had mass 1.1×10^{14} g today, the initial mass would have been just 7.5×10^{14} g. Clearly, there is extremely little difference between these two initial populations. For these two examples, we can compute $\Delta M_{\rm init}/M_{\rm crit} \sim 0.01$—so there is roughly a 1% difference in formation mass for a three orders-of-magnitude difference in black hole mass today. In Fig. 1, we show this percentage difference for a range of black hole masses today, using $M_{\rm crit}$ as the 'reference' mass. We can see that minuscule changes in formation mass have drastic impact on the remnant mass today. As a result, we have a kind of 'stability' constraint on a theory which predicts a specific range of black holes today with $M < M_{\rm crit}$, since it is so sensitive to the initial conditions—the precision required to source such a population would be smaller than the theoretical and observational uncertainties in our calculation.

Essentially, we can not expect to find a rapidly evaporating monochromatic distribution of black holes today. However, if the distribution was instead extended, this becomes a more interesting question to ask, since the tail of an extended distribution near the critical mass would seed an evaporating PBH population today. We investigate such a distribution in the following sections.

3 Evolving PBH distributions

With the increased interest in primordial black holes in recent years, constraining extended distributions has become a more pressing task. In Ref. [10], Carr et al. derived the constraints on extended distributions by interpolating the monochromatic distribution constraints. In Ref. [13], Arbey et al. argue that this method will not work for small black holes, since Hawking evaporation changes the PBH distribution between formation and today. Arbey et al. rederived the PBH constrains from isotropic gamma rays numerically, by simulating the evaporation of a number of black holes using the program BlackHawk [17, 18]. Here, we will show that the method of Carr can still be rescued for evolved distributions, as long as one uses the correct distribution at relevant epochs. We show how to derive this distribution and later rederive the galactic center γ-ray bounds. We define the fraction of total PBHs in the range $[M, M + \mathrm{d}M]$ as,

$$\phi(M) \equiv \frac{1}{n_{\rm BH}} \frac{\mathrm{d}n(M)}{\mathrm{d}M} \, , \tag{2}$$

where $n_{\rm BH}$ is the total PBH number density and $n(M)$ is the number density of PBHs in the mass range $[M, M + \mathrm{d}M]$. The physical interpretation of ϕ is that if you had a population of a certain number of black holes, the fraction of this population in a particular mass range (by number) can be found by integrating ϕ over the mass range. This is to be compared to the often-used definition $\psi \equiv M\mathrm{d}n/\mathrm{d}M$, which would give you the fraction of energy density in some mass range. Defining the quantity as in Eq. 2 is perhaps more useful, however, in our case. This is because the Hawking process happens to each black hole separately, rather than to the black hole population as a whole. Then $\phi(M)$, at PBH formation, would be normalized as,

$$\int \mathrm{d}M \; \phi(M) = 1 \, , \tag{3}$$

and we could compute,

$$\rho_{\rm BH} = n_{\rm BH} \int \mathrm{dM} \; M\phi(M) \, , \tag{4}$$

for some choice of volume V. However, we are interested in the time evolution of this distribution. In this case, the fraction of black holes in the range $[M, M + \mathrm{d}M]$ at a particular time has two arguments, $\phi(M, t)$. We assume that the initial distribution $\mathrm{d}n(M)/\mathrm{d}M$ is fixed by whatever physics produces the PBHs, and from then on is able to evolve. Then the fraction at

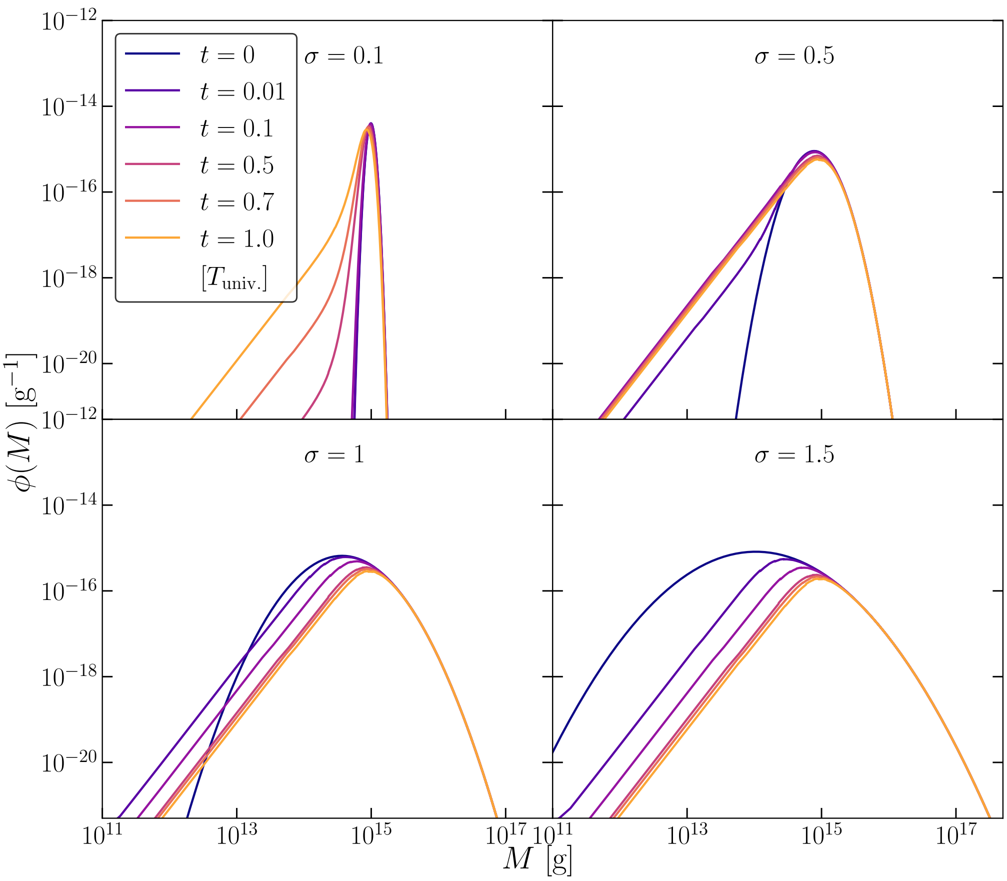

Figure 2: Four initial distributions with $M_* = 10^{15}$ g and different σ, evolved to later times. It is evident that the low-mass tail tends towards a slope $\propto M^2$, which means a suppression of masses around the peak, and an enhancement at low masses (the $\sigma = 1.5$ case is too wide to show this behaviour within the bounds of our data). Note that t is in units of the age of the universe.

a particular time is given by,

$$\phi(M,t) = \phi(M_0(M,t),t_0)\frac{\mathrm{d}M_0(M,t)}{\mathrm{d}M}\ , \tag{5}$$

where $M_0(M,t)$ is the formation mass corresponding to a black hole of mass M at time t, and t_0 is the time of formation. The second term can be thought of as a change of variable, since we need to preserve $\phi(M)\mathrm{d}M = \phi(M_0)\mathrm{d}M_0$.

Let us then compute the time-dependent distribution, assuming black holes without spin or charge (the arguments will not change drastically with the inclusion of this complication). Also, although black holes of different sizes form at different epochs in the early universe, accounting for this properly will only have a minuscule effect on the distribution, since we are considering black hole evolution times on the scale of the age of the universe. For simplicity, we can then define the time of formation as $t = 0$. The Hawking mass-loss equation [25] is given by,

$$\frac{dM}{dt} = -\frac{\hbar c^4}{G^2}\frac{\alpha}{M^2}, \tag{6}$$

where α is a coefficient depending on which species are possible to emit, which is determined by the mass. A black hole will spend the majority of its lifetime near its initial mass, so $\alpha \approx \alpha_0$ is a sufficiently good approximation for our purposes and allows for the analytic solution of the differential equation,

$$M(t) = \left(M_0^3 - 3\alpha_0\frac{\hbar c^4}{G^2}t\right)^{1/3}, \qquad t \leq \tau. \tag{7}$$

This equation can trivially be inverted to calculate the initial mass M_0 for a black hole of mass M at time t after formation:

$$M_0(M,t) = \left(M^3 + 3\alpha_0\frac{\hbar c^4}{G^2}t\right)^{1/3}\ . \tag{8}$$

However, determining α_0 is generally complicated, and so does not make for a nice analytic solution. One could use the 'classical' value $\alpha_{\text{classical}} = 1/15360\pi$, but this is not particularly accurate, since it only accounts for photon radiation. In order to proceed semi-analytically, we use the approximation $\alpha_0 = \alpha_{\text{eff}}$, which we define as,

$$\alpha_{\text{eff}} \equiv \frac{G^2}{\hbar c^4}\frac{M_0^3}{3\tau}\ , \tag{9}$$

where τ is the black hole lifetime, calculated numerically with BlackHawk. This means that α_{eff} guarantees we obtain the correct lifetime for any initial mass. We find that using this effective parameter instead of the numerical value gives a correct evolved mass to within a few percent at all times for the majority of initial masses. We show a plot of α_{eff} in Fig. 3, which must be derived numerically. If one is desperate for a purely analytical evolved distribution, this could be approximated with the following function, which we fit with a χ^2 regression:

$$\alpha_{\text{eff,fit}} = \begin{cases} c_1 + c_2 M_0^p & M_0 \lesssim 10^{18} g \\ 2.011\times 10^{-4} & M_0 \gtrsim 10^{18} g \end{cases}\ , \tag{10}$$

where $c_1 = -0.3015$, $c_2 = 0.3113$, and $p = -0.0008$ and the value for $M_0 \gtrsim 10^{18}$ is taken from Ref. [25]. This fit is shown in Fig. 3. Combining Eqs. 5 & 8, we find the time-dependent

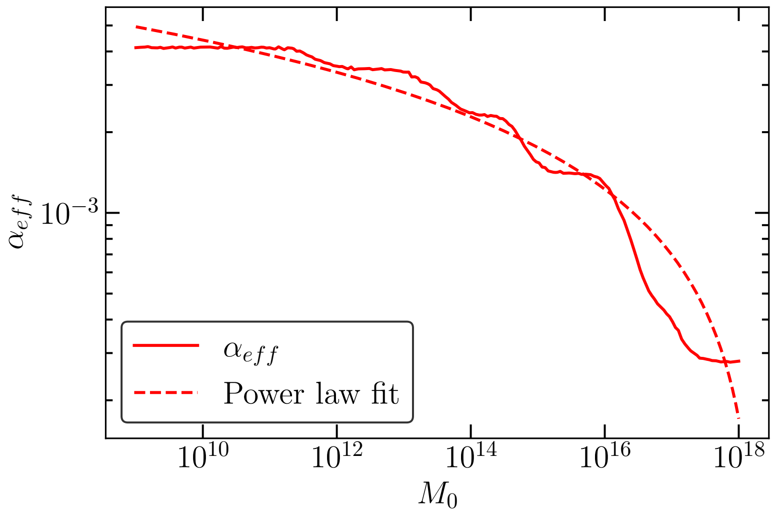

Figure 3: We show α_{eff} as a function of initial black hole mass. This parameter is defined in such a way that the Hawking lifetime of a black hole, calculated using α_{eff}, is the same as the lifetime of a black hole with that initial mass, fully calculated numerically with BlackHawk. For convenience, we also fit an approximate power law. Using the fit gives the correct black hole evolution to within a few percent for most of the distribution, diverging a bit more for larger masses.

evolved distribution,

$$\boxed{\phi(M,t) = \phi(M_0(M,t),t_0) \times M^2 \left(M^3 + \alpha_{\text{eff}}(M_0(M,t)) \frac{\hbar c^4}{G^2} t \right)^{-2/3}} \tag{11}$$

This equation can be used easily by anyone wishing to study extended PBH distributions which are effected by Hawking radiation. If one wishes to study a different kind of mass change, such as by accretion in the early universe, however, the second term would need to be modified according to the mass-change equations for that physical process.

There a few subtleties which should be addressed. Firstly, n_{BH} in Eq. 2 is defined at the black hole formation time. Since some black holes will completely evaporate, this means that the integral in Eq. 3 will be less than one as time goes on, as the $M = 0$ portion of the integral is lost. Secondly, two black holes with different initial masses, but which have eventually evaporated, will not be distinguishable (since there is nothing to distinguish)—so one must be careful when applying Eq. 8 for fully evaporated black holes. Finally, this method requires that the black hole masses evolve via a continuous function of mass and time. Here, that function was the black hole mass loss due to Hawking radiation in Eq. 8—but this method could equally be applied for anything else which affects the black hole mass in a continuous way (e.g., accretion in the early universe).

Eq. 11 also provides a simple explanation for the shape of the evolved distributions in Fig. 2. It is useful to examine the two extreme regimes. For $M^3 \gg \alpha_{\text{eff}} \frac{\hbar c^4}{G^2} t$, the second term is suppressed, leading to an essentially unchanged distribution. This is indeed the behaviour seen for large masses. Conversely, when $M^3 \ll \alpha_{\text{eff}} \frac{\hbar c^4}{G^2} t$, the second term dominates, scaling $\propto M^2$. Since evolved PBHs with small masses originate at almost the same initial mass, $\phi(M_0, t_0)$ and $\alpha_{\text{eff}}(M_0)$ become essentially constant in M, and the evolved distribution becomes $\propto M^2$, which is indeed also the behaviour seen for small masses at later times. Where the two terms are of comparable size, there will be an intermediate region. This analysis is independent of

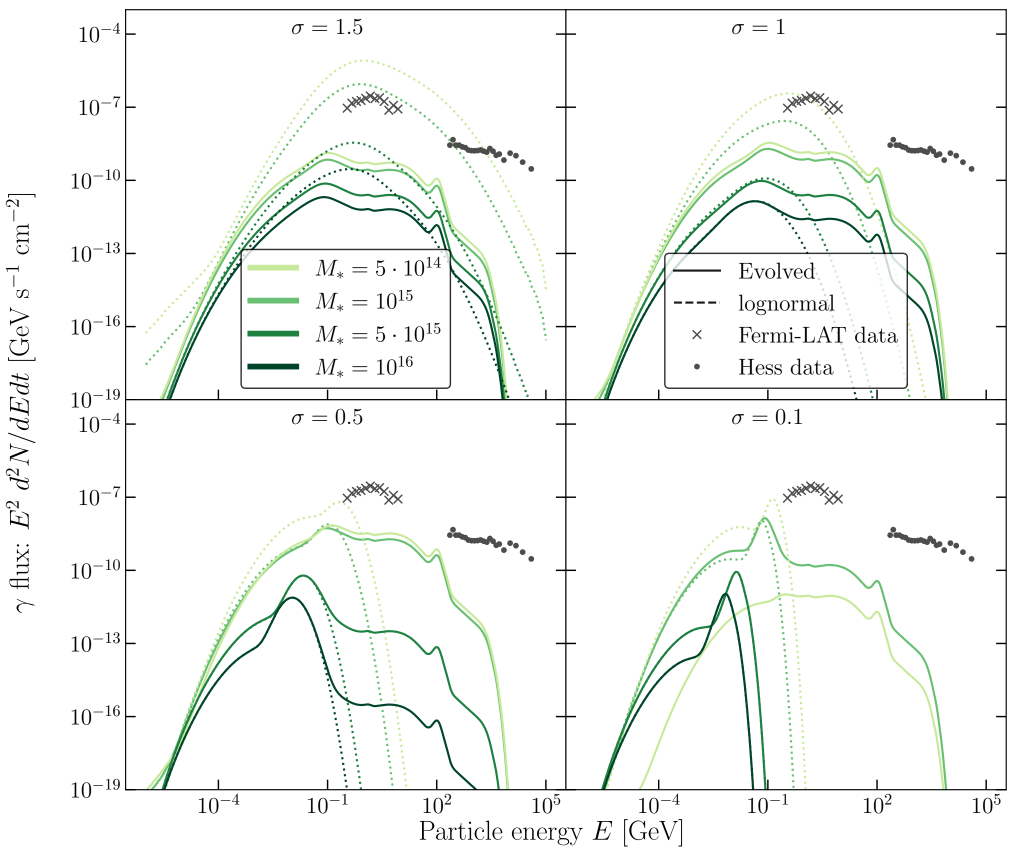

Figure 4: Expected signal from the galactic centre for evolved populations today, contrasted with the expected signal for unevolved lognormal distributions, for $f_{\rm pbh} = 10^{-8}$. Larger values of $f_{\rm pbh}$ produce signals roughly larger than observed by HESS [20] and Fermi-LAT data [21, 22], setting constraints on the PBHs.

the initial distribution, and will hold as long as $\phi(M_0, t_0)$ does not vary extremely quickly in mass.

It would certainly be interesting to examine in addition the evolution of the distribution from mergers, although it would not be trivial to calculate. The PBH-binary parameter distribution from even a monochromatic distribution [26, 27] is already somewhat complicated, and performing this calculation with an extended mass distribution is well beyond the scope of this paper, if it is even analytically tractable (and it is difficult to intuit which way the bounds would shift, after including this effect).

4 γ-ray constraints

For demonstrative purposes, we will recompute the γ-ray constraints from the galactic center, using a lognormal distribution. The lognormal distribution is relatively well-motivated, since many physically realistic processes are expected to result in such distributions [13, 28–33], and is given by,

$$\frac{\mathrm{d}n(M)}{\mathrm{d}M} = \frac{n_{\rm BH}}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{(\ln(M/M_*))^2}{2\sigma^2}\right) \tag{12}$$

The γ-ray flux from an extended distribution of black holes is given as

$$\frac{d^2N_\gamma}{dEdt}(E) = \int dM \frac{df}{dM}\frac{d^2N_\gamma}{dEdt}(M,E), \tag{13}$$

and given that the emission of low-mass black holes is much larger than that of larger masses, the low-mass tail of $\phi(M)$ becomes very important, as was noted in Ref. [13]. We use this to compute the expected flux in γ-rays, Φ, from the galactic centre, using

$$\frac{d\Phi_\gamma}{dE}(E) = f_{\rm pbh}\frac{D}{\bar{M}}\frac{d^2N_\gamma}{dEdt}(E), \tag{14}$$

where $\bar{M}$ is the initial mean PBH mass, and D is the D-factor commonly used for decaying dark matter predictions [34], given by

$$D = \int dl d\Omega\, \rho_{\rm dm}. \tag{15}$$

In Fig. 4 we show the expected signal from the galactic centre, using the Navarro–Frenk–White dark matter profile [35] for the PBHs, contrasted with the expected signal for a lognormal PBH distribution which does not evolve. Note that for evolved distributions, we use $f_{\rm pbh}$ to refer to the PBH fraction of dark matter at formation. For distributions with significant portions of low-mass PBHs, part of that mass would be evaporated at later times.

By requiring that the emission from PBHs does not exceed the observed flux from the galactic centre, we can constrain $f_{\rm pbh}$ for a specific distribution. An alternative method for doing this was proposed in [10], which does not require computing the signal from a given distribution, but instead adapting the constraints on monochromatic distributions. We find that using this 'adapted' method, but with our correctly evolved distribution, agrees excellently with the bounds computed numerically, by simulating an initial PBH distribution and computing the γ-ray spectrum. In addition, the correctly evolved bounds are very similar to those derived in Ref. [13] for isotropic γ-rays [2].

5 Black hole explosions

The end of life of an evaporating black hole is not entirely known. However, at least down to extremely small masses, it should be the case that the black holes will get hotter and brighter, emitting a huge spectrum of particles. For convenience, we call this end-of-life phenomenon an explosion, although we will not comment on whether or not the black hole is completely exhausted, or leaves behind some kind of remnant. The possibility of observing such an explosion would be very exciting. Since all possible particle species are emitted, we could not only probe the Standard Model more clearly, but we could possibly make statements about dark matter and beyond-the-Standard Model physics [23, 24].

We can straightforwardly convert the evolving PBH distribution to a plot of black hole explosions, per volume per year, which we plot in Fig. 6. We can see that there are actually quite a large quantity of explosions per year. However, and unfortunately, the distance between these explosions is still probably too small for observation from Earth—see Fig. 7, where we plot the average distance between these explosions as a function of the distribution parameters and $f_{\rm pbh}$.

[2] We choose not to reproduce these particular bounds, however, since the extragalactic flux must be integrated back in time—a slightly more complicated task when the evolution itself is evolving.

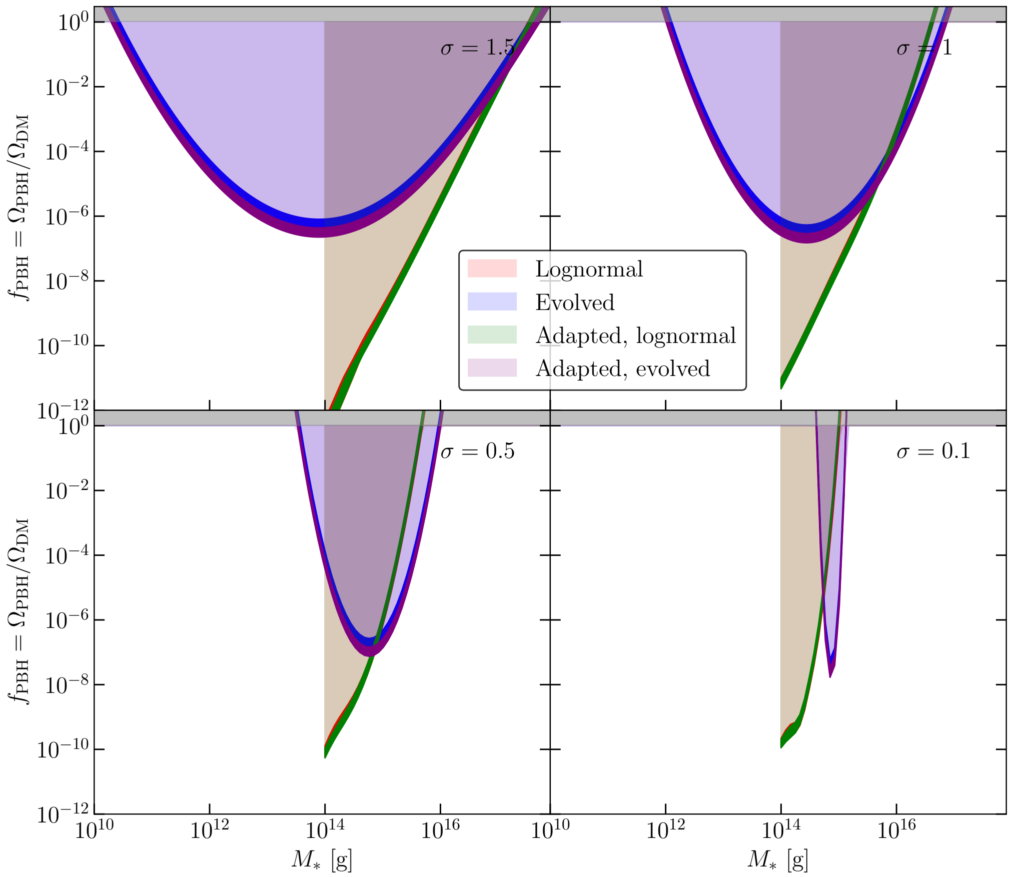

Figure 5: Bounds on $f_{\rm pbh}$ from galactic centre γ-ray observations. There are four bounds plotted here (the thickness of the borders accounts for observational uncertainty). The first 'lognormal' bound, is the spectrum computed produced today, if we do not account for the evolution of the spectrum. The second 'evolved' bound is numerically calculated from the γ-ray spectrum of black holes, distributed with our evolved spectrum today Eq. 11. The 'adapted' bound refers to the bound obtained using the method from Ref. [10], where the monochromatic constraints are interpolated to form the extended distribution bounds. The 'adapted, evolved' bound is calculated using the same method, but with the correct evolved distribution Eq. 11. In each case, the actual bound is placed by requiring that the signal be below the HESS and Fermi-LAT sensitivities. We can see that the adapted method is perfectly compatible with the numerical results, when using our evolved distribution. The reason our bounds for the evolved distribution are loosened for small values of M_* is that a large fraction of that population would have evaporated already, and thus would have no impact on the γ-rays from the galactic centre. A different probe, such as their impact on BBN or CMB would be needed to constrain this type of populations. The unevolved lognormal bounds must be arbitrarily cutoff at 10^{14} g, since it would not be consistent to have a lognormal distribution *today* consisting of tiny, rapidly-evaporating black holes (following the same logic as in Sec. 2).

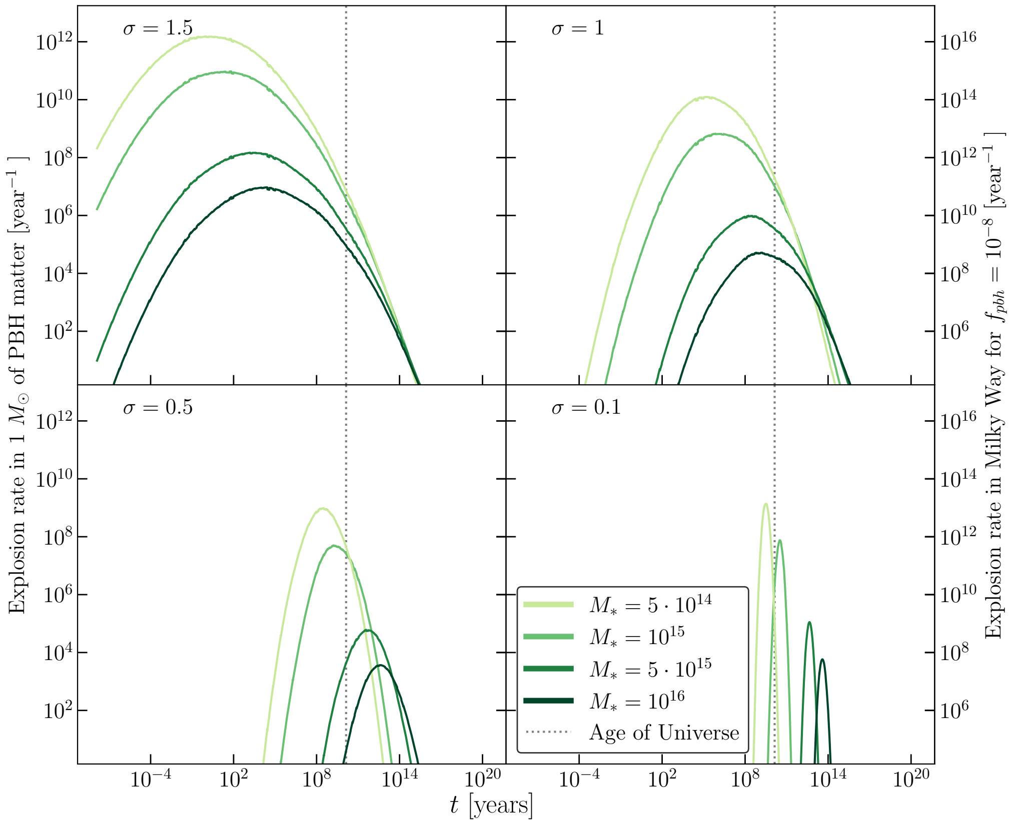

Figure 6: Explosion rate of PBHs over time, for a few values of σ and central mass M_*. The rate is given in terms of number of explosions per cumulative 'solar mass' of PBHs. To escape the previously derived bounds, we have $f_{\rm PBH} \lesssim 10^{-8}$. On the second vertical axis, then, we show what this explosion rate would imply in the Milky Way for a PBH distribution satisfying this constraint. We can see that even at this relatively low fraction, there is a significant quantity of explosions per year. For a different fraction, the event rate would change proportionately.

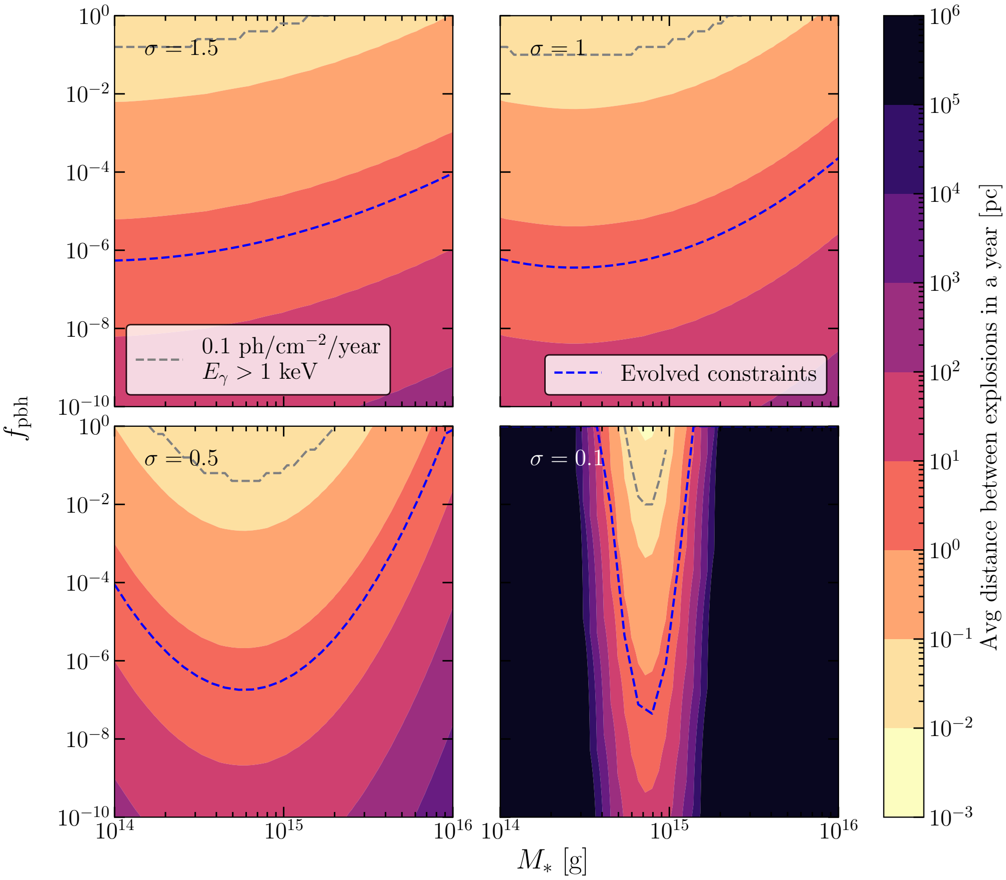

Figure 7: Average distance between black hole explosions, for the evolving lognormal distribution as above. For demonstrative purposes, we also plot a line showing where such a distance corresponds to a photon flux of 0.1 photons cm^{-2}yr^{-1}. We can see that even this very low flux is only satisfied for very large fractions $f_{\rm pbh}$. As tantalizing as the prospect of witnessing a black hole explosion would be, it appears to be unlikely in this scenario. Of course, the Earth would be located somewhere in between these explosions, but the expected distance to the nearest explosion would only be a factor of few smaller than this distance—unless we were exceptionally lucky.

As a representative observation, we can examine the γ-ray flux from one of these transient events. In the last year of the black hole's life, at a distance of ∼0.01 parsec, we only expect 1 photon per square cm per year. In order to observe a single photon from an explosion with Fermi[3], the black hole would need to be within a distance of ∼ 100 AU. A single photon, however, is hardly a positive detection. Ten detected photons requires a distance of ∼ 35 AU (∼ 10^{-4} pc), placing it firmly inside our solar system. One such event, unless we happen to be very lucky, corresponds to PBH fractions which are well excluded— in the monochromatic case, it would be excluded by the argument in Sec. 2, whereas the extended distributions are ruled out by the arguments in Secs. 3, 4. Perhaps there is some more creative way to observe these explosions as transient events (or even, a background) which we have not considered— after all, there is a lot of possibility when the entire particle spectrum is produced. However, for the moment, it does not appear that we will be witnessing any black hole explosions any time soon.

A different way of determining the presence of such explosions would be through investi-

[3]Assuming an effective area of 10^4 cm^2 for the relevant energies [36].

gating the energy injected into the interstellar medium. A conservative estimate of the energy emitted from PBHs in a given year is $\sim 10^{11}$ g (10^{32} ergs) per explosion, neglecting the emission from PBHs with more than a year of life left. As shown in Fig. 6, the explosion rate in a Milky Way-like galaxy can vary greatly over time, depending on the initial distribution. Assuming a Milky Way explosion rate of 10^{10} per year, this is 10^{42} ergs emitted per year, of which a large portion is in photons. In a similar naive analysis, a supernova will generally release $\sim 10^{51}$ ergs [37, 38]. If the supernova rate is one every 10 to 100 years in a MW-like galaxy, this means that supernovae will inject $\sim 10^{7}$ times more energy over that timespan compared to the black hole explosions. However, there may be some morphological differences, as a supernova will be very localised, whereas the energy injection from PBH explosions will be distributed with the halo density profile, and with roughly 'continuous' emission. Additionally, supernovae are often tied to star formation, since many supernova progenitors are short-lived high-mass stars, whereas PBH explosions are completely independent of star formation, and could even happen before stars are formed. A more thorough analysis of the energy injection by PBH explosions would be interesting, but beyond our scope here.

6 Conclusions

Small black holes can lose a significant fraction of their mass via Hawking radiation. Distributions of small black holes therefore evolve over time, as some black holes explode and some shrink considerably. We showed that for monochromatic distributions, it is extremely unlikely to find a population today which is rapidly evaporating, since the initial mass would have to be extremely fine-tuned to a small value above this critical mass. However, extended distributions centered near the critical mass would source a population of evaporating black holes. We demonstrated how to derive this distribution today, and that using the correctly evolved distributions saves the method of Ref. [10], which recasts monochromatic constraints into extended constraints. We then calculated the rate of PBH explosions for a lognormal distribution near the critical mass. Unfortunately, we found that although there are a significant quantity of these explosions, they are on average sufficiently far from Earth that we do not expect to see them.

Primordial black holes are seeing something of a Renaissance today, in large part due to the exciting observations of black holes from experiments such as LIGO/Virgo. As our understanding of their origins and astrophysics improves, the need to properly model extended mass distributions becomes more pressing. There is a lot to learn from these black holes, both cosmologically and astrophysically, as well as on the theoretical side—for gravity and particle physics.

During the preparation of this paper, a similar treatment of the evolved mass distribution was published in the context of PBH bubbles as cosmological standard timers [39]. We find that our results agree well.

Acknowledgements

We would like to thank Celine Boehm, Archil Kobakhidze, and Ciaran O'Hare for many useful discussions and insights throughout the research and writing of this paper.

Funding

The authors are funded by The University of Sydney.

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